LOGARITHM FUNCTIONS

A NATURAL LOGARITHM FUNCTION

A.1 DETERMINING DOMAINS OF LOGARITHMIC FUNCTIONS

MCQ 1: What is the domain of the function $f(x) = \ln(x+1)$?

☐ $(-\infty, \infty)$

☐ $(-\infty, -1]$

☐ $(-1, \infty)$

☐ $[0, \infty)$

MCQ 2: Find the domain of the function $f : x \mapsto \ln(2 - x)$.

☐ $\mathbb{R}$

☐ $[-2, +\infty)$

☐ $(2, +\infty)$

☐ $(-\infty, 2)$

MCQ 3: Find the domain of the function $f : x \mapsto \ln(2x - 6)$.

☐ $\mathbb{R}$

☐ $[3, +\infty)$

☐ $(3, +\infty)$

☐ $(-\infty, 3)$

MCQ 4: Find the domain of the function $f : x \mapsto \ln(9 - 3x)$.

☐ $\mathbb{R}$

☐ $[3, +\infty)$

☐ $(3, +\infty)$

☐ $(-\infty, 3)$

MCQ 5: What is the domain of the function $f(x) = \ln(x^2+1)$?

☐ $(-\infty, \infty)$

☐ $(-\infty, 0) \cup (0, \infty)$

☐ $(-1, 1)$

☐ $[-1, 1]$

MCQ 6: What is the domain of the function $f(x) = \ln(-x^2 + 3x - 2)$?

☐ $(-\infty, \infty)$

☐ $(-\infty, 1] \cup [2, \infty)$

☐ $(1, 2)$

☐ $[1, 2]$

A.2 CALCULATING $f(x)$

Ex 7: For $f : x \mapsto 3\ln(x)$, find in simplest form:

1. $f(1) =$ ☐
2. $f(e) =$ ☐

Ex 8: For $f : x \mapsto \frac{1}{1+\ln(x)}$, find in simplest form:

1. $f(1) =$ ☐
2. $f(e) =$ ☐

Ex 9: For $f : x \mapsto x\ln(x + 1)$, find in simplest form:

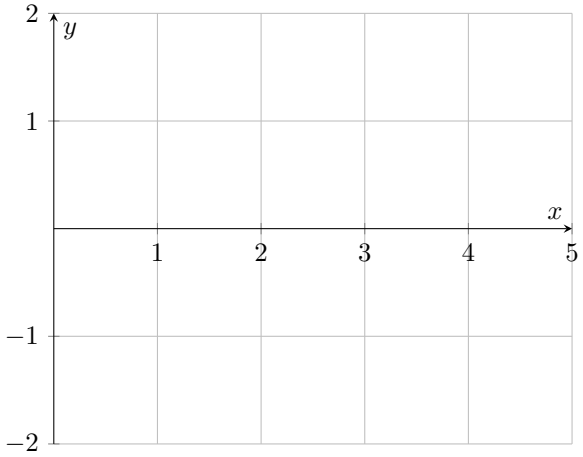
1. $f(0) =$ ☐
2. $f(1) =$ ☐

A.3 PLOTTING GRAPHS OF THE NATURAL LOGARITHM

Ex 10: Here is a table of values for the function $f(x) = \ln(x)$:

x	0.2	0.5	1	2	5
$\ln(x)$	-1.61	-0.69	0	0.69	1.61

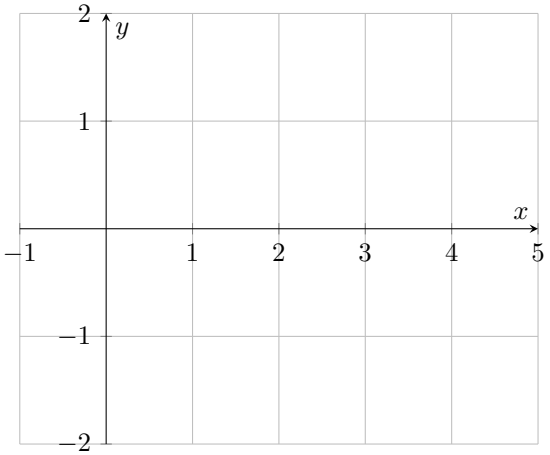
Plot the graph of the function.



Ex 11: Here is a table of values for the function $f(x) = \ln(1+x)$:

x	-0.8	-0.5	0	1	4
$\ln(1 + x)$	-1.61	-0.69	0	0.69	1.61

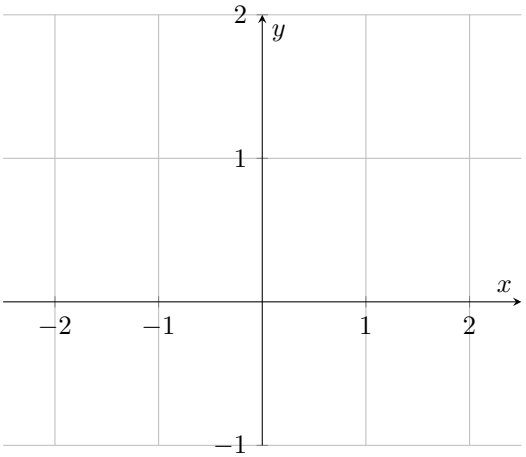
Plot the graph of the function.



Ex 12: Here is a table of values for the function $f(x) = \ln(1 + x^2)$:

x	-2	-1	-0.5	0	0.5	1	2
$\ln(1+x^2)$	1.61	0.69	0.22	0	0.22	0.69	1.61

Plot the graph of the function.



A.4 FINDING INVERSE FUNCTIONS

Ex 13: For $f : x \mapsto 3\ln(x)$, find the inverse function:

$$f^{-1}(x) = \square$$

Ex 14: For $f : x \mapsto \ln(x + 2) - 3$, find the inverse function:

$$f^{-1}(x) = \square$$

A.5 FINDING ASYMPTOTES

Ex 15: For the function $f(x) = \ln(x - 3)$, find the equation of the vertical asymptote:

$$x = \square$$

Ex 16: For the function $f(x) = 3\ln(x^2) - 1$, find the equation of the vertical asymptote:

$$x = \square$$

Ex 17: For the function $f(x) = \ln(e^x - 1)$, find the equation of the vertical asymptote:

$$x = \square$$

A.6 FINDING $f(g(x))$

Ex 18: For the function $f(x) = e^x$ and $g(x) = \ln(x - 3)$, find and simplify:

$$(f \circ g)(x) = \square$$

Ex 19: For the function $f(x) = \ln(x)$ and $g(x) = x^2 + 4$, find and simplify:

$$(f \circ g)(x) = \square$$

Ex 20: For the function $f(x) = \ln(x)$ and $g(x) = e^{2x}$, find and simplify:

$$(g \circ f)(x) = \square$$

A.7 ANALYZING LOGARITHMIC FUNCTIONS

Ex 21: For the function $f(x) = \ln(x - 3)$:

1. Find the domain and range.
2. Find any asymptotes and axes intercepts.
3. Sketch the graph of $y = f(x)$, showing all important features.
4. Solve $f(x) = -1$ algebraically and check the solution on your graph.
5. Find the inverse function f^{-1}.

Ex 22: For the function $f : x \mapsto 2 - \ln(x - 1)$:

1. Find the domain and range.
2. Find any asymptotes and axes intercepts.
3. Sketch the graph of $y = f(x)$, showing all important features.
4. Solve $f(x) = -1$ algebraically and check the solution on your graph.
5. Find the inverse function f^{-1}.

Ex 23: For the function $f(x) = (\ln(x))^2$:

1. Find the domain and range.
2. Find any asymptotes and axes intercepts.
3. Sketch the graph of $y = f(x)$, showing all important features.
4. Solve $f(x) = 4$ algebraically and check the solution on your graph.
5. Find the inverse function f^{-1}, for $\boldsymbol{x \geq 1}$

B LOGARITHMIC FUNCTION IN BASE a

B.1 DETERMINING DOMAINS OF LOGARITHMIC FUNCTIONS

MCQ 24: Find the domain of the function $f : x \mapsto \log_2(x-4)$.

- ☐ $\mathbb{R}$
- ☐ $[-4, +\infty)$
- ☐ $(4, +\infty)$
- ☐ $(-\infty, 4)$

MCQ 25: Find the domain of the function $f : x \mapsto \log_5(2-x)$.

☐ $\mathbb{R}$

☐ $[-2, +\infty)$

☐ $(2, +\infty)$

☐ $(-\infty, 2)$

MCQ 26: Find the domain of the function $f : x \mapsto \log(2x-6)$.

☐ $\mathbb{R}$

☐ $[3, +\infty)$

☐ $(3, +\infty)$

☐ $(-\infty, 3)$

MCQ 27: Find the domain of the function $f : x \mapsto \log_{10}(9 - 3x)$.

☐ $\mathbb{R}$

☐ $[3, +\infty)$

☐ $(3, +\infty)$

☐ $(-\infty, 3)$

B.2 CALCULATING $f(x)$

Ex 28: For $f : x \mapsto 3\log(x)$, find in simplest form:

1. $f(1) =$ ☐
2. $f(10) =$ ☐

Ex 29: For $f : x \mapsto \frac{1}{1+\log_2(x)}$, find in simplest form:

1. $f(1) =$ ☐
2. $f(2) =$ ☐

Ex 30: For $f : x \mapsto x\log(x+1)$, find in simplest form:

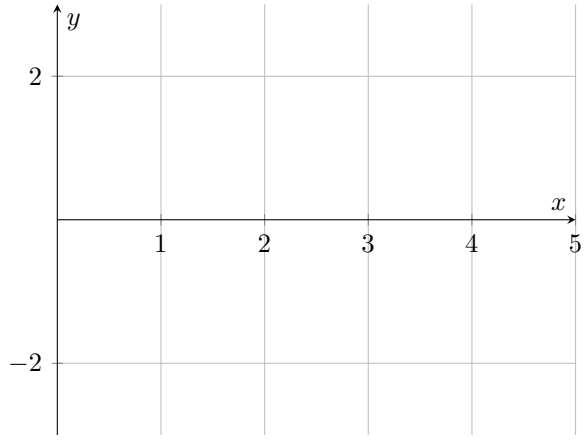
1. $f(0) =$ ☐
2. $f(1) =$ ☐

B.3 PLOTTING LOGARITHMIC GRAPHS

Ex 31: Here is a table of values for the function $f(x) = \log_2(x)$:

x	0.25	0.5	1	2	4
$\log_2(x)$	-2	-1	0	1	2

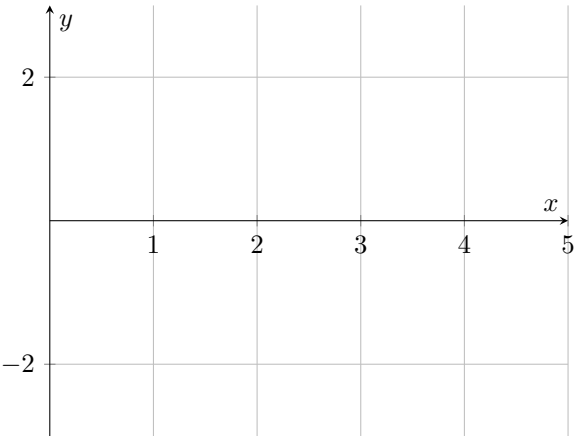
Plot the graph of the function.



Ex 32: Here is a table of values for the function $f(x) = \log_{0.5}(x)$:

x	0.25	0.5	1	2	4
$\log_{0.5}(x)$	2	1	0	-1	-2

Plot the graph of the function.



B.4 FINDING INVERSE FUNCTIONS

Ex 33: For $f : x \mapsto \log_2(x-1)$, find the inverse function:

$$f^{-1}(x) = \square$$

Ex 34: For $f : x \mapsto 5\log_3(2x)$, find the inverse function:

$$f^{-1}(x) = \square$$

B.5 FINDING ASYMPTOTES

Ex 35: For the function $f(x) = \log_3(x-3)$, find the equation of the vertical asymptote:

$$x = \square$$

Ex 36: For the function $f(x) = \log_2(x+1)$, find the equation of the vertical asymptote:

$$x = \square$$

B.6 FINDING $f(g(x))$

Ex 37: For the function $f(x) = \log_2(x)$ and $g(x) = 4^x$, find and simplify:

$$(f \circ g)(x) = \square$$

Ex 38: For the function $f(x) = 2^x$ and $g(x) = \log_4(x)$, find and simplify:

$$(f \circ g)(x) = \square$$

B.7 ANALYZING LOGARITHMIC FUNCTIONS

Ex 39: For the function $f(x) = \log_2(x-3)$:

1. Find the domain and range.
2. Find any asymptotes and axes intercepts.
3. Sketch the graph of $y = f(x)$, showing all important features.
4. Solve $f(x) = -1$ algebraically and check the solution on your graph.
5. Find the inverse function f^{-1}.

Ex 40: For the function $f(x) = 2 - \log_3(x-1)$:

1. Find the domain and range.
2. Find any asymptotes and axes intercepts.
3. Sketch the graph of $y = f(x)$, showing all important features.
4. Solve $f(x) = -1$ algebraically and check the solution on your graph.
5. Find the inverse function f^{-1}.