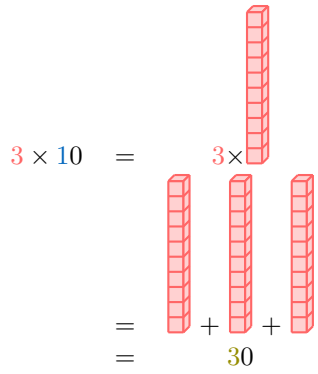
# LONG MULTIPLICATION

Long multiplication is a systematic method for multiplying numbers, particularly those with multiple digits. It relies on a foundational understanding of single-digit multiplication (times tables) and place value. This chapter will detail the standard algorithm for this procedure.

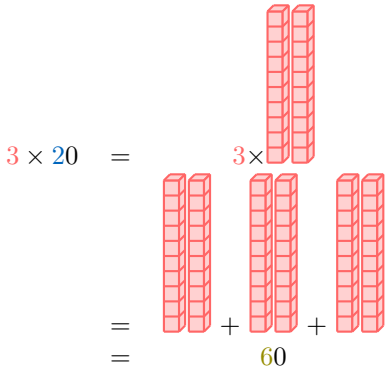
## A A PATTERN WITH TENS

**Discover:** To get ready for long multiplication, let's explore a pattern. What happens when we multiply a number by 10, 20, 30, or other multiples of 10?

- **3 groups of 10** ($3 \times 10$):




- **3 groups of 20** ($3 \times 20$):




Do you see a pattern emerging?

- $3 \times 4 = 12 \quad \rightarrow \quad 3 \times 40 = 120$
- $3 \times 5 = 15 \quad \rightarrow \quad 3 \times 50 = 150$
- $3 \times 6 = 18 \quad \rightarrow \quad 3 \times 60 = 180$

The pattern is that multiplying by a multiple of 10 is like using the basic times table, but then adding a zero to the end of the answer!

**Proposition The "Add a Zero" Rule**

To multiply by a multiple of 10, you can multiply the non-zero digits first, then place a zero at the end of your answer.

$$
\begin{array}{lcl}
3 \times 1 = 3 & & 3 \times 10 = 30 \\
3 \times 2 = 6 & & 3 \times 20 = 60 \\
3 \times 3 = 9 & & 3 \times 30 = 90 \\
3 \times 4 = 12 & & 3 \times 40 = 120 \\
3 \times 5 = 15 & \rightarrow & 3 \times 50 = 150 \\
3 \times 6 = 18 & & 3 \times 60 = 180 \\
3 \times 7 = 21 & & 3 \times 70 = 210 \\
3 \times 8 = 24 & & 3 \times 80 = 240 \\
3 \times 9 = 27 & & 3 \times 90 = 270
\end{array}
$$

## B LONG MULTIPLICATION BY ONE-DIGIT NUMBERS

**Discover:** The expanded method of multiplication involves calculating partial products for each place value and then summing them. For example, to calculate $764 \times 2$:

$$\begin{array}{rll} & 764 & \\ \times & \quad 2 & \\ \hline & \quad 8 & (4 \times 2 = 8) \\ + & 120 & (60 \times 2 = 120) \\ + & 1400 & (700 \times 2 = 1\,400) \\ \hline & 1528 & (8 + 120 + 1\,400 = 1\,528) \end{array}$$

While accurate, this method can be inefficient. The standard algorithm for column multiplication streamlines this process by incorporating a carry-over technique, analogous to that used in column addition.

### Method **Column Multiplication by a Single Digit**

To calculate $23 \times 7$, the procedure is as follows:

1. **Step 1: Align the numbers.** Position the numbers vertically, aligning them by place value.

$$\begin{array}{r} 23 \\ \times \quad 7 \\ \hline \end{array}$$

2. **Step 2: Multiply the ones digit.**

$$3 \text{ ones} \times 7 \text{ ones} = 21 \text{ ones} = 2 \text{ tens} + 1 \text{ one}$$

This result is composed of 1 one and 2 tens. Write the 1 in the ones column of the result. Carry over the 2 to the tens column.

$$\begin{array}{r} {\scriptstyle 2\phantom{3}} \\ 23 \\ \times \quad 7 \\ \hline \quad 1 \end{array}$$

3. **Step 3: Multiply the tens digit and add the carry-over.**

$$2 \text{ tens} \times 7 \text{ ones} + 2 \text{ tens (carry-over)} = 16 \text{ tens}$$

Write 16 in the tens column of the result (no carry because no more calculation).

$$\begin{array}{r} {\scriptstyle 2\phantom{3}} \\ 23 \\ \times \quad 7 \\ \hline 161 \end{array}$$

4. **Result**: $23 \times 7 = 161$.

## C LONG MULTIPLICATION BY MULTI-DIGIT NUMBERS

**Discover:** To calculate a product such as $23 \times 37$, the distributive property is applied. The multiplication is broken into two simpler products:

$$\begin{aligned} 23 \times 37 &= 23 \times (7 + 30) \\ &= (23 \times 7) + (23 \times 30) \\ &= 161 + 690 \\ &= 851 \end{aligned}$$

The column multiplication method organizes this process into a standard algorithm. Note that when multiplying by the tens digit (3), the partial product (690) is shifted one place to the left. This is indicated by a placeholder zero.

$$\begin{array}{r} 23 \\ \times\, 37 \\ \hline 161 \\ 690 \\ \hline 851 \end{array} \quad \begin{array}{l} \\ \\ (23 \times 7 = 161) \\ (23 \times 30 = 690) \\ (161 + 690 = 851) \end{array}$$

Method **Column Multiplication by a Two-Digit Number**

To calculate $23 \times 37$:

1. **Step 1: Align the numbers** vertically by place value.

$$\begin{array}{r} 23 \\ \times\, 37 \\ \hline \end{array}$$

2. **Step 2: Multiply by the ones digit.** Multiply the top number (23) by the ones digit of the bottom number (7): $23 \times 7 = 161$.

$$\begin{array}{r} {\scriptstyle 2} \\ 23 \\ \times\, 37 \\ \hline 161 \end{array}$$

3. **Step 3: Multiply by the tens digit.**
   - First, place a **placeholder** 0 in the ones column of the second row. This is because we are now multiplying by the tens digit (3, which represents 30). This placeholder shifts our answer one place to the left.
   - Next, multiply the top number (23) by the tens digit (3). Calculate $23 \times 3 = 69$ and write it to the left of the placeholder.

$$\begin{array}{r} 23 \\ \times\, 37 \\ \hline 161 \\ 690 \end{array}$$

4. **Step 4: Sum the partial products.** Add the results from Step 2 and Step 3: $161 + 690 = 851$

$$\begin{array}{r} 23 \\ \times\, 37 \\ \hline 161 \\ 690 \\ \hline 851 \end{array}$$

5. **Result**: $23 \times 37 = 851$.