OPERATIONS WITH WHOLE NUMBERS

A ADDITION

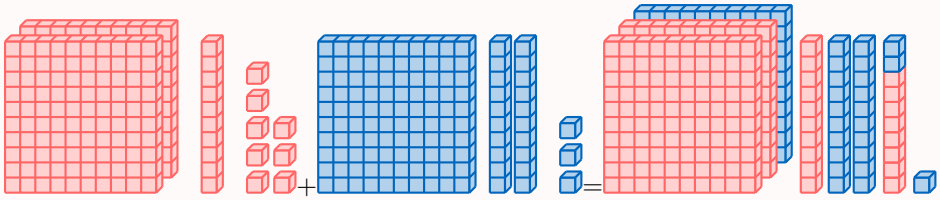
Definition Addition

Addition is joining groups together to find the total, or **sum**. The **plus sign (+)** tells us to add.
We can show "two hundred eighteen plus one hundred twenty-three equals three hundred forty-one" in many ways:

- **With numbers**:

$$218 + 123 = 341$$

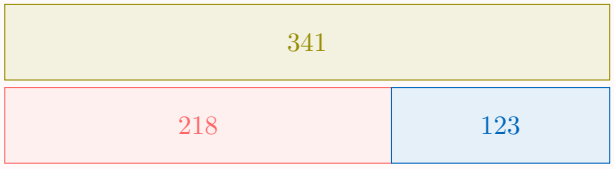
- **With cubes**:



- **With a column addition**:

$$\begin{array}{r} {\scriptstyle 1} \\ 2\,1\,8 \\ +\,1\,2\,3 \\ \hline 3\,4\,1 \end{array}$$

- **With a part-whole model**:



Method Column Addition with Regrouping

Let's calculate:

$$288 + 545$$

- **Step 1: Line up the numbers by place value.**
  Write the numbers one on top of the other. Make sure the ones, tens, and hundreds are in straight columns.

$$\begin{array}{r} 2\,8\,8 \\ +\,5\,4\,5 \\ \hline \end{array}$$

- **Step 2: Add the ones.**

$$8 \text{ ones} + 5 \text{ ones} = 13 \text{ ones}$$

  13 ones is the same as 1 ten and 3 ones. We write the 3 in the ones place and **carry** the 1 ten over to the tens column.

$$\begin{array}{r} {\scriptstyle 1} \\ 2\,8\,8 \\ +\,5\,4\,5 \\ \hline 3 \end{array}$$

- **Step 3: Add the tens.**

$$1 \text{ ten (carried)} + 8 \text{ tens} + 4 \text{ tens} = 13 \text{ tens}$$

  13 tens is the same as 1 hundred and 3 tens. We write the 3 in the tens place and **carry** the 1 hundred over.

$$\begin{array}{r} {\scriptstyle 1\ 1\ \ } \\ 2\,8\,8 \\ +\,5\,4\,5 \\ \hline 3\,3 \end{array}$$

- **Step 4: Add the hundreds.**

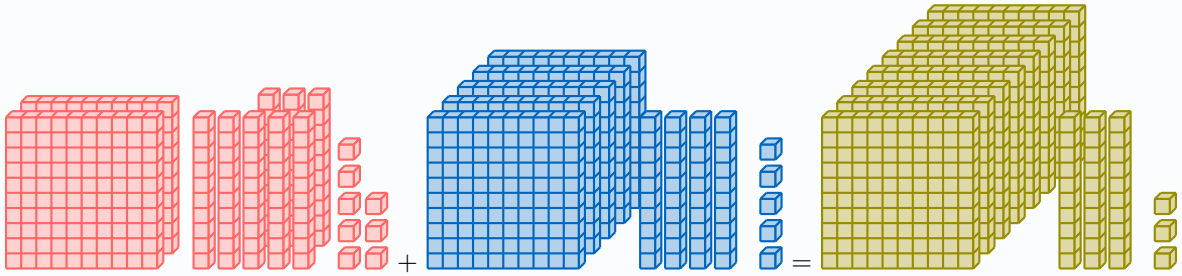
  $$1 \text{ hundred (carried)} + 2 \text{ hundreds} + 5 \text{ hundreds} = 8 \text{ hundreds}$$

  Write the 8 in the hundreds place.

$$\begin{array}{r} {\scriptstyle 1\ 1\ \ } \\ 2\,8\,8 \\ +\,5\,4\,5 \\ \hline 8\,3\,3 \end{array}$$

- **The Result**:

$$288 + 545 = 833$$



Ex: Calculate $189 + 784$

Answer:

$$\begin{array}{r} {\scriptstyle 1\ 1\ \ } \\ 1\,8\,9 \\ +\,7\,8\,4 \\ \hline 9\,7\,3 \end{array}$$

B SUBTRACTION

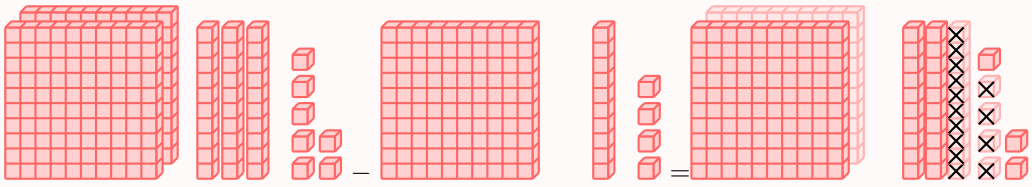
Definition Subtraction

Subtraction means taking an amount away from a group to find out what is left. This result is called the **difference**. The **minus sign** $(-)$ tells us to subtract.
We can show "two hundred thirty-seven minus one hundred fourteen equals one hundred twenty-three" in different ways:

- **With Numbers**:

$$237 - 114 = 123$$

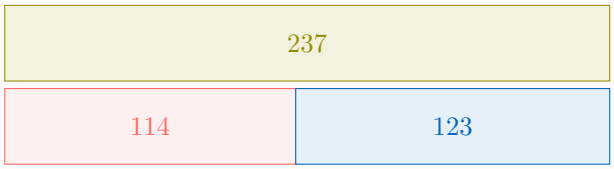
- **With Cubes**:



- **With Column Subtraction**:

$$\begin{array}{r} 2\,3\,7 \\ -\,1\,1\,4 \\ \hline 1\,2\,3 \end{array}$$

- **With a Part-Whole Model**:

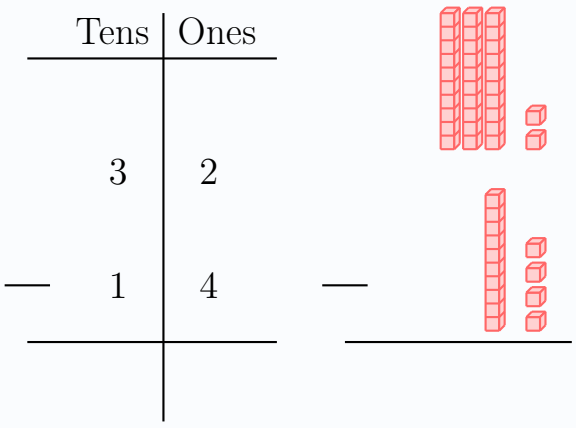


Method Subtraction by Compensation

Let's calculate:

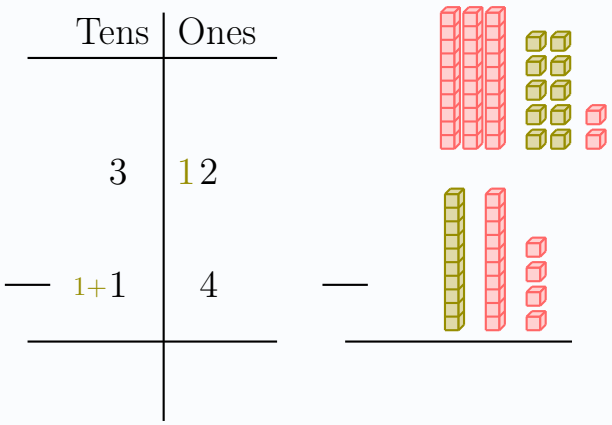
$$32 - 14$$

- **Step 1: Set up the subtraction.**



- **Step 2: Check the ones place.** We need to do $2 - 4$, but we don't have enough ones.
- **Step 3: Compensate.** Add 10 to both numbers (10 ones to the top; 1 ten to the bottom)

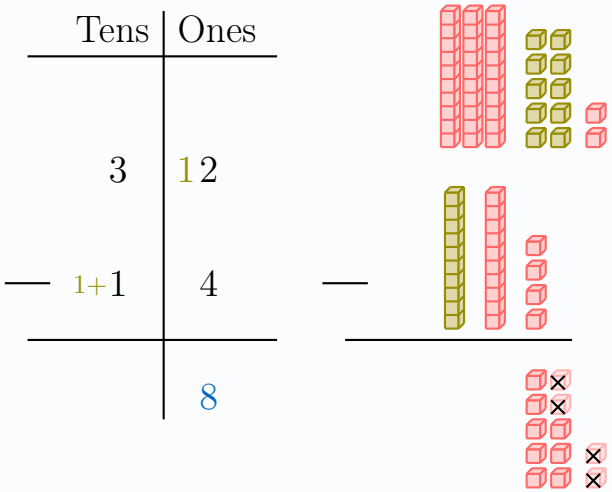
Tens	Ones
3	12
− 1+1	4



- **Step 4: Subtract the ones.** Now it's easy:

$$12 \text{ ones} - 4 \text{ ones} = 8 \text{ ones}$$

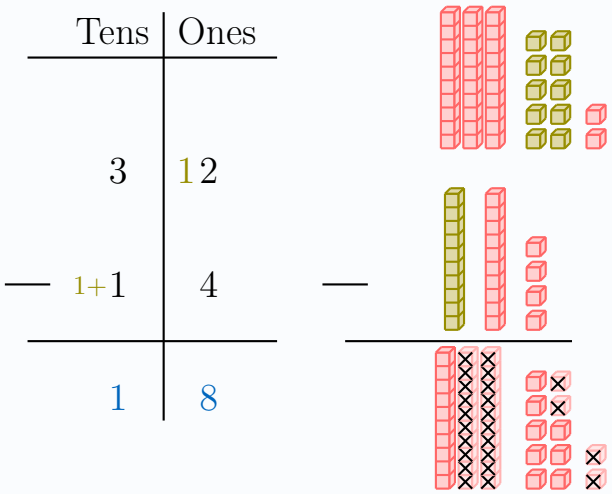
Tens	Ones
3	12
− 1+1	4
	8



- **Step 5: Subtract the tens.**

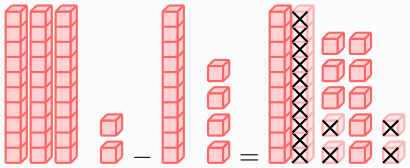
$$3 \text{ tens (from 32)} - 1 \text{ ten (from 14)} - 1 \text{ ten (compensated)} = 1 \text{ ten}$$

Tens	Ones
3	12
− 1+1	4
1	8



- **Result**: The difference is 1 ten and 8 ones. So,

$$32 - 14 = 18$$



Ex: Calculate $784 - 189$

Answer:

$$\begin{array}{r} 7\,{}^{1}8\,{}^{1}4 \\ -1\,1\,8\;9 \\ \hline 5\;9\;5 \end{array}$$

C MULTIPLICATION

Definition Multiplication

Multiplication is a fast way to show repeated addition. We can show the idea of "**four** times **three** equals **twelve**" in many different ways:

- **With Numbers**:

$$4 \times 3 = 12$$

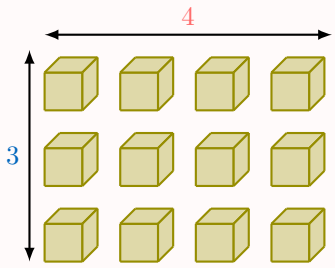
- **In Groups**:

$$4 \text{ groups of } 3 = 12$$

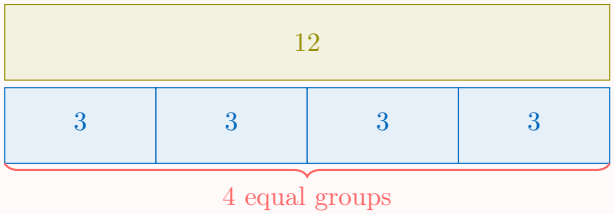
- **As Repeated Addition**:

$$3 + 3 + 3 + 3 = 12$$

- **With Cubes**:



- **With a Part-Whole Model**:



Method Column Multiplication by a Two-Digit Number

To calculate 23×37:

1. **Step 1: Align the numbers** vertically by place value.

$$\begin{array}{r} 2\,3 \\ \times\; 3\,7 \\ \hline \end{array}$$

2. **Step 2: Multiply by the ones digit.** Multiply the top number (23) by the ones digit of the bottom number (7): $23 \times 7 = 161$.

$$\begin{array}{r} {\scriptstyle 2} \\ 2\,3 \\ \times\; 3\,7 \\ \hline 1\;6\;1 \end{array}$$

3. **Step 3: Multiply by the tens digit.**
   - First, place a **placeholder** 0 in the ones column of the second row. This is because we are now multiplying by the tens digit (3, which represents 30). This placeholder shifts our answer one place to the left.

- Next, multiply the top number (23) by the tens digit (3). Calculate $23 \times 3 = 69$ and write it to the left of the placeholder.

$$\begin{array}{r} 23 \\ \times\, 37 \\ \hline 161 \\ 690 \end{array}$$

4. **Step 4: Sum the partial products.** Add the results from Step 2 and Step 3: $161 + 690 = 851$

$$\begin{array}{r} 23 \\ \times\, 37 \\ \hline 161 \\ 690 \\ \hline 851 \end{array}$$

5. **Result**: $23 \times 37 = 851$.

Ex: Calculate 123×21

Answer:

$$\begin{array}{r} 123 \\ \times\ \ 21 \\ \hline 123 \leftarrow 123 \times 1 = 123 \\ 246\,\cdot \leftarrow 123 \times 20 = 2460 \\ \hline 2583 \leftarrow 123 + 2460 = 2583 \end{array}$$

Definition **Division**

Division is the **inverse operation** of multiplication. It is the process of determining how many times one number is contained within another.
The components of a division expression are formally named:

- The **dividend**: the number that is being divided.
- The **divisor**: the number by which the dividend is divided.
- The **quotient**: the result of the division.

The operation is denoted by the **division symbol** ($\div$).

$$\text{Dividend} \div \text{Divisor} = \text{Quotient}$$

For example, the multiplication fact $3 \times 2 = 6$ corresponds to:

$$\underbrace{6}_{\text{Dividend}} \div \underbrace{3}_{\text{Divisor}} = \underbrace{2}_{\text{Quotient}} .$$

Division can be represented in several ways:

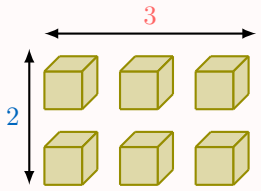
- **Numerical Form**:

$$6 \div 3 = 2$$

- **Word Form**:

Six divided by **three** equals **two**.

- **Grid Model**:



Definition **Euclidean Division**

Euclidean Division is the process of dividing one integer (the dividend) by another (the divisor) when the division is not exact. This process yields an integer quotient and a remainder.
The components of a Euclidean division expression are formally named:

- The **dividend**: the number that is being divided.
- The **divisor**: the number by which the dividend is divided.
- The **quotient**: the whole number of times the divisor fits into the dividend.
- The **remainder**: the amount left over after the division.

This relationship is defined by the identity:

$$\text{Dividend} = (\text{Divisor} \times \text{Quotient}) + \text{Remainder}$$

Important rules:

- The remainder is always **smaller** than the divisor. (If it isn't, you can still make another group!)
- If the remainder is 0, the division is **exact** (no remainder).

Euclidean division can be represented in several ways:

- **Word Form**:

Seven divided by **three** equals **two**, with a remainder of one

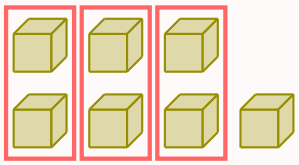
- **Division Sentence**:

$$\underbrace{7}_{\text{Dividend}} \div \underbrace{3}_{\text{Divisor}} = \underbrace{2}_{\text{Quotient}} \text{ R } \underbrace{1}_{\text{Remainder}}$$

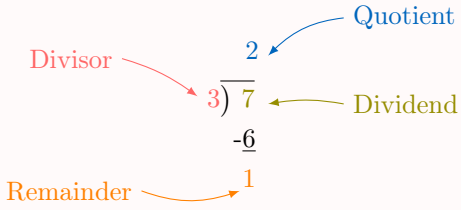
- **Euclidean Identity**:

$$\underbrace{7}_{\text{Dividend}} = (\underbrace{3}_{\text{Divisor}} \times \underbrace{2}_{\text{Quotient}}) + \underbrace{1}_{\text{Remainder}}$$

- **Group Model**:



- **Long Division Algorithm**:



Method **The Long Division Algorithm: Multi-Steps**

To divide with a remainder, like $130 \div 4$, follow these steps:

- **Set up:** Write the dividend (130) inside the bracket and the divisor (4) outside.

$$4\overline{)130}$$

- **Divide the first part (13):** "How many times does 4 go into 13?"

$$4 \times 3 = \boxed{12} \ (\leq 13), \qquad 4 \times 4 = \cancel{16} \ (> 13).$$

Write 3 above and 12 under 13; then subtract.

$$\begin{array}{r} 3 \\ \hline 4\,)\,130 \\ -12 \end{array}$$

- **Subtract and Bring down the next digit:** $13 - 12 = 1$; bring down 0 to make 10.

$$\begin{array}{r} 3 \\ \hline 4\,)\,130 \\ -\underline{12}\downarrow \\ 10 \end{array}$$

- **Divide the new number (10):** "How many times does 4 go into 10?"

$$4 \times 2 = \boxed{8}\ (\leq 10), \qquad 4 \times 3 = \cancel{12}\ (> 10).$$

Write 2 above, put 8 under 10, and subtract to get the remainder.

$$\begin{array}{r} 32 \\ \hline 4\,)\,130 \\ -\underline{12}\downarrow \\ 10 \\ -\underline{8} \\ 2 \end{array}$$

- **Final answer:** $130 \div 4 = 32\text{R}2$, and$130 = 4 \times 32 + 2$.

Ex: Calculate $125 \div 4$

Answer: Using long division,

$$\begin{array}{r} 31 \\ \hline 4\,)\,125 \\ \underline{12} \\ 05 \\ \underline{4} \\ 1 \end{array}$$

we find:

$$125 \div 4 = 31\text{R}1.$$

E ORDER OF OPERATIONS

Method **Order of operations**

To evaluate an expression, we follow these steps in order:

1. **Parentheses ()**: Always solve what's inside parentheses first.
2. **Multiplication ($\times$) and Division ($\div$)**: Do them next, working from left to right.
3. **Addition ($+$) and Subtraction (-)**: Do them last, also working from left to right.

Ex: Calculate $4 + 2 \times 3$

Answer: We follow the order:

$$\begin{aligned} 4 + 2 \times 3 &= 4 + 6 \quad \text{(Do the multiplication } 2 \times 3\text{)} \\ &= 10 \quad \text{(Do the addition } 4 + 6\text{)} \end{aligned}$$

F SOLVING PROBLEMS

Method **A Procedure for Solving Word Problems**

To solve a word problem systematically, the following five-step procedure can be applied:

1. **Understand the Goal**: Read the problem carefully to identify the main question and the information provided.
2. **Plan the Steps**: Determine the sequence of mathematical operations required to reach the solution.
3. **Write the Expression**: Translate the planned steps into a single mathematical expression, using parentheses where necessary to ensure the correct order of operations.
4. **Calculate the Solution**: Evaluate the expression to find the numerical answer.
5. **State the Conclusion**: Write a final sentence that directly answers the question asked in the problem.

Ex: Hugo is planning his birthday party and needs to purchase a cake and juice.The items on his list are:

1. **Cakes**: 2 cakes at a cost of \$10 each.
2. **Juice**: 4 cans of juice at a cost of \$3 each.

Hugo has a budget of \$30. Determine if he can afford to purchase all the items.

Answer: We will apply the five-step procedure to solve Hugo's problem.

- **1. Understand the Goal**: We need to calculate the total cost of the cakes and juice, and then compare that total to Hugo's budget of \$30.
- **2. Plan the Steps**:
  1. Calculate the cost of the cakes (multiplication).
  2. Calculate the cost of the juice (multiplication).
  3. Add the two costs together to find the total cost (addition).
  4. Compare the total cost to \$30.
- **3. Write the Expression**: The total cost can be represented as:

$$(2 \times 10) + (4 \times 3)$$

- **4. Calculate the Solution**:

$$\begin{aligned} \text{Total cost} &= (2 \times 10) + (4 \times 3) \\ &= 20 + 12 \\ &= 32 \end{aligned}$$

- **5. State the Conclusion**: The total cost is \$32. Since \$32 is greater than \$30, Hugo cannot afford to buy everything.