LONG MULTIPLICATION

Long multiplication is a systematic method for multiplying numbers, particularly those with multiple digits. It relies on a foundational understanding of single-digit multiplication (times tables) and place value. This chapter will detail the standard algorithm for this procedure.

A A PATTERN WITH TENS

Proposition The "Add a Zero" Rule

To multiply by a multiple of 10, you can multiply the non-zero digits first, then place a zero at the end of your answer.

$$\begin{aligned} 3 \times 1 &= 3 & 3 \times 10 &= 30 \\ 3 \times 2 &= 6 & 3 \times 20 &= 60 \\ 3 \times 3 &= 9 & 3 \times 30 &= 90 \\ 3 \times 4 &= 12 & 3 \times 40 &= 120 \\ 3 \times 5 &= 15 \quad \rightarrow & 3 \times 50 &= 150 \\ 3 \times 6 &= 18 & 3 \times 60 &= 180 \\ 3 \times 7 &= 21 & 3 \times 70 &= 210 \\ 3 \times 8 &= 24 & 3 \times 80 &= 240 \\ 3 \times 9 &= 27 & 3 \times 90 &= 270 \end{aligned}$$

B LONG MULTIPLICATION BY ONE-DIGIT NUMBERS

Method Column Multiplication by a Single Digit

To calculate 23×7, the procedure is as follows:

1. **Step 1: Align the numbers.** Position the numbers vertically, aligning them by place value.

$$\begin{array}{r} 23 \\ \times\ \ 7 \\ \hline \end{array}$$

2. **Step 2: Multiply the ones digit.**

$$3 \text{ ones} \times 7 \text{ ones} = 21 \text{ ones} = 2 \text{ tens} + 1 \text{ one}$$

This result is composed of 1 one and 2 tens. Write the 1 in the ones column of the result. Carry over the 2 to the tens column.

$$\begin{array}{r} {\scriptstyle 2\ \ } \\ 23 \\ \times\ \ 7 \\ \hline 1 \end{array}$$

3. **Step 3: Multiply the tens digit and add the carry-over.**

$$2 \text{ tens} \times 7 \text{ ones} + 2 \text{ tens (carry-over)} = 16 \text{ tens}$$

Write 16 in the tens column of the result (no carry because no more calculation).

$$\begin{array}{r} {\scriptstyle 2\ \ } \\ 23 \\ \times\ \ 7 \\ \hline 161 \end{array}$$

4. **Result**: $23 \times 7 = 161$.

C LONG MULTIPLICATION BY MULTI-DIGIT NUMBERS

Method **Column Multiplication by a Two-Digit Number**

To calculate 23×37:

1. **Step 1: Align the numbers** vertically by place value.

$$\begin{array}{r} 23 \\ \times\ 37 \\ \hline \end{array}$$

2. **Step 2: Multiply by the ones digit.** Multiply the top number (23) by the ones digit of the bottom number (7): $23 \times 7 = 161$.

$$\begin{array}{r} {\scriptstyle 2} \\ 23 \\ \times\ 37 \\ \hline 161 \end{array}$$

3. **Step 3: Multiply by the tens digit.**

   - First, place a **placeholder** 0 in the ones column of the second row. This is because we are now multiplying by the tens digit (3, which represents 30). This placeholder shifts our answer one place to the left.
   - Next, multiply the top number (23) by the tens digit (3). Calculate $23 \times 3 = 69$ and write it to the left of the placeholder.

$$\begin{array}{r} 23 \\ \times\ 37 \\ \hline 161 \\ 690 \end{array}$$

4. **Step 4: Sum the partial products.** Add the results from Step 2 and Step 3: $161 + 690 = 851$

$$\begin{array}{r} 23 \\ \times\ 37 \\ \hline 161 \\ 690 \\ \hline 851 \end{array}$$

5. **Result**: $23 \times 37 = 851$.