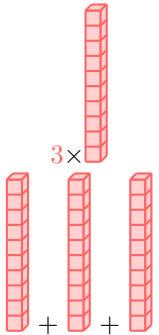# LONG MULTIPLICATION

Long multiplication is a method used for multiplying larger numbers. It requires knowledge of the multiplication table for single digits.

A MULTIPLICATION TABLES FOR MULTIPLES OF 10

Discover: Imagine starting with a simple one-digit number, such as 3. What happens when we multiply by multiples of 10? Let's explore step by step:
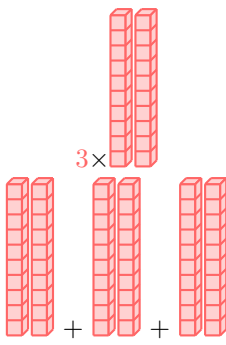

- $3 \times 10 = 3 \times$
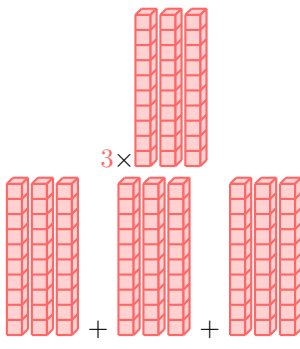
  $=$ $+$ $+$

  $= 30$

- $3 \times 20 = 3 \times$

  $=$ $+$ $+$

  $= 60$

- $3 \times 30 = 3 \times$

  $=$ $+$ $+$

  $= 90$

- $3 \times 40 = 120$

- $3 \times 50 = 150$

- $3 \times 60 = 180$

- $3 \times 70 = 210$

- $3 \times 80 = 240$

- $3 \times 90 = 270$

Notice that the multiplication table for multiples of 10 looks similar to the regular table, but you just add a zero at the end.

Proposition **Multiplication Table for Multiple of 10**

To multiply by multiples of 10, use the regular multiplication table and add the zeros from the factors to the result:

$$\begin{array}{ll}
\color{red}3 \color{black}\times 1 = 3 & \color{red}3\color{black}00 \times 10 = \color{teal}3\color{black}000 \\
\color{red}3 \color{black}\times 2 = 6 & \color{red}3\color{black}00 \times 20 = \color{teal}6\color{black}000 \\
\color{red}3 \color{black}\times 3 = 9 & \color{red}3\color{black}00 \times 30 = \color{teal}9\color{black}000 \\
\color{red}3 \color{black}\times 4 = 12 & \color{red}3\color{black}00 \times 40 = \color{teal}12\color{black}000 \\
\color{red}3 \color{black}\times 5 = 15 & \color{red}3\color{black}00 \times 50 = \color{teal}15\color{black}000 \\
\color{red}3 \color{black}\times 6 = 18 & \color{red}3\color{black}00 \times 60 = \color{teal}18\color{black}000 \\
\color{red}3 \color{black}\times 7 = 21 & \color{red}3\color{black}00 \times 70 = \color{teal}21\color{black}000 \\
\color{red}3 \color{black}\times 8 = 24 & \color{red}3\color{black}00 \times 80 = \color{teal}24\color{black}000 \\
\color{red}3 \color{black}\times 9 = 27 & \color{red}3\color{black}00 \times 90 = \color{teal}27\color{black}000
\end{array}$$

B LONG MULTIPLICATION BY ONE-DIGIT NUMBERS

Discover: When multiplying a large number by a single-digit number, it can be tedious and time-consuming to handle each step separately. For example:

$$\begin{array}{rl}
7\,6\,4 & \\
\times \quad\quad 2 & \\
\hline
8 & 4 \times 2 = 8 \\
+ \quad 1\,2\,0 & 60 \times 2 = 120 \\
+ 1\,4\,0\,0 & 700 \times 2 = 1\,400 \\
\hline
1\,5\,2\,8 & 8 + 120 + 1\,400 = 1\,528
\end{array}$$

To simplify, you can perform the additions step by step using the carry-over method, similar to column addition.

Method Column Multiplication

To calculate 23×7, follow these steps:

1. **Step 1: Write the multiplication in columns**
   Write the numbers in columns, aligning the digits based on their place value (units, tens, hundreds).

$$\begin{array}{r}
2\,3 \\
\times \quad 7 \\
\hline
\end{array}$$

2. **Step 2: Multiply the ones**

$$\color{red}3 \text{ ones} \times \color{blue}7 \text{ ones} = 21 \text{ ones} = \color{blue}2 \text{ tens} + \color{blue}1 \text{ one}$$

   Write the carry-over ($\color{olive}2$) above the tens column.

$$\begin{array}{r}
\color{olive}2 \\
2\,\color{red}3 \\
\times \quad \color{blue}7 \\
\hline
\color{blue}1
\end{array}$$

3. **Step 3: Multiply the tens**

$$\color{red}2 \text{ tens} \times \color{blue}7 \text{ ones} + \color{olive}2 \text{ tens (carry-over)} = \color{blue}16 \text{ tens}$$

   Write $\color{blue}16$ in the tens and hundreds columns.

$$\begin{array}{r}
\color{olive}2 \\
\color{red}2\,3 \\
\times \quad \color{blue}7 \\
\hline
\color{blue}1\,6\,1
\end{array}$$

4. **Result**: $23 \times 7 = 161$.

C LONG MULTIPLICATION BY MULTI-DIGIT NUMBERS

Discover: To calculate 23×37, we can use the distributive property:

$$\begin{aligned} 23 \times 37 &= 23 \times 7 + 23 \times 30 &&\text{(by distributivity: } 37 = 7 + 30) \\ &= 161 + 690 &&\text{(intermediate products)} \\ &= 851 &&\text{(add the intermediate results)} \end{aligned}$$

This approach forms the basis of column multiplication for multi-digit numbers. The process organizes the steps efficiently:

$$\begin{array}{r} 2\,3 \\ \times\,3\,7 \\ \hline 1\,6\,1 \\ 6\,9\,0 \\ \hline 8\,5\,1 \end{array} \quad \begin{array}{l} 23 \times 7 = 161 \\ 23 \times 30 = 690 \\ 161 + 690 = 851 \end{array}$$

Method Column Multiplication for Multi-Digit Numbers

To calculate 23×37, follow these steps:

1. **Step 1: Write the multiplication in columns**

$$\begin{array}{r} 2\,3 \\ \times\,3\,7 \\ \hline \end{array}$$

2. **Step 2: Multiply the ones digit (7)**

   (a) Multiply the ones: $3 \times 7 = 21$

$$\begin{array}{r} {\scriptstyle 2} \\ 2\,3 \\ \times\,3\,7 \\ \hline 1 \end{array}$$

   (b) Multiply the tens: $2 \times 7 + 2{\scriptstyle(\text{carried})} = 14 + 2 = 16$

$$\begin{array}{r} {\scriptstyle 2} \\ 2\,3 \\ \times\,3\,7 \\ \hline 1\,6\,1 \end{array}$$

3. **Step 3: Multiply the tens digit (3)**

   (a) Add a placeholder . (or 0) as a for the multiplication with a tens digit

$$\begin{array}{r} 2\,3 \\ \times\,3\,7 \\ \hline 1\,6\,1 \\ . \end{array}$$

   (b) Multiply the ones: $3 \times 3 = 9$

$$\begin{array}{r} 2\,3 \\ \times\,3\,7 \\ \hline 1\,6\,1 \\ 9\,. \end{array}$$

   (c) Multiply the tens: $2 \times 3 = 6$

$$\begin{array}{r} 2\,3 \\ \times\,3\,7 \\ \hline 1\,6\,1 \\ 6\,9\,. \end{array}$$

4. **Step 4: Add the intermediate results** $161 + 690 = 851$

$$\begin{array}{r} 2\,3 \\ \times\,3\,7 \\ \hline 1\,6\,1 \\ 6\,9\,. \\ \hline 8\,5\,1 \end{array}$$

5. **Result**: $23 \times 37 = 851$.