INTERESTS

A DEFINITIONS

Definition Principal

The **principal** is the original amount of money that is either invested or loaned.

Definition Interest

Interest is the cost paid for borrowing money or the amount earned from lending or investing money.

B SIMPLE INTEREST

Definition Simple Interest

The **simple interest** is calculated each year as a fixed percentage on the principal (original amount) of money borrowed or invested.

Proposition Simple Interest Formula

The simple interest, denoted by I, is calculated as:

$$I = t \times r \times P$$

where:

- P is the principal (original amount)
- r is the interest rate per year
- t is the time (in years)

The final amount, denoted by A, is:

$$\begin{aligned} A &= P + I \\ &= P + t \times r \times P \\ &= (1 + t \times r) \times P \end{aligned}$$

Ex: Find the simple interest on a principal of \$500 at a rate of 3% per year over 5 years.

Solution:

$$\begin{aligned} \text{Interest} &= 5 \times 3\% \text{ of } 500 \\ &= 5 \times \frac{3}{100} \times 500 \\ &= 75 \text{ dollars} \end{aligned}$$

Method Finding the Principal

To find the principal P, use the formula:

$$P = \frac{I}{t \times r}$$

Ex: An investment earns \$100 over 5 years at a rate of 5% per year. Find the principal.

Solution:

$$\begin{aligned} P &= \frac{I}{t \times r} \\ &= \frac{100}{5 \times \frac{5}{100}} \\ &= 400 \text{ dollars} \end{aligned}$$

The principal is \$400.

Method Finding the Rate

To find the rate r, use the formula:

$$r = \frac{I}{t \times P}$$

Ex: A principal of \$500 earns \$60 over 5 years. Find the interest rate per year.

Solution:

$$\begin{aligned} r &= \frac{I}{t \times P} \\ &= \frac{60}{5 \times 500} \\ &= 0.024 \\ &= \frac{2.4}{100} \\ &= 2.4\% \end{aligned}$$

The interest rate per year is 2.4%.

Method Finding the Number of Years

To find the number of years t, use the formula:

$$t = \frac{I}{r \times P}$$

Ex: A principal of \$1 000 earns \$300 at an interest rate of 3% per year. Find the number of years.

Solution:

$$\begin{aligned} t &= \frac{I}{r \times P} \\ &= \frac{300}{\frac{3}{100} \times 1\,000} \\ &= 10 \text{ years} \end{aligned}$$

The number of years is 10.

C COMPOUND INTEREST

Definition Compound Interest

Compound interest is interest that accumulates on both the principal sum and the previously accumulated interest.

Proposition Annual Compound Interest Formula

The final amount of an investment with interest compounded annually is:

$$A = P(1+r)^t$$

where:

- P is the principal,
- r is the annual interest rate,
- t is the time (in years).

Ex: Find the final amount for compound interest on a principal of \$500 at a rate of 3% per year over 5 years.

Solution:

$$\begin{aligned} A &= P(1+r)^t \\ &= 500 \times (1+0.03)^5 \\ &\approx \$580.81 \end{aligned}$$

The final amount is approximately \$580.81.

D COMPOUND INTEREST BY PERIOD

Definition Compounding by Period

Interest can be compounded in various periods, including:

- monthly (12 times per year)
- semi-annually (2 times per year)
- weekly (52 times per year)

Proposition Periodic Compound Interest Formula

The final amount of an investment with interest compounded periodically is:

$$A = P\left(1 + \frac{r}{c_y}\right)^{c_y t}$$

where:

- P is the principal,
- r is the annual interest rate,
- t is the time (in years),
- c_y is the number of times the compound interest is applied per year.

Ex: Calculate the final amount of a $5\,000 principal invested at an annual interest rate of 2%, compounded monthly, over a period of 10 years.

Solution:

$$\begin{aligned} A &= P\left(1 + \frac{r}{c_y}\right)^{c_y t} \\ &= 5\,000\left(1 + \frac{0.02}{12}\right)^{12\times 10} \\ &\approx \$6\,106.71 \end{aligned}$$

The final amount is approximately $6 106.71.

Method Finding Any Value Using a Graphing Calculator

The TVM Solver (Time Value of Money) can be used to find any variable if all the other variables are given.

- PV is the principal, considered as an outgoing, and is entered as a negative value ($PV = -P$).
- FV represents the final amount ($FV = A$).
- C/Y is the number of compounding periods per year ($C/Y = c_y$).
- n represents the number of compounding periods, not the number of years ($n = c_y \times t$).
- PMT and P/Y are not used in this case. $P/Y = C/Y$ and $PMT = 0$.

Ex: Find the final amount on a principal of $23 000 at a rate of 3.45% over 6 years compounded quarterly.

Solution: Using the TVM Solver with:

$$n = 6 \times 4 = 24, \quad I\% = 3.45, \quad PV = -23\,000, \quad C/Y = 4, \quad (PMT = 0, \quad P/Y = 4)$$

we find the final amount:

$$FV \approx \$28\,264.50$$

E VARIABLE RATE INVESTMENTS

Method Calculation for Variable Rate Investments

If the interest rate varies over the term of the investment, separate calculations must be performed for each interest rate period.

Ex: Louis invested \$200 in a variable rate investment account for 8 years. The interest rates were:

- for the first six years: 3% compounded yearly.
- for the last two years: 2% compounded yearly.

Find the final amount after 8 years.

Solution: We can solve this problem using two methods: the compound interest formula and the TVM Solver.

- **Method 1: Using the Compound Interest Formula**
  - First, calculate the amount after the first 6 years at 3% interest compounded yearly:

$$\begin{aligned} A_1 &= P \times (1+r)^{t_1} \\ &= 200 \times (1+0.03)^6 \\ &= 200 \times (1.03)^6 \\ &\approx 200 \times 1.194052 \\ &\approx \$238.81 \end{aligned}$$

  - Next, calculate the amount after the last 2 years at 2% interest compounded yearly, using A_1 as the principal:

$$\begin{aligned} A &= A_1 \times (1+r)^{t_2} \\ &= 238.81 \times (1+0.02)^2 \\ &= 238.81 \times (1.02)^2 \\ &= 238.81 \times 1.0404 \\ &\approx \$248.46 \end{aligned}$$

  - Therefore, the final amount after 8 years is approximately \$248.46.

- **Method 2: Using the TVM Solver**
  - First period (first 6 years at 3% interest compounded yearly):
    - $PV = -200$ (initial investment)
    - $I\% = 3$
    - $n = 6$
    - $PMT = 0$
    - $FV = ?$
    - $P/Y = 1$
    - $C/Y = 1$

    Calculating FV, we get $FV \approx \$238.81$.
  - Second period (next 2 years at 2% interest compounded yearly):
    - $PV = -238.81$
    - $I\% = 2$
    - $n = 2$
    - $PMT = 0$
    - $FV = ?$
    - $P/Y = 1$
    - $C/Y = 1$

    Calculating FV, we get $FV \approx \$248.46$.
  - Thus, the final amount after 8 years is approximately \$248.46.

F PERIODIC PAYMENT

Method Using a Graphing Calculator to Find Any Value

The TVM Solver can be used to find any unknown variable if the other variables are known. This tool is helpful when calculating the periodic payment, principal, or interest rate in financial problems.The TVM Solver works as follows:

- PV (Present Value) is the principal loan amount. It is considered an outgoing and is entered as a negative value ($PV = -P$).
- FV (Future Value) is the final amount, typically zero for loans ($FV = 0$).
- $I\%$ is the annual interest rate.
- n is the total number of payments ($n = c_y \times t$).
- PMT represents the periodic payment (what we aim to find in many cases).
- P/Y represents the number of payment periods per year. For monthly payments, $P/Y = 12$.
- C/Y represents the number of compounding periods per year ($C/Y = c_y$).

By inputting these values, you can use the TVM Solver to compute any unknown variable.

Ex: Maria takes out a loan of \$15 000 to buy a car. The loan has an interest rate of 5.5% per annum, compounded monthly, and she agrees to repay the loan over 5 years.
Calculate the monthly payment Maria needs to make using the TVM Solver on your calculator.

Solution: We will use the TVM Solver to find the monthly payment.

- $PV = -15\,000$ (the loan amount is an outgoing, so it's negative).
- $FV = 0$ (since there is no remaining balance at the end of the loan).
- $I\% = 5.5$ (the annual interest rate).
- $n = 12 \times 5 = 60$ (monthly payments over 5 years).
- $PMT = ?$ (the value we are solving for).
- $P/Y = 12$ (monthly payments).
- $C/Y = 12$ (compounded monthly).

Entering these values into your calculator's TVM Solver, we calculate:

$$PMT \approx \$286.52$$

Therefore, Maria will need to make monthly payments of approximately \$286.52 to repay her car loan over 5 years.