LINE EQUATIONS
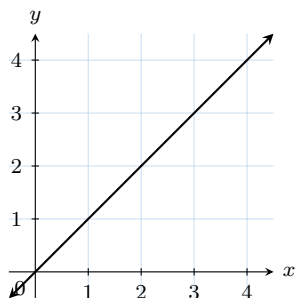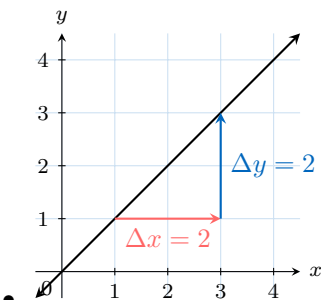
A SLOPES

A.1 FINDING SLOPES OF LINES

Ex 1:

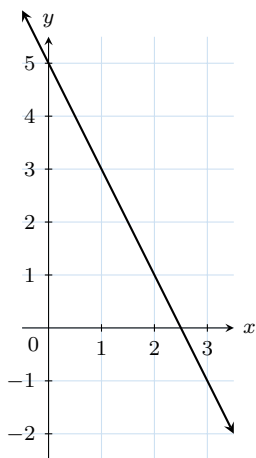Find the slope of the line:

$$\text{slope} = \boxed{1}$$

Answer:

$$\text{slope} = \frac{\Delta y}{\Delta x}$$
$$= \frac{2}{2}$$
$$= 1$$

Ex 2:

Find the slope of the line:

$$\text{slope} = \boxed{-2}$$

Answer:

$$\text{slope} = \frac{\Delta y}{\Delta x}$$
$$= \frac{-2}{1}$$
$$= -2$$

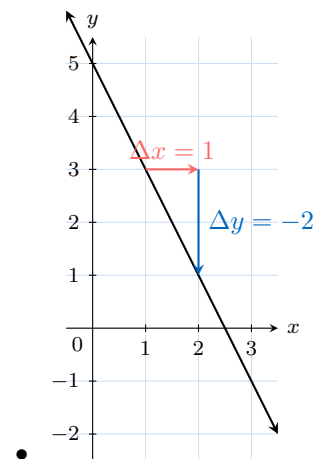**Ex 3:**
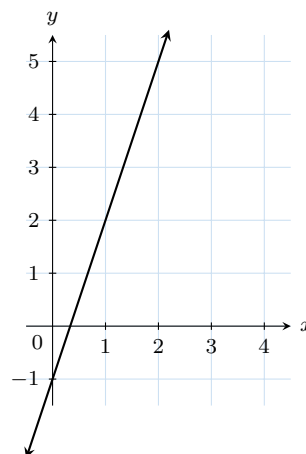
Find the slope of the line:

$$\text{slope} = \boxed{3}$$

Answer:

-
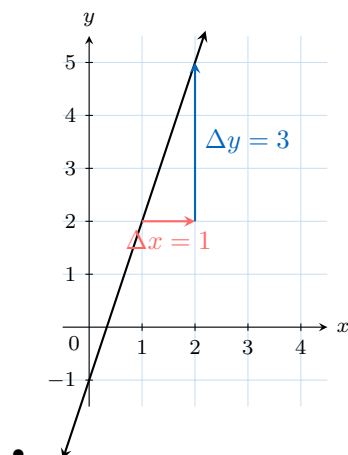
$$\text{slope} = \frac{\Delta y}{\Delta x}$$
$$= \frac{3}{1}$$
$$= 3$$

Ex 4:
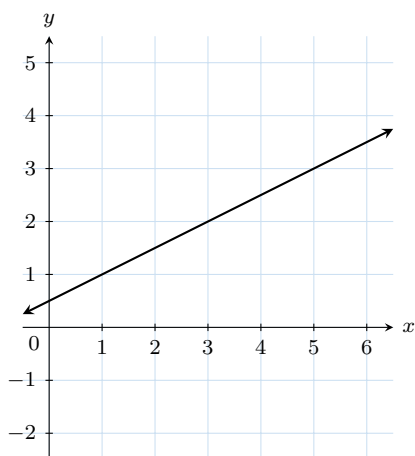
Find the slope of the line:

$$\text{slope} = \boxed{\frac{1}{2}}$$

Answer:

-

$$\text{slope} = \frac{\Delta y}{\Delta x}$$
$$= \frac{1}{2}$$

Ex 5:
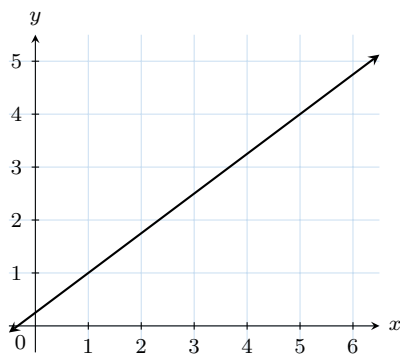
Find the slope of the line:

$$\text{slope} = \boxed{\frac{3}{4}}$$

Answer:

-

$$\text{slope} = \frac{\Delta y}{\Delta x}$$
$$= \frac{3}{4}$$

Ex 6:
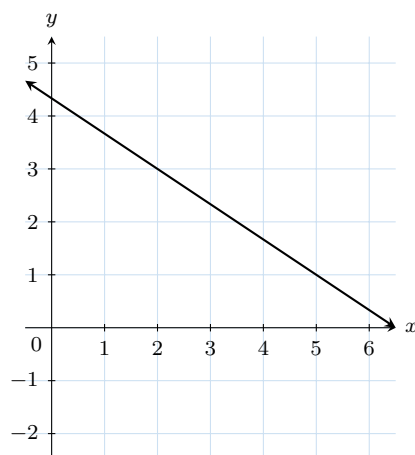
Find the slope of the line:

$$\text{slope} = \boxed{-\frac{2}{3}}$$

Answer:

-

$$\text{slope} = \frac{\Delta y}{\Delta x}$$
$$= \frac{-2}{3}$$
$$= -\frac{2}{3}$$

A.2 INTERPRETING SLOPE IN CONTEXT

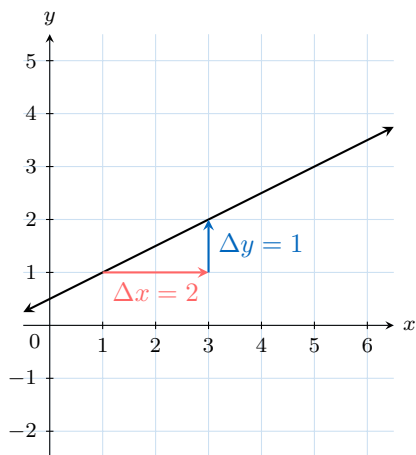**Ex 7:**

Find the slope of the road:

$$\text{slope} = \boxed{\frac{1}{4}}$$

Answer:

$$\text{slope} = \frac{\Delta y}{\Delta x}$$
$$= \frac{1}{4}$$

Ex 8:

Find the slope of the road:

$$\text{slope} = \boxed{-1}$$

Answer:

$$\text{slope} = \frac{\Delta y}{\Delta x}$$
$$= \frac{-3}{3} \quad \text{(vertical change is downward, so negative)}$$
$$= -1$$

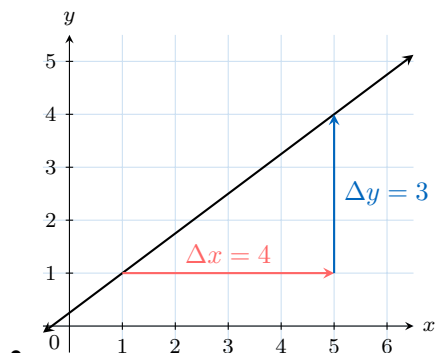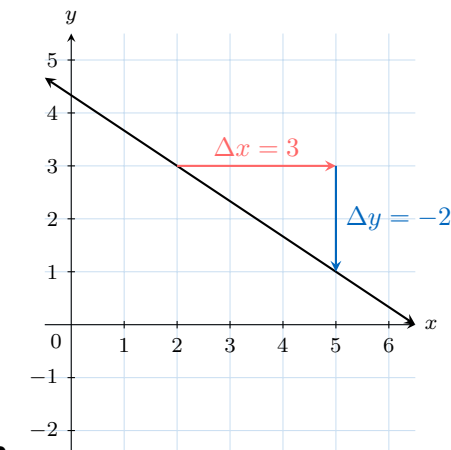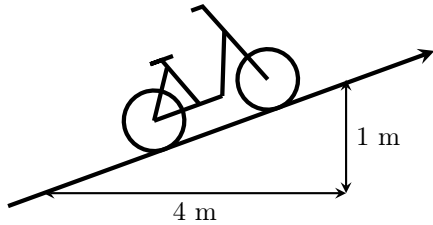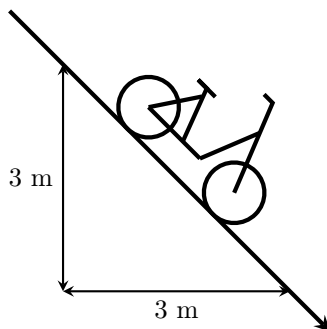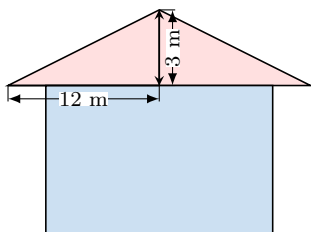**Ex 9:**

Find the slope of the roof:

$$\text{slope} = \boxed{\frac{1}{4}}$$

Answer:

$$\text{slope} = \frac{\Delta y}{\Delta x}$$
$$= \frac{3}{12}$$
$$= \frac{1}{4}$$

MCQ 10: You are following a trail through the mountains from point A to point E, traveling from left to right.
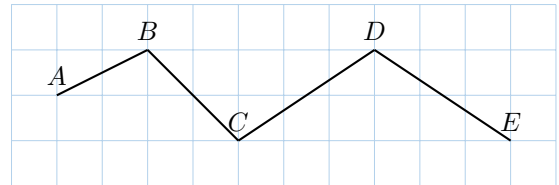
Identify the steepest upward segment.

☐ $\overline{AB}$

☐ $\overline{BC}$

☒ $\overline{CD}$

☐ $\overline{DE}$

Answer:

- An upward segment corresponds to a positive slope. There are two positive slopes: $\overline{AB}$ and $\overline{CD}$.

-

$$\text{slope of } \overline{AB} = \frac{1}{2}$$
$$\text{slope of } \overline{CD} = \frac{2}{3}$$

- Since $\frac{2}{3} > \frac{1}{2}$, the steepest positive slope is along segment $\overline{CD}$.

MCQ 11: You are following a trail through the mountains from point A to point E, traveling from left to right.

Identify the steepest downward segment.

☐ $\overline{AB}$

☒ $\overline{BC}$

☐ $\overline{CD}$

3

☐ $\overline{DE}$

Answer:

- A downward segment corresponds to a negative slope. There are two negative slopes: $\overline{BC}$ and $\overline{DE}$.

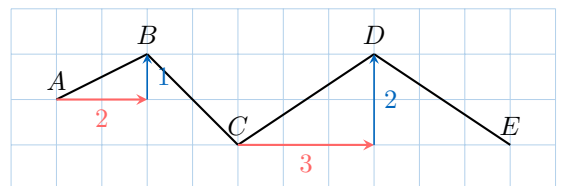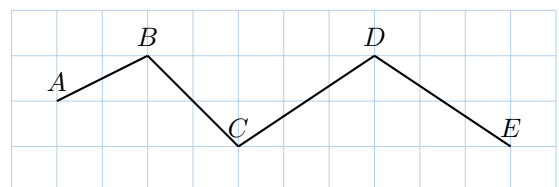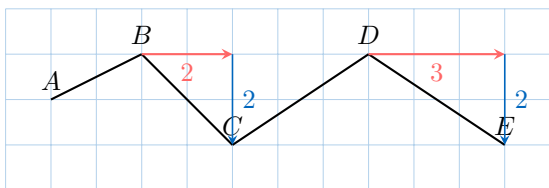$$\text{slope of } \overline{BC} = \frac{-2}{2}$$
$$= -1$$
$$\text{slope of } \overline{DE} = \frac{-2}{3}$$
$$= -\frac{2}{3}$$

- Since $-1 < -\frac{2}{3}$, the steepest negative slope is along segment $\overline{BC}$.

B SLOPE FORMULA

B.1 CALCULATING THE SLOPE

Ex 12: For $A(1, 2)$ and $B(5, 4)$, find the slope of the line $\overleftrightarrow{AB}$.

$$\text{Slope of } \overleftrightarrow{AB} = \boxed{\frac{1}{2}}$$

Answer:
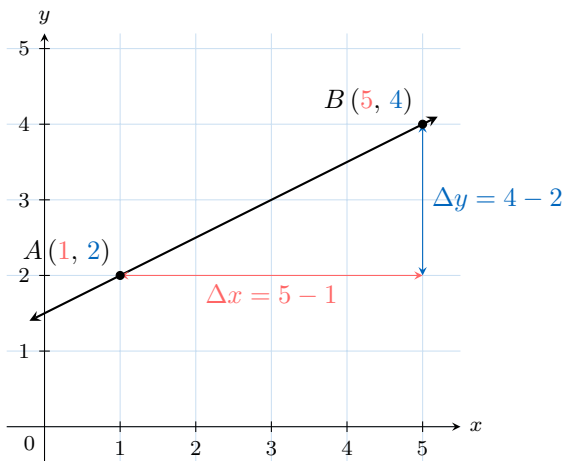
$$\text{slope of } \overleftrightarrow{AB} = \frac{y_B - y_A}{x_B - x_A}$$
$$= \frac{4 - 2}{5 - 1}$$
$$= \frac{2}{4}$$
$$= \frac{1}{2}$$

Ex 13: For $A(3, 0)$ and $B(4, 4)$, find the slope of the line $\overleftrightarrow{AB}$.

$$\text{Slope of } \overleftrightarrow{AB} = \boxed{4}$$

Answer:

$$\text{slope of } \overleftrightarrow{AB} = \frac{y_B - y_A}{x_B - x_A}$$
$$= \frac{4 - 0}{4 - 3}$$
$$= \frac{4}{1}$$
$$= 4$$

Ex 14: For $A(-1, 3)$ and $B(2, -1)$, find the slope of the line $\overleftrightarrow{AB}$.

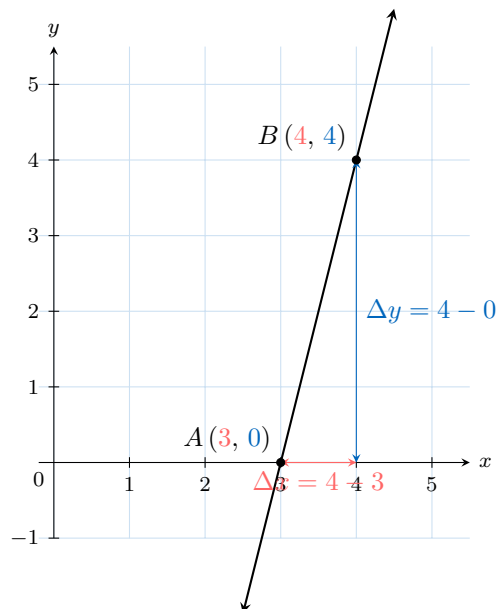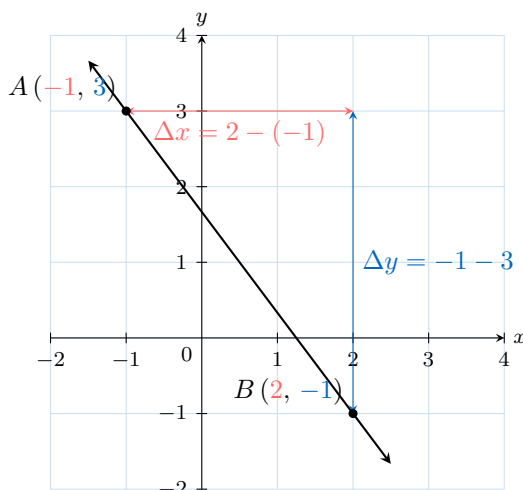$$\text{Slope of } \overleftrightarrow{AB} = \boxed{-\frac{4}{3}}$$

Answer:

$$\text{slope of } \overleftrightarrow{AB} = \frac{y_B - y_A}{x_B - x_A}$$
$$= \frac{-1 - 3}{2 - (-1)}$$
$$= \frac{-4}{3}$$
$$= -\frac{4}{3}$$

Ex 15: For $C\,(3,\,1)$ and $D\,(1,\,3)$, find the slope of the line $\overleftrightarrow{CD}$. *Answer:*

$$\text{Slope of } \overleftrightarrow{CD} = \boxed{-1}$$

Answer:

$$\text{slope of } \overleftrightarrow{CD} = \frac{y_B - y_A}{x_B - x_A}$$
$$= \frac{3 - 1}{1 - 3}$$
$$= \frac{2}{-2}$$
$$= -1$$

y-intercept $= -2$

C y-INTERCEPT

C.1 FINDING THE y-INTERCEPT

Ex 16:

$$y\text{-intercept} = \boxed{1}$$

Answer:

y-intercept $= 1$

Ex 17:

$$y\text{-intercept} = \boxed{-2}$$

Ex 18:

$$y\text{-intercept} = \boxed{1.5}$$

Answer:

y-intercept $= 1.5$

Ex 19:

$$y\text{-intercept} = \boxed{-2.5}$$

Answer:

y-intercept $= -2.5$
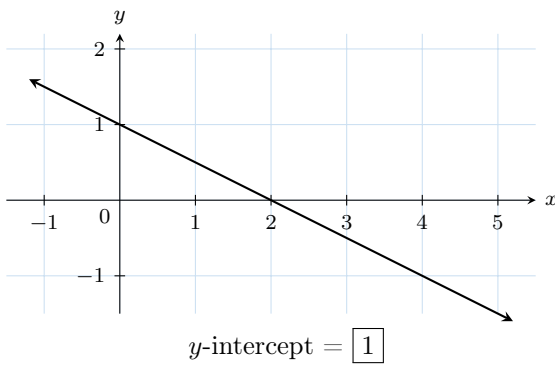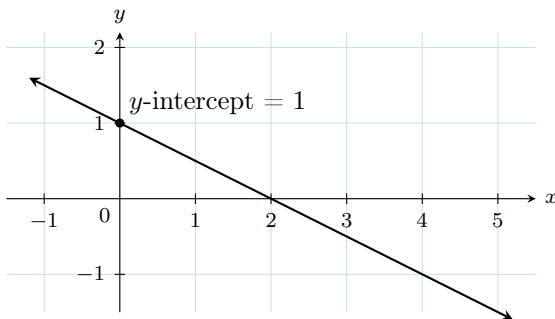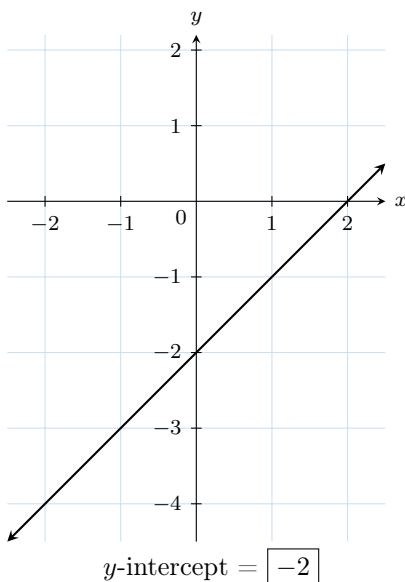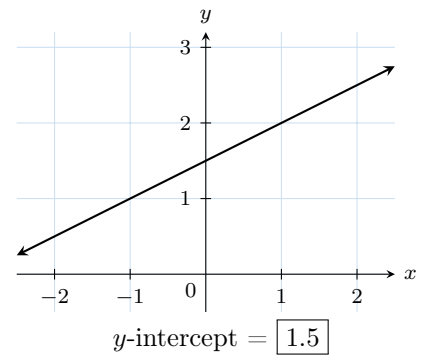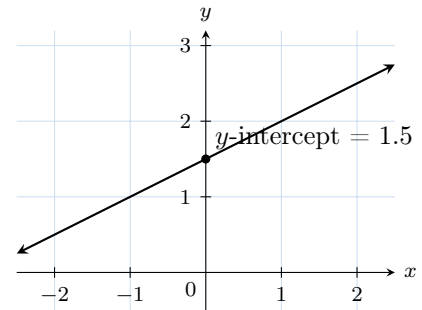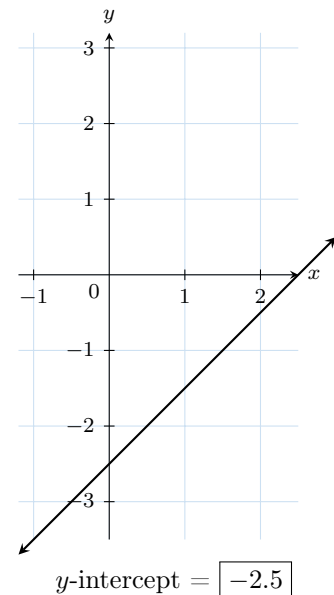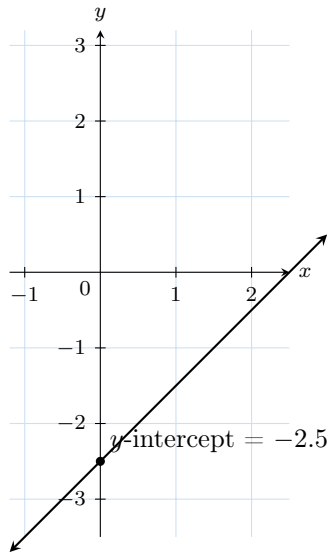
D LINE EQUATIONS

D.1 COMPLETING A TABLE OF VALUES

Ex 20: For $y = x + 3$, fill in the table:

x	-2	-1	0	1	2
y	1	2	3	4	5

Answer:

- For $x = -2$,
$$y = (-2) + 3$$
$$= 1$$

- For $x = -1$,
$$y = (-1) + 3$$
$$= 2$$

- For $x = 0$,
$$y = 0 + 3$$
$$= 3$$

- For $x = 1$,
$$y = 1 + 3$$
$$= 4$$

- For $x = 2$,
$$y = 2 + 3$$
$$= 5$$

So the completed table is:

x	-2	-1	0	1	2
y	1	2	3	4	5

Ex 21: For $y = -2x + 1$, fill in the table:

x	-2	-1	0	1	2
y	5	3	1	-1	-3

Answer:

- For $x = -2$,
$$y = -2 \times (-2) + 1$$
$$= 4 + 1$$
$$= 5$$

- For $x = -1$,
$$y = -2 \times (-1) + 1$$
$$= 2 + 1$$
$$= 3$$

- For $x = 0$,
$$y = -2 \times 0 + 1$$
$$= 0 + 1$$
$$= 1$$

- For $x = 1$,
$$y = -2 \times 1 + 1$$
$$= -2 + 1$$
$$= -1$$

- For $x = 2$,
$$y = -2 \times 2 + 1$$
$$= -4 + 1$$
$$= -3$$

So the completed table is:

x	-2	-1	0	1	2
y	5	3	1	-1	-3

Ex 22: For $y = 3x - 5$, fill in the table:

x	-2	-1	0	1	2
y	-11	-8	-5	-2	1

Answer:

- For $x = -2$,
$$y = 3 \times (-2) - 5$$
$$= -6 - 5$$
$$= -11$$

- For $x = -1$,
$$y = 3 \times (-1) - 5$$
$$= -3 - 5$$
$$= -8$$

- For $x = 0$,
$$y = 3 \times 0 - 5$$
$$= 0 - 5$$
$$= -5$$

- For $x = 1$,
$$y = 3 \times 1 - 5$$
$$= 3 - 5$$
$$= -2$$

- For $x = 2$,
$$y = 3 \times 2 - 5$$
$$= 6 - 5$$
$$= 1$$

So the completed table is:

x	-2	-1	0	1	2
y	-11	-8	-5	-2	1

Ex 23: 🖩 For $y = -2.5x - 2$, fill in the table:

x	-2	-1	0	1	2
y	3	0.5	-2	-4.5	-7

Answer:

- For $x = -2$,
$$y = -2.5 \times (-2) - 2$$
$$= 5 - 2$$
$$= 3$$

- For $x = -1$,
$$y = -2.5 \times (-1) - 2$$
$$= 2.5 - 2$$
$$= 0.5$$

- For $x = 0$,
$$y = -2.5 \times 0 - 2$$
$$= 0 - 2$$
$$= -2$$

- For $x = 1$,
$$y = -2.5 \times 1 - 2$$
$$= -2.5 - 2$$
$$= -4.5$$

- For $x = 2$,
$$y = -2.5 \times 2 - 2$$
$$= -5 - 2$$
$$= -7$$

So the completed table is:

x	-2	-1	0	1	2
y	3	0.5	-2	-4.5	-7

Ex 24: 🖩 For $y = 0.5x + 1$, fill in the table:

x	-2	-1	0	1	2
y	0	0.5	1	1.5	2

Answer:

- For $x = -2$,
$$y = 0.5 \times (-2) + 1$$
$$= -1 + 1$$
$$= 0$$

- For $x = -1$,
$$y = 0.5 \times (-1) + 1$$
$$= -0.5 + 1$$
$$= 0.5$$

- For $x = 0$,
$$y = 0.5 \times 0 + 1$$
$$= 0 + 1$$
$$= 1$$

- For $x = 1$,
$$y = 0.5 \times 1 + 1$$
$$= 0.5 + 1$$
$$= 1.5$$

- For $x = 2$,
$$y = 0.5 \times 2 + 1$$
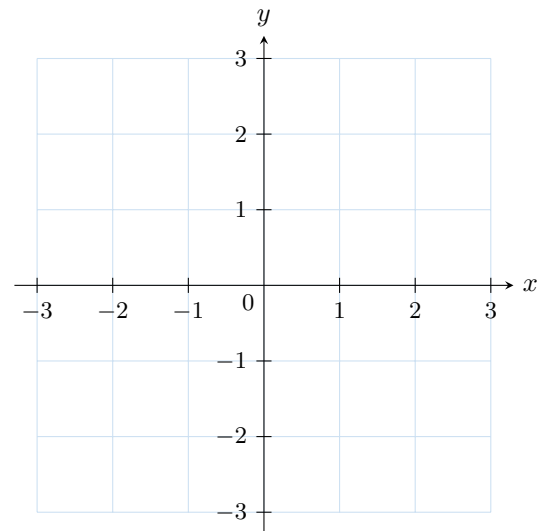$$= 1 + 1$$
$$= 2$$

So the completed table is:

x	-2	-1	0	1	2
y	0	0.5	1	1.5	2

D.2 GRAPHING A LINE FROM TWO POINTS

Ex 25: Here is a table of values for the line equation $y = x - 1$:

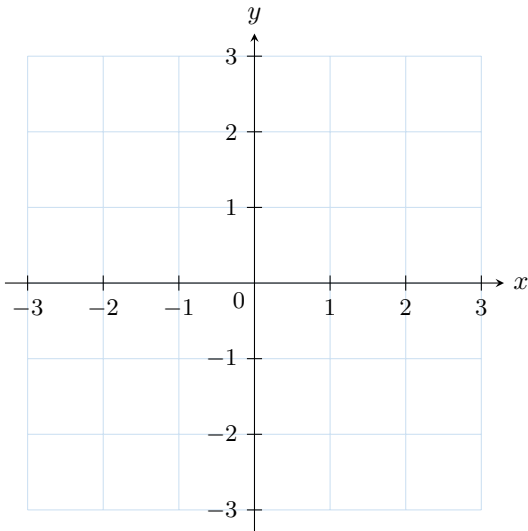| x	0	2
y	-1	1

Plot the line.

Answer: Plot the points $(0, -1)$ and $(2, 1)$. Then, draw the line passing through the two points.
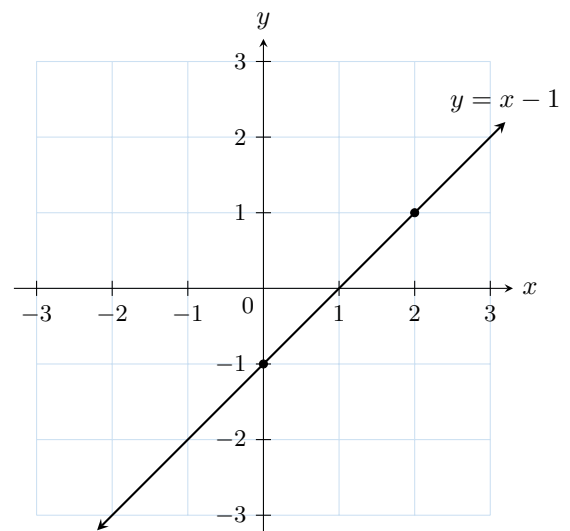
Ex 26: Here is a table of values for the line equation $y = 0.5x + 1$:

x	0	2
y	1	2

Plot the line.

Answer: Plot the points $(0, 1)$ and $(2, 2)$. Then, draw the line passing through the two points.
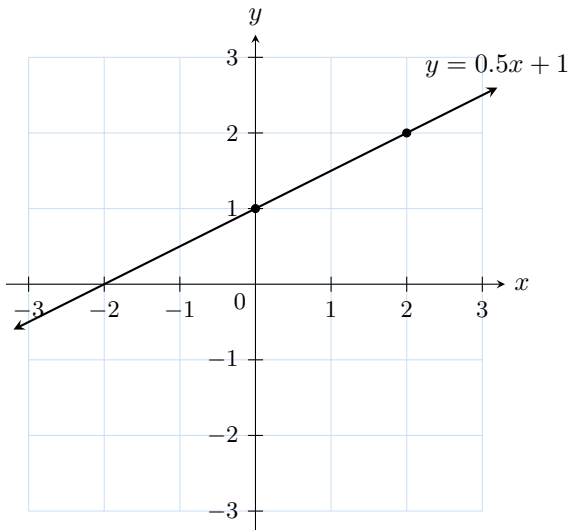
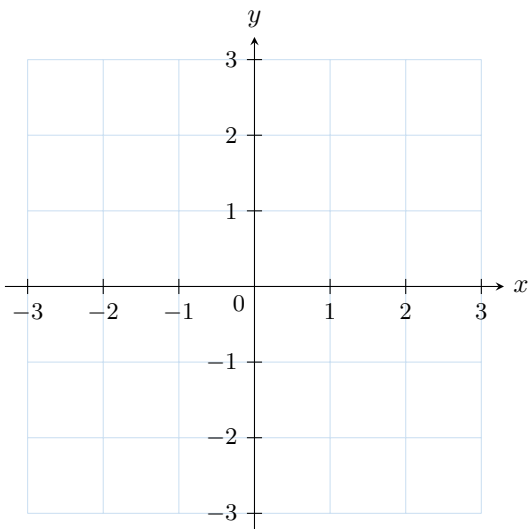**Ex 27:** Here is a table of values for the line equation $y = -2x+2$:

x	0	2
y	2	-2

Plot the line.

Answer: Plot the points $(0, 2)$ and $(2, -2)$. Then, draw the line passing through the two points.
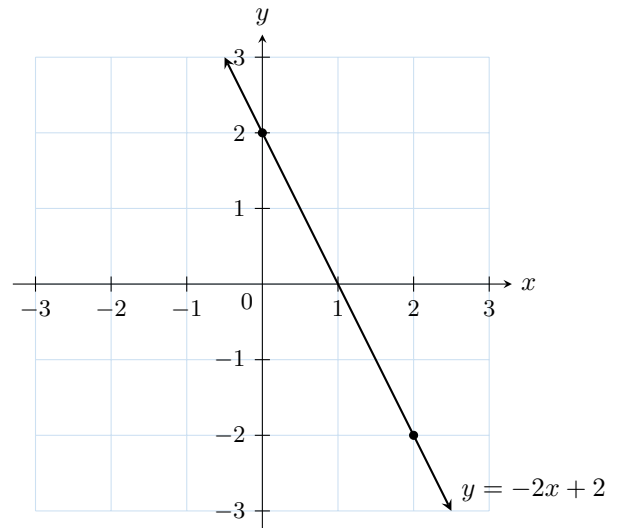
D.3 FINDING COORDINATE POINTS

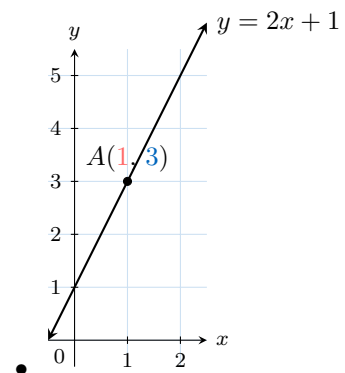**Ex 28:** Find the coordinates of the point A on the line with the equation $y = 2x + 1$:

$$A(1, \boxed{3})$$

Answer:

- For $x = 1$,

$$y = 2 \times 1 + 1$$
$$= 3$$

- The coordinates of A are $A(1, 3)$.

Ex 29: Find the coordinates of the point A on the line with the equation $y = -x + 2$:

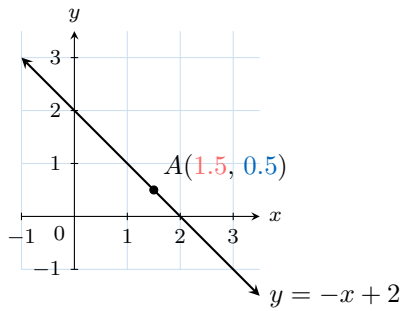$$A(1.5, \boxed{0.5})$$

Answer:

- For $x = 1.5$,

$$y = -1.5 + 2$$
$$= 0.5$$

- The coordinates of A are $A(1.5, 0.5)$.

$y = -x + 2$

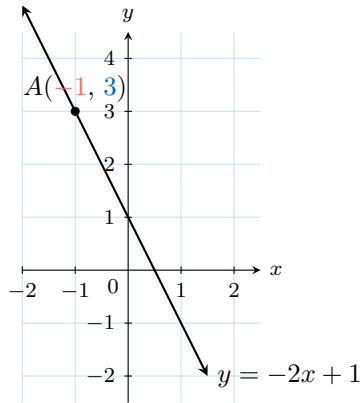**Ex 30:** Find the coordinates of the point A on the line with the equation $y = -2x + 1$:

$$A(-1, \boxed{3})$$

Answer:

- For $x = -1$,
$$y = -2 \times -1 + 1$$
$$= 3$$

- The coordinates of A are $A(-1, 3)$.

$y = -2x + 1$

D.4 DETERMINING WHETHER A POINT IS ON A LINE

MCQ 31: Determine whether the point $(3, 6)$ lies on the line with the equation $y = 2x + 1$.

☐ Yes

☒ No

Answer:

- For $x = 3$ in the equation $y = 2x + 1$:
$$y = 2 \times 3 + 1$$
$$= 7 \neq 6$$

- Therefore, the point $(3, 6)$ does **not** lie on the line.

MCQ 32: Determine whether the point $(4, -3)$ lies on the line with the equation $y = -2x + 5$.

☒ Yes

☐ No

Answer:

- For $x = 4$ in the equation $y = -2x + 5$:
$$y = -2 \times 4 + 5$$
$$= -8 + 5$$
$$= -3$$

- Therefore, the point $(4, -3)$ **does** lie on the line.

MCQ 33: Determine whether the point $(2, 2)$ lies on the line with the equation $y = x - 1$.

☐ Yes

☒ No

Answer:

- For $x = 2$ in the equation $y = x - 1$:
$$y = 2 - 1$$
$$= 1 \neq 2$$

- Therefore, the point $(2, 2)$ does **not** lie on the line.

MCQ 34: Determine whether the point $(0, -2)$ lies on the line with the equation $y = 3x - 2$.

☒ Yes

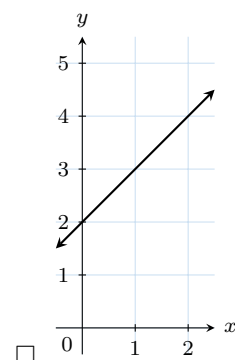☐ No

Answer:

- For $x = 0$ in the equation $y = 3x - 2$:
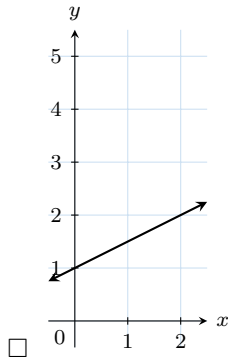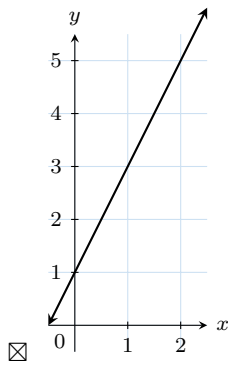$$y = 3 \times 0 - 2$$
$$= 0 - 2$$
$$= -2$$

- Therefore, the point $(0, -2)$ **does** lie on the line.

E GRAPHING LINE EQUATIONS

E.1 MATCHING EQUATIONS AND GRAPHS

MCQ 35: Choose the graph corresponding to the line with the equation $y = 2x + 1$.
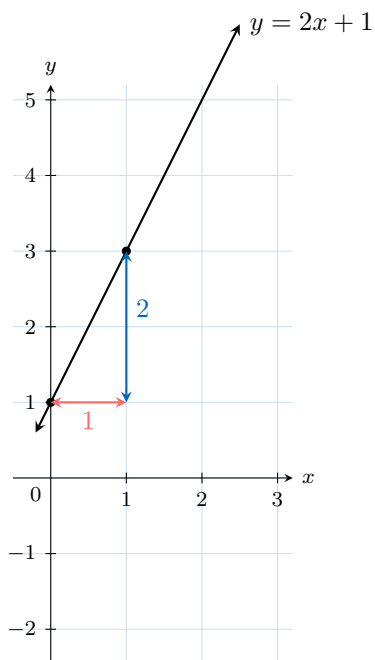

☐

MCQ 36: Choose the graph corresponding to the line with the equation $y = -x + 2$.

Answer:

- **y-Intercept and Slope Method:**
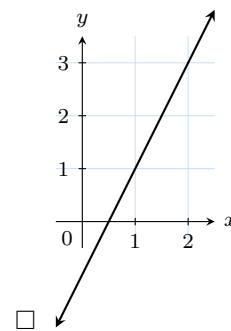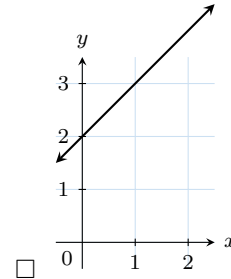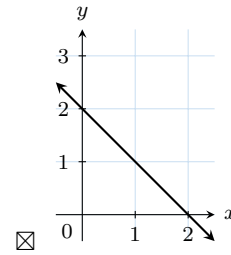   - The y-intercept is 1, so plot the point $(0,1)$.
    - The slope is 2. From $(0,1)$, move 1 unit right ($\Delta x = 1$), then 2 units up ($\Delta y = 2$), to reach $(1,3)$.
    - Draw the line through these points. This matches the **second graph**.

- **Two Points Method:**
    - Choose two values for x (e.g., $x = 0$ and $x = 1$).
    - When $x = 0$, $y = 2 \times 0 + 1 = 1 \rightarrow$ point $(0,1)$.
    - When $x = 1$, $y = 2 \times 1 + 1 = 3 \rightarrow$ point $(1,3)$.
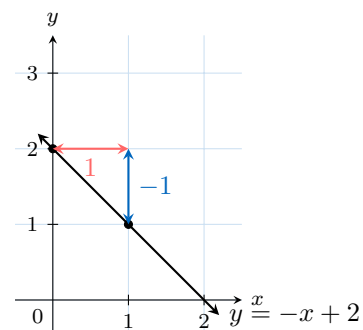   - The correct graph passes through both these points, which is the **second graph**.

Answer: **There are two methods to recognize the correct graph:**

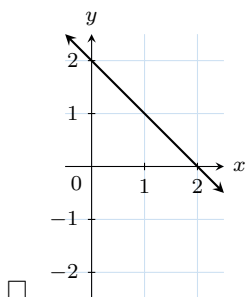- **1. y-Intercept and Slope Method:**
    - The y-intercept is 2, so plot the point $(0,2)$.
    - The slope is -1: from $(0,2)$, move 1 unit right ($\Delta x = 1$), then 1 unit down ($\Delta y = -1$), reaching $(1,1)$.
    - Draw the line through these points. This matches the **first graph**.
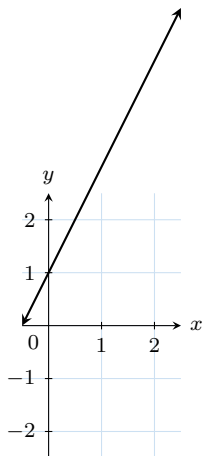
- **2. Two Points Method:**
    - Choose two values for x (e.g., $x = 0$ and $x = 1$).
    - When $x = 0$, $y = -0 + 2 = 2 \rightarrow$ point $(0,2)$.
    - When $x = 1$, $y = -1 + 2 = 1 \rightarrow$ point $(1,1)$.
    - The correct graph passes through both these points, which is the **first graph**.
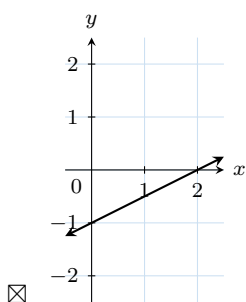

$y = -x + 2$

$y = 2x + 1$

MCQ 37: Choose the graph corresponding to the line with the equation $y = 0.5x - 1$.

□

□

⊠

Answer: **There are two methods to recognize the correct graph:**

- **1. y-Intercept and Slope Method:**

    - The y-intercept is -1: plot the point $(0, -1)$.

    - The slope is 0.5 (or $\frac{1}{2}$): from $(0, -1)$, move 1 unit right ($\Delta x = 1$), and 0.5 units up ($\Delta y = 0.5$), reaching $(1, -0.5)$.

    - Draw the line through these points. This matches the **third graph**.
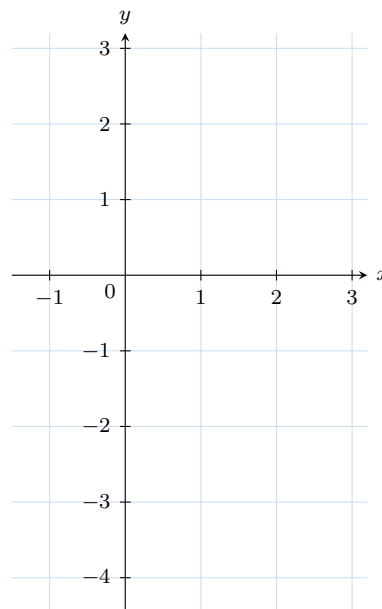
- **2. Two Points Method:**

    - Choose two values for x (e.g., $x = 0$ and $x = 2$).

    - When $x = 0$, $y = 0.5 \times 0 - 1 = -1 \rightarrow$ point $(0, -1)$.

    - When $x = 2$, $y = 0.5 \times 2 - 1 = 1 - 1 = 0 \rightarrow$ point $(2, 0)$.

    - The correct graph passes through $(0, -1)$ and $(2, 0)$, which is the **third graph**.
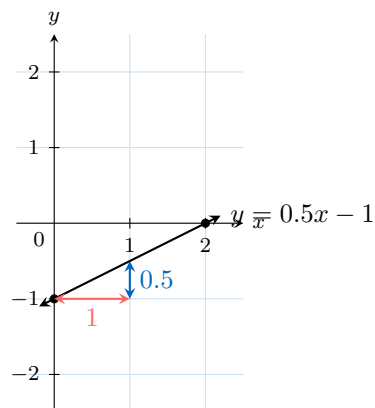
E.2 PLOTTING LINES FROM EQUATIONS
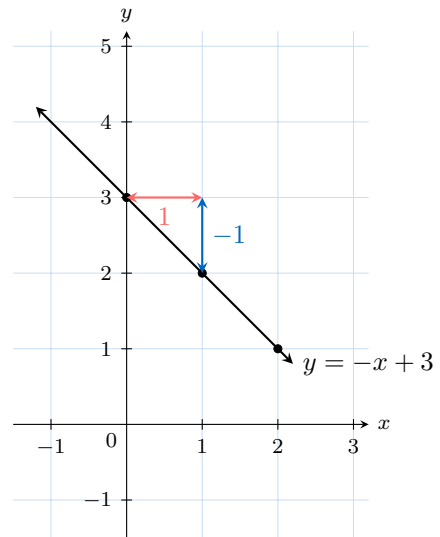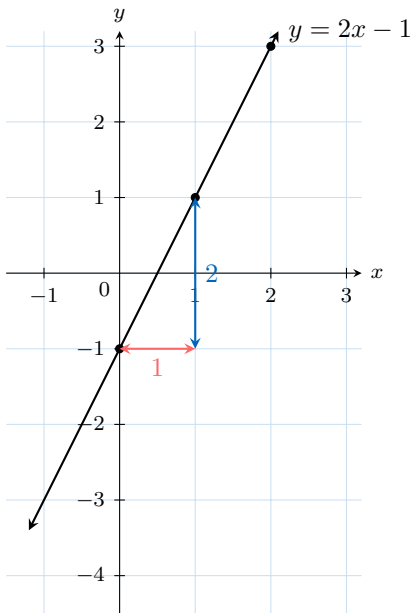
Ex 38: Plot the line equation $y = 2x - 1$:

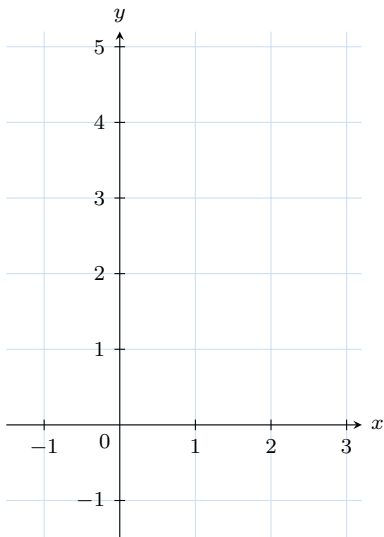*Answer:*

- **y-Intercept and Slope Method:**

    - The y-intercept is -1, so plot the point $(0, -1)$.

    - The slope is 2. From $(0, -1)$, move 1 unit right ($\Delta x = 1$), then 2 units up ($\Delta y = 2$), to reach $(1, 1)$.

- **Two Points Method:**
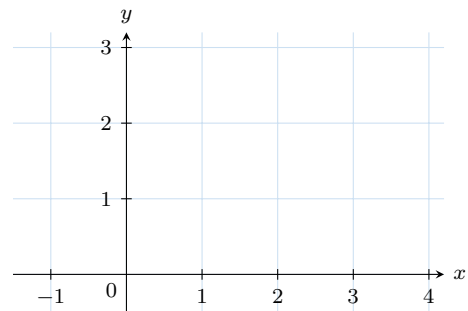
    - Choose two values for x (e.g., $x = 0$ and $x = 2$).

    - When $x = 0$, $y = 2 \times 0 - 1 = -1 \rightarrow$ point $(0, -1)$.

    - When $x = 2$, $y = 2 \times 2 - 1 = 4 - 1 = 3 \rightarrow$ point $(2, 3)$.

$$y = 2x - 1$$

$$y = -x + 3$$

Ex 39: Plot the line equation $y = -x + 3$:

Answer:

- *y*-**Intercept and Slope Method:**

  - The y-intercept is 3, so plot the point $(0, 3)$.

  - The slope is -1. From $(0, 3)$, move 1 unit right ($\Delta x = 1$), then 1 unit down ($\Delta y = -1$), to reach $(1, 2)$.

- **Two Points Method:**

  - Choose two values for x (e.g., $x = 0$ and $x = 2$).

  - When $x = 0$, $y = -0 + 3 = 3 \rightarrow$ point $(0, 3)$.

  - When $x = 2$, $y = -2 + 3 = 1 \rightarrow$ point $(2, 1)$.

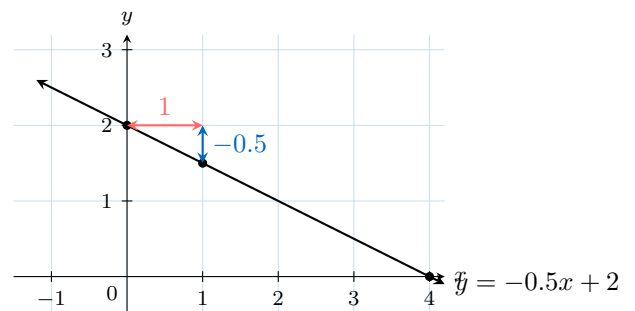**Ex 40:** Plot the line equation $y = -0.5x + 2$:

Answer:

- *y*-**Intercept and Slope Method:**

  - The y-intercept is 2: plot the point $(0, 2)$.

  - The slope is -0.5 (or $-\frac{1}{2}$): from $(0, 2)$, move 1 units right ($\Delta x = 1$), then 0.5 unit down ($\Delta y = -0.5$), to reach $(1, 1.5)$.

- **Two Points Method:**

  - Choose two values for x (e.g., $x = 0$ and $x = 4$).

  - When $x = 0$, $y = -0.5 \times 0 + 2 = 2 \rightarrow$ point $(0, 2)$.

  - When $x = 4$, $y = -0.5 \times 4 + 2 = -2 + 2 = 0 \rightarrow$ point $(4, 0)$.

$$y = -0.5x + 2$$