VECTOR EQUATIONS OF LINES

A VECTOR EQUATION

A.1 LOCATING POINTS ON A LINE

Ex 1: For the vector equation, $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \end{pmatrix}, \quad \lambda \in \mathbb{R}$, locate the point on the line for which $\lambda = 0$.

$A(\square, \square)$

Ex 2: For the vector equation, $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \end{pmatrix}, \quad \lambda \in \mathbb{R}$, locate the point on the line for which $\lambda = 2$.

$A(\square, \square)$

Ex 3: For the vector equation, $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \end{pmatrix}, \quad \lambda \in \mathbb{R}$, locate the point on the line for which $\lambda = \frac{1}{2}$.

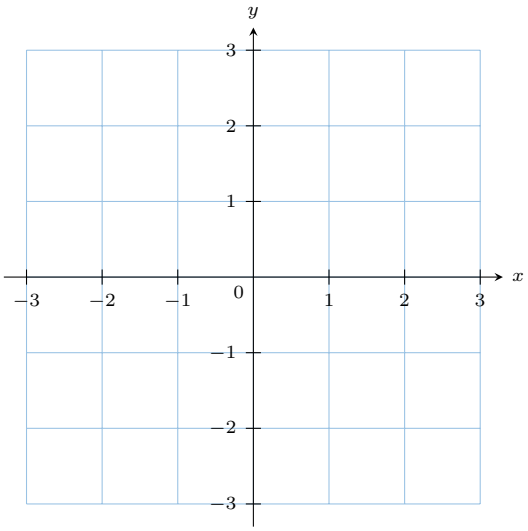
$A(\square, \square)$

Ex 4: For the vector equation, $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \end{pmatrix}, \quad \lambda \in \mathbb{R}$, locate the point on the line for which $\lambda = -\frac{3}{2}$.

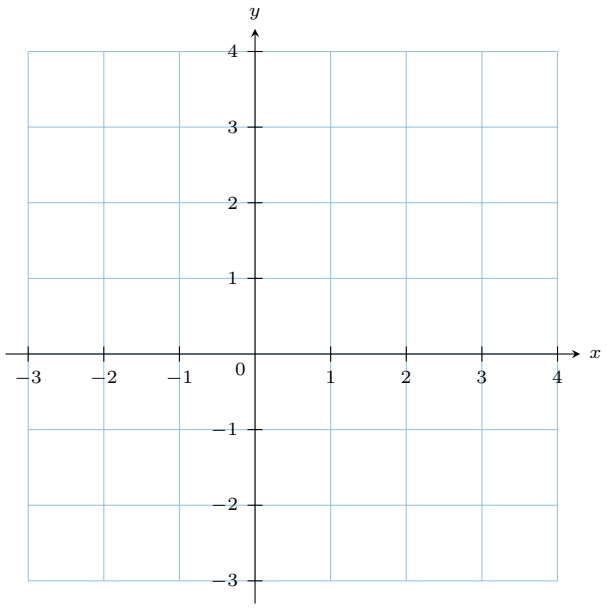
$A(\square, \square)$

A.2 PLOTTING A LINE FROM ITS VECTOR EQUATION

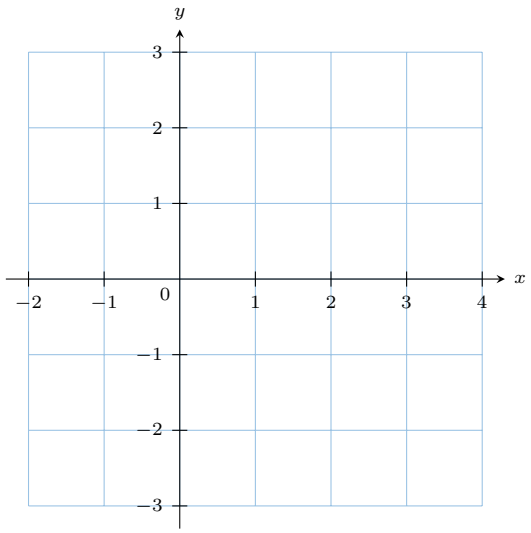
Ex 5: For the vector equation, $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \quad \lambda \in \mathbb{R}$, plot the line.



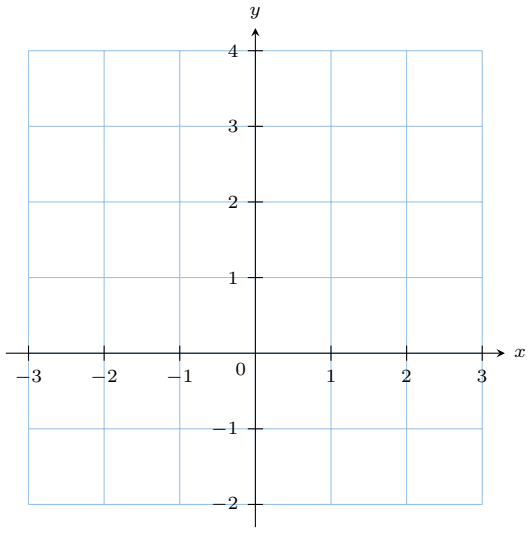
Ex 6: For the vector equation, $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \end{pmatrix}, \quad \lambda \in \mathbb{R}$, plot the line.



Ex 7: A line passes through the point $A(1, -2)$ with direction vector $\vec{b} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$. Plot the point A, the direction vector originating from A, and the resulting line.



Ex 8: A line passes through the point $A(-2, 2)$ with direction vector $\overrightarrow{b} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$. Plot the point A, the direction vector originating from A, and the resulting line.



A.3 WRITING EQUATIONS OF LINES

Ex 9: A line in space passes through the point $(1, -2, 3)$ in the direction $\begin{pmatrix} 4 \\ 5 \\ -6 \end{pmatrix}$.
Describe the line using a vector equation.

Ex 10: A line in space passes through the points $A(2, -1, 4)$ and $B(-1, 0, 2)$.
Describe the line using a vector equation.

Ex 11: A line in a plane passes through the point $A(1,4)$ and is perpendicular to the vector $\overrightarrow{n} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$.
Describe the line using a vector equation.

B PARAMETRIC EQUATIONS

B.1 FINDING INTERSECTIONS WITH COORDINATE AXES AND PLANES

Ex 12: A line has parametric equations:

$$\begin{cases} x = -4 + 2\lambda \\ y = 9 - 3\lambda \end{cases} \quad , \quad \lambda \in \mathbb{R}$$

The line intersects the x-axis at point P.

1. What is the value of the y-coordinate at point P?
2. Find the value of the parameter λ at point P.
3. Determine the coordinates of the point of intersection P.

Ex 13: A line has parametric equations:

$$\begin{cases} x = 5 + \lambda \\ y = 1 - 2\lambda \\ z = -3 + 4\lambda \end{cases} \quad , \quad \lambda \in \mathbb{R}$$

The line intersects the YZ-plane at point Q.

1. What is the value of the x-coordinate at point Q?
2. Find the value of the parameter λ at point Q.
3. Determine the coordinates of the point of intersection Q.

Ex 14: A line has parametric equations:

$$\begin{cases} x = 7 - 2\lambda \\ y = -4 + 3\lambda \\ z = 10 - 5\lambda \end{cases} \quad , \quad \lambda \in \mathbb{R}$$

The line intersects the XY-plane at point R.

1. What is the value of the z-coordinate at point R?

2. Find the value of the parameter λ at point R.

3. Determine the coordinates of the point of intersection R.

B.2 VERIFYING IF A POINT LIES ON A LINE

Ex 15: A line is defined by the parametric equations:

$$\begin{cases} x = 2 - t \\ y = 3 + 2t \end{cases}, \quad t \in \mathbb{R}$$

Determine if the point $Q(-1, 9)$ lies on the line.

Ex 16: A line is defined by the parametric equations:

$$\begin{cases} x = 4 - 2t \\ y = 1 + 3t \end{cases}, \quad t \in \mathbb{R}$$

Determine if the point $P(2, 5)$ lies on the line.

Ex 17: A line is defined by the parametric equations:

$$\begin{cases} x = 1 + 2t \\ y = 5 - t \\ z = -2 + 4t \end{cases}, \quad t \in \mathbb{R}$$

Determine if the point $P(5, 3, 6)$ lies on the line.

C CARTESIAN EQUATION IN PLANE

C.1 FINDING THE NORMAL VECTOR FROM A CARTESIAN EQUATION

Ex 18: State the normal vector for the line with equation $2x - 5y = 8$.

$$\vec{n} = \begin{pmatrix} \square \\ \square \end{pmatrix}$$

Ex 19: State the normal vector for the line with equation $y = 4x - 1$.

$$\overrightarrow{n} = \begin{pmatrix} \square \\ 1 \end{pmatrix}$$

Ex 20: State the normal vector for the line with equation $y = -\frac{2}{3}x + 5$.

$$\overrightarrow{n} = \begin{pmatrix} \square \\ 3 \end{pmatrix}$$

C.2 USING THE NORMAL VECTOR TO FIND THE CARTESIAN EQUATION

Ex 21: Find the Cartesian equation of the line that passes through the point $P(-3, 5)$ and has a normal vector of $\overrightarrow{n} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$.

Ex 22: Find the Cartesian equation of the line that passes through the point $P(1, 2)$ and has a normal vector of $\overrightarrow{n} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$.