LOGARITHMS

A LOGARITHMS IN BASE 10

A.1 EVALUATING LOGARITHMS

Ex 1: Evaluate:

$$\log 100 = \square$$

Ex 2: Evaluate:

$$\log 0.1 = \square$$

Ex 3: Evaluate:

$$\log\left(\frac{1}{100}\right) = \square$$

Ex 4: Evaluate:

$$\log\sqrt{10} = \square$$

Ex 5: Evaluate:

$$\log 1 = \square$$

A.2 EVALUATING USING A CALCULATOR

Ex 6: Evaluate (round to 2 decimal places).

$$\log(2) \approx \square$$

Ex 7: Evaluate (round to 2 decimal places).

$$\log(0.2) \approx \square$$

Ex 8: Evaluate (round to 2 decimal places).

$$\log(2 \times 10^9) \approx \square$$

A.3 SOLVING EXPONENTIAL EQUATIONS USING LOGARITHMS

Ex 9: Find x such that $8 = 10^x$.

$x \approx \square$ (rounded to 3 decimal places)

Ex 10: Find x such that $0.4 = 10^x$.

$x \approx \square$ (rounded to 3 decimal places)

Ex 11: Find x such that $250 = 10^x$.

$x \approx \square$ (rounded to 3 decimal places)

A.4 SOLVING FOR x WHEN $\log(x)$ IS GIVEN

Ex 12: Find x such that $\log(x) = 3$.

$$x = \square$$

Ex 13: Find x such that $\log(x) = -1$.

$$x = \square$$

Ex 14: Find x such that $\log(x) = 0$.

$$x = \square$$

Ex 15: Find x such that $\log(x) = 7$.

$$x = \square$$

B LOGARITHMS IN BASE a

B.1 EVALUATING LOGARITHMS

Ex 16: Evaluate:

$$\log_2 8 = \square$$

Ex 17: Evaluate:

$$\log_3 1/9 = \square$$

Ex 18: Evaluate:

$$\log_5\left(\frac{1}{25}\right) = \square$$

Ex 19: Evaluate:

$$\log_4\sqrt{4} = \square$$

Ex 20: Evaluate:

$$\log_7 1 = \square$$

B.2 SOLVING FOR x WHEN $\log_a(x)$ IS GIVEN

Ex 21: Find x such that $\log_2(x) = 3$.

$$x = \square$$

Ex 22: Find x such that $\log_2(x) = -1$.

$$x = \square$$

Ex 23: Find x such that $\log_2(x) = 0$.

$$x = \square$$

Ex 24: Find x such that $\log_2(x) = 7$.

$$x = \square$$

C NATURAL LOGARITHM

C.1 EVALUATING NATURAL LOGARITHMS

Ex 25: Evaluate:

$$\ln e^3 = \square$$

Ex 26: Evaluate:

$$\ln(1/e) = \square$$

Ex 27: Evaluate:

$$\ln\left(\frac{1}{e^2}\right) = \square$$

Ex 28: Evaluate:

$$\ln\sqrt{e} = \square$$

Ex 29: Evaluate:

$$\ln 1 = \square$$

D LAWS OF LOGARITHMS

D.1 WRITING AS A SINGLE LOGARITHM: LEVEL 1

Ex 30: Write as a single logarithm

$$\log(5) + \log(3) = \square$$

Ex 31: Write as a single logarithm in the form $\log k$:

$$\log(15) - \log(5) = \square$$

Ex 32: Write as a single logarithm in the form $\log k$:

$$\log(4) + \log\left(\frac{1}{2}\right) = \square$$

Ex 33: Write as a single logarithm in the form $\log k$:

$$\log(18) - \log(3) = \square$$

D.2 WRITING AS A SINGLE LOGARITHM: LEVEL 2

Ex 34: Write as a single logarithm in the form $\log k$:

$$\log(8) + 1 = \square$$

Ex 35: Write as a single logarithm in the form $\log k$:

$$\log(3) + 2 = \square$$

Ex 36: Write as a single logarithm in the form $\log k$:

$$2 - \log(25) = \square$$

Ex 37: Write as a single logarithm in the form $\log k$:

$$\log(200) - 2 = \square$$

D.3 WRITING AS A SINGLE LOGARITHM: LEVEL 3

Ex 38: Write as a single logarithm in the form $\log k$:

$$2\log(3) + 1 = \square$$

Ex 39: Write as a single logarithm in the form $\log k$:

$$3\log(2) - \log(4) = \square$$

Ex 40: Write as a single logarithm in the form $\log k$:

$$2\log(20) - 2 = \square$$

Ex 41: Write as a single logarithm in the form $\log k$:

$$2\log(30) - 1 = \square$$

D.4 WRITING AS A SINGLE NATURAL LOGARITHM: LEVEL 1

Ex 42: Write as a single logarithm

$$\ln(5) + \ln(3) = \square$$

Ex 43: Write as a single logarithm in the form $\ln k$:

$$\ln(15) - \ln(5) = \square$$

Ex 44: Write as a single logarithm

$$\ln(x) + \ln(x) = \square$$

Ex 45: Write as a single logarithm in the form $\ln k$:

$$\ln(20) - \ln(4) = \square$$

D.5 WRITING AS A SINGLE NATURAL LOGARITHM: LEVEL 2

Ex 46: Write as a single logarithm in the form $\ln k$:

$$\ln(8) + 1 = \square$$

Ex 47: Write as a single logarithm in the form $\ln k$:

$$2 - \ln(5) = \square$$

Ex 48: Write as a single logarithm:

$$1 - \ln(x) = \square$$

Ex 49: Write as a single logarithm:

$$\ln(2) + x = \square$$

D.6 WRITING AS A SINGLE NATURAL LOGARITHM: LEVEL 3

Ex 50: Write as a single logarithm in the form $\ln k$:

$$2\ln(3) + 1 = \boxed{}$$

Ex 51: Write as a single logarithm in the form $\ln k$:

$$3\ln(2) - \ln(4) = \boxed{}$$

Ex 52: Write as a single logarithm:

$$2\ln(x) + \ln(4) = \boxed{}$$

Ex 53: Write as a single logarithm:

$$3\ln(x) - 2\ln(\sqrt{x}) = \boxed{}$$

D.7 WRITING AS A SINGLE LOGARITHM IN BASE A: LEVEL 1

Ex 54: Write as a single logarithm

$$\log_2(5) + \log_2(3) = \boxed{}$$

Ex 55: Write as a single logarithm in the form $\log_a k$:

$$\log_3(18) - \log_3(6) = \boxed{}$$

D.8 WRITING AS A SINGLE LOGARITHM IN BASE A: LEVEL 2

Ex 56: Write as a single logarithm in the form $\log_a k$:

$$\log_5(4) + 1 = \boxed{}$$

Ex 57: Write as a single logarithm in the form $\log_a k$:

$$2 - \log_3(5) = \boxed{}$$

Ex 58: Write as a single logarithm:

$$\log_2(3) + x = \boxed{}$$

D.9 WRITING AS A SINGLE LOGARITHM IN BASE A: LEVEL 3

Ex 59: Write as a single logarithm in the form $\log_a k$:

$$2\log_6(3) + \log_6(4) = \boxed{}$$

Ex 60: Write as a single logarithm in the form $\log_a k$:

$$3\log_2(4) - \log_2(8) = \boxed{}$$

Ex 61: Write as a single logarithm:

$$\log_2(5) - 2x = \boxed{}$$

E CHANGE OF BASE RULE

E.1 EVALUATING LOGARITHMS USING CHANGE OF BASE FORMULA

Ex 62: Evaluate in changing to base 10(round to 2 decimal places).

$$\log_3(2) \approx \boxed{}$$

Ex 63: Evaluate by changing to base 10 (round to 2 decimal places).

$$\log_5(8) \approx \boxed{}$$

Ex 64: Evaluate by changing to base 10 (round to 2 decimal places).

$$\log_7(0.5) \approx \boxed{}$$

Ex 65: Evaluate by changing to base 10 (round to 2 decimal places).

$$\log_2(100) \approx \boxed{}$$

E.2 PROVING CHANGE OF BASE IDENTITIES

Ex 66: Prove that $\log_a b \cdot \log_b a = 1$.

Ex 67: Prove that $\log_a(b)\log_b(c) = \log_a(c)$.

Ex 68: Prove that $\log_{a^n}(b) = \frac{1}{n}\log_a(b)$.

Ex 69: Prove that $\log_{1/a}(b) = -\log_a(b)$.

F USING LOGARITHMS TO SOLVE EXPONENTIAL EQUATIONS

F.1 SOLVING EXPONENTIAL EQUATIONS: LEVEL 1

Ex 70: Solve $2^x = 7$ (give your answer to 3 decimal places).

$x =$

Ex 71: Solve $3^x = 15$ (give your answer to 3 decimal places).

$x =$

Ex 72: Solve $5^x = 100$ (give your answer to 3 decimal places).

$x =$

Ex 73: Solve $6^x = 80$ (give your answer to 3 decimal places).

$x =$

F.2 SOLVING EXPONENTIAL EQUATIONS: LEVEL 2

Ex 74: Solve $5 \cdot 2^x = 7$ (give your answer to 3 decimal places).

$x =$

Ex 75: Solve $-(2^x) = -10$ (give your answer to 3 decimal places).

$x =$

Ex 76: Solve $4 \cdot 3^x = 60$ (give your answer to 3 decimal places).

$x =$

Ex 77: Solve $-2 \cdot (0.5)^x = -4$.

$x =$

F.3 APPLYING EXPONENTIAL FUNCTIONS

Ex 78: The population of a town, P, is growing exponentially. The population can be modelled by the function $P(t) = 25000 \times 1.035^t$, where t is the number of years after the 1st of January 2020.

1. Write down the population of the town on the 1st of January 2020.
2. Calculate the population of the town after 5 years, giving your answer to the nearest whole number.
3. Determine the number of years it will take for the population to double. Give your answer to the nearest year.
4. Another town's population is modelled by the function $Q(t) = 40000 \times 1.018^t$. After how many years will the population of both towns be equal?

Ex 79: The mass, M, in grams of a radioactive substance is modelled by the function $M(t) = 150 \times (0.88)^t$, where t is the time in years.

1. Write down the initial mass of the substance.

2. Calculate the mass of the substance remaining after 10 years, giving your answer to two decimal places.

3. Find the half-life of the substance. Give your answer to the nearest year.

4. Another radioactive substance has its mass modelled by the function $N(t) = 200 \times (0.85)^t$. Find the time it takes for the mass of both substances to be equal.

Ex 80: Laura invests \$8000 in a savings account that pays a nominal annual interest rate of 4.2%, compounded annually. The value of her investment, V, after t years is given by the formula $V(t) = 8000(1.042)^t$.

1. Find the value of Laura's investment after 7 years. Give your answer to two decimal places.

2. Determine the number of years it will take for the value of the investment to exceed \$15,000.

3. Marco also invests in an account with an initial amount of \$7500. After 10 years, his investment is worth \$11,000. Assuming the interest is also compounded annually, find the annual interest rate for Marco's investment.

G APPLICATIONS OF LOGARITHMS

G.1 APPLYING LOGARITHMS IN SCIENCE

Ex 81: The pH scale in chemistry is $\text{pH} = -\log_{10}[H^+]$ where $[H^+]$ is the hydrogen ion concentration in moles per litre. The pH of a solution is 3.2. Find the hydrogen ion concentration $[H^+]$ (give your answer in scientific notation with 3 significant digits).

$$\square \times \square \text{ mol/L}$$

Ex 82: The Richter scale measures earthquake intensity using the formula $M = \log_{10}\left(\frac{I}{I_0}\right)$, where M is the magnitude, I is the intensity of the earthquake, and I_0 is the intensity of a standard earthquake.
An earthquake has a magnitude of 4.5 on the Richter scale. Find the intensity ratio $\frac{I}{I_0}$ (give your answer in scientific notation with 3 significant digits).

$$\frac{I}{I_0} = \square \times \square$$

Ex 83: The intensity of sound is measured in decibels (dB) using the formula $L = 10\log_{10}\left(\frac{I}{I_0}\right)$, where L is the sound level in decibels, I is the intensity of the sound, and I_0 is the reference intensity (threshold of human hearing).
A sound has a level of 75 decibels. Find the intensity ratio $\frac{I}{I_0}$ (give your answer in scientific notation with 3 significant digits).

$$\frac{I}{I_0} = \square \times \square$$