# INTERESTS

## A DEFINITIONS

### A.1 FINDING THE INTEREST

**Ex 1:** Louis lends Hugo 100 dollars. After one year, Hugo repays Louis 110 dollars.
Find the interest paid.

$\boxed{10}$ dollars

*Solution:* The interest paid is the difference between the amount repaid and the original amount lent:

$$\begin{aligned}\text{Interest} &= \text{Amount repaid} - \text{Original amount}\\ &= 110 - 100\\ &= 10 \text{ dollars}\end{aligned}$$

**Ex 2:** Maria borrows 200 dollars from John. After one year, Maria repays John 230 dollars.
Find the interest paid.

$\boxed{30}$ dollars

*Solution:* The interest paid is the difference between the amount repaid and the original amount lent:

$$\begin{aligned}\text{Interest} &= \text{Amount repaid} - \text{Original amount}\\ &= 230 - 200\\ &= 30 \text{ dollars}\end{aligned}$$

**Ex 3:** Jack lends Sarah 500 dollars. After one year, Sarah repays Jack 525 dollars.
Find the interest paid.

$\boxed{25}$ dollars

*Solution:* The interest paid is the difference between the amount repaid and the original amount lent:

$$\begin{aligned}\text{Interest} &= \text{Amount repaid} - \text{Original amount}\\ &= 525 - 500\\ &= 25 \text{ dollars}\end{aligned}$$

**Ex 4:** A bank lends 1 000 dollars to a customer. After one year, the customer repays the bank 1 080 dollars.
Find the interest paid.

$\boxed{80}$ dollars

*Solution:* The interest paid is the difference between the amount repaid and the original amount lent:

$$\begin{aligned}\text{Interest} &= \text{Amount repaid} - \text{Original amount}\\ &= 1\,080 - 1\,000\\ &= 80 \text{ dollars}\end{aligned}$$

### A.2 FINDING THE TOTAL AMOUNT

**Ex 5:** A customer borrows 2 500 dollars from a bank, with 150 dollars of interest.
Find the total amount the customer needs to repay the bank.

$\boxed{2650}$ dollars

*Solution:* The total amount to be repaid is the sum of the original amount borrowed (the principal) and the interest:

$$\begin{aligned}\text{Amount to repay} &= \text{Principal} + \text{Interest}\\ &= 2\,500 + 150\\ &= 2\,650 \text{ dollars}\end{aligned}$$

**Ex 6:** Maria borrows 300 dollars from John with 30 dollars of interest.
Find the amount Maria needs to repay.

$\boxed{330}$ dollars

*Solution:* The total amount to be repaid is the sum of the original amount borrowed (the principal) and the interest:

$$\begin{aligned}\text{Amount to repay} &= \text{Principal} + \text{Interest}\\ &= 300 + 30\\ &= 330 \text{ dollars}\end{aligned}$$

**Ex 7:** Jack lends Sarah 500 dollars with 50 dollars of interest.
Find the total amount Sarah needs to repay Jack.

$\boxed{550}$ dollars

*Solution:* The total amount to be repaid is the sum of the original amount borrowed (the principal) and the interest:

$$\begin{aligned}\text{Amount to repay} &= \text{Principal} + \text{Interest}\\ &= 500 + 50\\ &= 550 \text{ dollars}\end{aligned}$$

**Ex 8:** A bank lends 1 000 dollars to a customer with 80 dollars of interest.
Find the total amount the customer needs to repay the bank.

$\boxed{1080}$ dollars

*Solution:* The total amount to be repaid is the sum of the original amount borrowed (the principal) and the interest:

$$\begin{aligned}\text{Amount to repay} &= \text{Principal} + \text{Interest}\\ &= 1\,000 + 80\\ &= 1\,080 \text{ dollars}\end{aligned}$$

### A.3 FINDING THE PRINCIPAL

**Ex 9:** Emma repaid 330 dollars in total, including 30 dollars of interest.Find the original amount (principal) that Emma borrowed.

$\boxed{300}$ dollars

*Solution:* The principal is the difference between the total amount repaid and the interest paid:

$$\begin{aligned}\text{Principal} &= \text{Amount repaid} - \text{Interest}\\ &= 330 - 30\\ &= 300 \text{ dollars}\end{aligned}$$

**Ex 10:** Lucas repaid 550 dollars in total, including 50 dollars of interest.Find the original amount (principal) that Lucas borrowed.

$\boxed{500}$ dollars

*Solution:* The principal is the difference between the total amount repaid and the interest paid:

$$\begin{aligned}\text{Principal} &= \text{Amount repaid} - \text{Interest}\\ &= 550 - 50\\ &= 500 \text{ dollars}\end{aligned}$$

**Ex 11:** Sophia repaid 1,080 dollars in total, including 80 dollars of interest.Find the original amount (principal) that Sophia borrowed.

$\boxed{1000}$ dollars

*Solution:* The principal is the difference between the total amount repaid and the interest paid:

$$\begin{aligned}\text{Principal} &= \text{Amount repaid} - \text{Interest}\\ &= 1,080 - 80\\ &= 1,000 \text{ dollars}\end{aligned}$$

**Ex 12:** Mia repaid 750 dollars in total, including 150 dollars of interest.Find the original amount (principal) that Mia borrowed.

$\boxed{600}$ dollars

*Solution:* The principal is the difference between the total amount repaid and the interest paid:

$$\begin{aligned}\text{Principal} &= \text{Amount repaid} - \text{Interest}\\ &= 750 - 150\\ &= 600 \text{ dollars}\end{aligned}$$

# B SIMPLE INTEREST

## B.1 FINDING THE INTEREST

**Ex 13:** Find the simple interest on a principal of \$500 at a rate of 3% per year over 5 years (you can use a calculator).

$\boxed{75}$ dollars

*Solution:*

$$\begin{aligned}\text{Interest} &= \text{Number of years} \times \text{Percentage of the principal}\\ &= 5 \times 3\% \text{ of } 500\\ &= 5 \times \frac{3}{100} \times 500\\ &= 75 \text{ dollars}\end{aligned}$$

**Ex 14:** Find the simple interest on a principal of \$1 000 at a rate of 4% per year over 3 years (you can use a calculator).

$\boxed{120}$ dollars

*Solution:*

$$\begin{aligned}\text{Interest} &= \text{Number of years} \times \text{Percentage of the principal}\\ &= 3 \times 4\% \text{ of } 1\,000\\ &= 3 \times \frac{4}{100} \times 1\,000\\ &= 120 \text{ dollars}\end{aligned}$$

**Ex 15:** Find the simple interest on a principal of \$750 at a rate of 5% per year over 2 years (you can use a calculator).

$\boxed{75}$ dollars

*Solution:*

$$\begin{aligned}\text{Interest} &= \text{Number of years} \times \text{Percentage of the principal}\\ &= 2 \times 5\% \text{ of } 750\\ &= 2 \times \frac{5}{100} \times 750\\ &= 75 \text{ dollars}\end{aligned}$$

**Ex 16:** Find the simple interest on a principal of \$1 200 at a rate of 6% per year over 4 years (you can use a calculator).

$\boxed{288}$ dollars

*Solution:*

$$\begin{aligned}\text{Interest} &= \text{Number of years} \times \text{Percentage of the principal}\\ &= 4 \times 6\% \text{ of } 1\,200\\ &= 4 \times \frac{6}{100} \times 1\,200\\ &= 288 \text{ dollars}\end{aligned}$$

## B.2 FINDING THE INTEREST OVER MIXED TIME PERIODS

**Ex 17:** Find the simple interest on a principal of \$600 at a rate of 4% per year over 18 months (you can use a calculator).

$\boxed{36}$ dollars

*Solution:*

- Convert the time from months to years:

$$\begin{aligned}18 \text{ months} &= \frac{18}{12} \text{ years}\\ &= 1.5 \text{ years}\end{aligned}$$

- Calculate the interest:

$$\begin{aligned}\text{Interest} &= \text{Number of years} \times \text{Percentage of the principal}\\ &= 1.5 \times 4\% \text{ of } 600\\ &= 1.5 \times \frac{4}{100} \times 600\\ &= 36 \text{ dollars}\end{aligned}$$

**Ex 18:** Find the simple interest on a principal of \$700 at a rate of 5% per year over 180 days (you can use a calculator).

$\boxed{17.26}$ dollars (round at two decimal place)

*Solution:*

- Convert the time from days to years:

$$\begin{aligned} 180 \text{ days} &= \frac{180}{365} \text{ years} \\ &\approx 0.493 \text{ years} \end{aligned}$$

- Calculate the interest:

$$\begin{aligned} \text{Interest} &= \text{Number of years} \times \text{Percentage of the principal} \\ &= 0.493 \times 5\% \text{ of } 700 \\ &= 0.493 \times \frac{5}{100} \times 700 \\ &= 17.26 \text{ dollars} \end{aligned}$$

**Ex 19:** Find the simple interest on a principal of \$800 at a rate of 4% per year over 9 months (you can use a calculator).

$\boxed{24}$ dollars

*Solution:*

- Convert the time from months to years:

$$\begin{aligned} 9 \text{ months} &= \frac{9}{12} \text{ years} \\ &= 0.75 \text{ years} \end{aligned}$$

- Calculate the interest:

$$\begin{aligned} \text{Interest} &= \text{Number of years} \times \text{Percentage of the principal} \\ &= 0.75 \times 4\% \text{ of } 800 \\ &= 0.75 \times \frac{4}{100} \times 800 \\ &= 24 \text{ dollars} \end{aligned}$$

**Ex 20:** Find the simple interest on a principal of \$1 200 at a rate of 4% per year over 2 years and 6 months (you can use a calculator).

$\boxed{120}$ dollars

*Solution:*

- Convert the time from years and months to just years:

$$\begin{aligned} 2 \text{ years } 6 \text{ months} &= 2 + \frac{6}{12} \text{ years} \\ &= 2 + 0.5 \text{ years} \\ &= 2.5 \text{ years} \end{aligned}$$

- Calculate the interest:

$$\begin{aligned} \text{Interest} &= \text{Number of years} \times \text{Percentage of the principal} \\ &= 2.5 \times 4\% \text{ of } 1\,200 \\ &= 2.5 \times \frac{4}{100} \times 1\,200 \\ &= 120 \text{ dollars} \end{aligned}$$

## B.3 FINDING THE TOTAL AMOUNT

**Ex 21:** Jack lends Sarah 500 dollars with simple interest over 3 years at a rate of 3% per year.
Find the total amount Sarah needs to repay Jack (you can use a calculator).

$\boxed{545}$ dollars

*Solution:*

- The total amount to be repaid is the sum of the original amount borrowed (the principal) and the interest.
- Calculate the interest

$$\begin{aligned} \text{Interest} &= \text{Number of years} \times \text{Percentage of the principal} \\ &= 3 \times \frac{3}{100} \times 500 \\ &= 45 \text{ dollars} \end{aligned}$$

- Calculate the total amount to repay:

$$\begin{aligned} \text{Amount to repay} &= \text{Principal} + \text{Interest} \\ &= 500 + 45 \\ &= 545 \text{ dollars} \end{aligned}$$

**Ex 22:** Emma borrows 600 dollars from a bank with simple interest over 4 years at a rate of 2.5% per year.
Find the total amount Emma needs to repay the bank (you can use a calculator).

$\boxed{660}$ dollars

*Solution:*

- The total amount to be repaid is the sum of the original amount borrowed (the principal) and the interest.
- Calculate the interest

$$\begin{aligned} \text{Interest} &= \text{Number of years} \times \text{Percentage of the principal} \\ &= 4 \times \frac{2.5}{100} \times 600 \\ &= 60 \text{ dollars} \end{aligned}$$

- Calculate the total amount to repay:

$$\begin{aligned} \text{Amount to repay} &= \text{Principal} + \text{Interest} \\ &= 600 + 60 \\ &= 660 \text{ dollars} \end{aligned}$$

**Ex 23:** Michael lends 800 dollars to a friend with simple interest over 2 years at a rate of 4% per year.
Find the total amount the friend needs to repay Michael (you can use a calculator).

$\boxed{864}$ dollars

*Solution:*

- The total amount to be repaid is the sum of the original amount borrowed (the principal) and the interest.

- Calculate the interest

$$\begin{aligned}\text{Interest} &= \text{Number of years} \times \text{Percentage of the principal}\\ &= 2 \times \frac{4}{100} \times 800\\ &= 64 \text{ dollars}\end{aligned}$$

- Calculate the total amount to repay:

$$\begin{aligned}\text{Amount to repay} &= \text{Principal} + \text{Interest}\\ &= 800 + 64\\ &= 864 \text{ dollars}\end{aligned}$$

**Ex 24:** Sophia borrows 1 200 dollars with simple interest over 5 years at a rate of 2.5% per year.
Find the total amount Sophia needs to repay (you can use a calculator).

$\boxed{1350}$ dollars

*Solution:*

- The total amount to be repaid is the sum of the original amount borrowed (the principal) and the interest.
- Calculate the interest

$$\begin{aligned}\text{Interest} &= \text{Number of years} \times \text{Percentage of the principal}\\ &= 5 \times \frac{2.5}{100} \times 1\,200\\ &= 150 \text{ dollars}\end{aligned}$$

- Calculate the total amount to repay:

$$\begin{aligned}\text{Amount to repay} &= \text{Principal} + \text{Interest}\\ &= 1\,200 + 150\\ &= 1\,350 \text{ dollars}\end{aligned}$$

## B.4 FINDING THE PRINCIPAL

**Ex 25:** Find the original amount invested if a flat rate of 4% per year produces \$1 800 interest in 5 years (you can use a calculator).

$\boxed{9000}$ dollars

*Solution:* We can use the simple interest formula to find the original amount (principal):

$$\text{Interest} = \text{Time} \times \text{Rate} \times \text{Principal}$$

Rearranging to solve for the principal:

$$\text{Principal} = \frac{\text{Interest}}{\text{Time} \times \text{Rate}}$$

Substituting the values:

$$\begin{aligned}\text{Principal} &= \frac{1800}{5 \times \frac{4}{100}}\\ &= \frac{1800}{5 \times 0.04}\\ &= \frac{1800}{0.2}\\ &= 9\,000 \text{ dollars}\end{aligned}$$

**Ex 26:** Find the original amount invested if a flat rate of 5% per year produces \$2 500 interest in 10 years (you can use a calculator).

$\boxed{5000}$ dollars

*Solution:* We can use the simple interest formula to find the original amount (principal):

$$\text{Interest} = \text{Time} \times \text{Rate} \times \text{Principal}$$

Rearranging to solve for the principal:

$$\text{Principal} = \frac{\text{Interest}}{\text{Time} \times \text{Rate}}$$

Substituting the values:

$$\begin{aligned}\text{Principal} &= \frac{2\,500}{10 \times \frac{5}{100}}\\ &= \frac{2\,500}{10 \times 0.05}\\ &= \frac{2\,500}{0.5}\\ &= 5\,000 \text{ dollars}\end{aligned}$$

**Ex 27:** Find the original amount invested if a flat rate of 6% per year produces \$720 interest in 4 years (you can use a calculator).

$\boxed{3000}$ dollars

*Solution:* We can use the simple interest formula to find the original amount (principal):

$$\text{Interest} = \text{Time} \times \text{Rate} \times \text{Principal}$$

Rearranging to solve for the principal:

$$\text{Principal} = \frac{\text{Interest}}{\text{Time} \times \text{Rate}}$$

Substituting the values:

$$\begin{aligned}\text{Principal} &= \frac{720}{4 \times \frac{6}{100}}\\ &= \frac{720}{4 \times 0.06}\\ &= \frac{720}{0.24}\\ &= 3\,000 \text{ dollars}\end{aligned}$$

**Ex 28:** Find the original amount invested if a flat rate of 5% per year produces \$1 250 interest in 2 years (you can use a calculator).

$\boxed{12500}$ dollars

*Solution:* We can use the simple interest formula to find the original amount (principal):

$$\text{Interest} = \text{Time} \times \text{Rate} \times \text{Principal}$$

Rearranging to solve for the principal:

$$\text{Principal} = \frac{\text{Interest}}{\text{Time} \times \text{Rate}}$$

Substituting the values:

$$\begin{aligned}\text{Principal} &= \frac{1\,250}{2 \times \frac{5}{100}}\\ &= \frac{1\,250}{2 \times 0.05}\\ &= \frac{1\,250}{0.1}\\ &= 12\,500 \text{ dollars}\end{aligned}$$

## B.5 FINDING THE INTEREST RATE

**Ex 29:** Find the interest rate per year if an original investment of \$8 000 earns \$960 in interest over 3 years (you can use a calculator).

$\boxed{4}\%$

*Solution:* We can use the simple interest formula to find the interest rate:

$$\text{Interest} = \text{Time} \times \text{Rate} \times \text{Principal}$$

Rearranging to solve for the rate:

$$\text{Rate} = \frac{\text{Interest}}{\text{Time} \times \text{Principal}}$$

Substituting the values:

$$\begin{aligned}\text{Rate} &= \frac{960}{3 \times 8\,000}\\ &= \frac{960}{24\,000}\\ &= 0.04\\ &= 4\%\end{aligned}$$

**Ex 30:** Find the interest rate per year if an original investment of \$5 000 earns \$600 in interest over 4 years (you can use a calculator).

$\boxed{3}\%$

*Solution:* We can use the simple interest formula to find the interest rate:

$$\text{Interest} = \text{Time} \times \text{Rate} \times \text{Principal}$$

Rearranging to solve for the rate:

$$\text{Rate} = \frac{\text{Interest}}{\text{Time} \times \text{Principal}}$$

Substituting the values:

$$\begin{aligned}\text{Rate} &= \frac{600}{4 \times 5\,000}\\ &= \frac{600}{20\,000}\\ &= 0.03\\ &= 3\%\end{aligned}$$

**Ex 31:** Find the interest rate per year if an original investment of \$7 500 earns \$900 in interest over 5 years (you can use a calculator).

$\boxed{2.4}\%$

*Solution:* We can use the simple interest formula to find the interest rate:

$$\text{Interest} = \text{Time} \times \text{Rate} \times \text{Principal}$$

Rearranging to solve for the rate:

$$\text{Rate} = \frac{\text{Interest}}{\text{Time} \times \text{Principal}}$$

Substituting the values:

$$\begin{aligned}\text{Rate} &= \frac{900}{5 \times 7\,500}\\ &= \frac{900}{37\,500}\\ &= 0.024\\ &= 2.4\%\end{aligned}$$

**Ex 32:** Find the interest rate per year if an original investment of \$10 000 earns \$1 200 in interest over 4 years (you can use a calculator).

$\boxed{3}\%$

*Solution:* We can use the simple interest formula to find the interest rate:

$$\text{Interest} = \text{Time} \times \text{Rate} \times \text{Principal}$$

Rearranging to solve for the rate:

$$\text{Rate} = \frac{\text{Interest}}{\text{Time} \times \text{Principal}}$$

Substituting the values:

$$\begin{aligned}\text{Rate} &= \frac{1\,200}{4 \times 10\,000}\\ &= \frac{1\,200}{40\,000}\\ &= 0.03\\ &= 3\%\end{aligned}$$

## B.6 FINDING THE TIME

**Ex 33:** Find the time required for an original investment of \$6 000 to earn \$720 in interest at an interest rate of 4% per year (you can use a calculator).

$\boxed{3}$ years

*Solution:* We can use the simple interest formula to find the time:

$$\text{Interest} = \text{Time} \times \text{Rate} \times \text{Principal}$$

Rearranging to solve for the time:

$$\text{Time} = \frac{\text{Interest}}{\text{Rate} \times \text{Principal}}$$

Substituting the values:

$$\begin{aligned}\text{Time} &= \frac{720}{0.04 \times 6\,000}\\ &= \frac{720}{240}\\ &= 3 \text{ years}\end{aligned}$$

**Ex 34:** Find the time required for an original investment of \$4 500 to earn \$540 in interest at an interest rate of 3% per year (you can use a calculator).

$\boxed{4}$ years

*Solution:* We can use the simple interest formula to find the time:

$$\text{Interest} = \text{Time} \times \text{Rate} \times \text{Principal}$$

Rearranging to solve for the time:

$$\text{Time} = \frac{\text{Interest}}{\text{Rate} \times \text{Principal}}$$

Substituting the values:

$$\begin{aligned}\text{Time} &= \frac{540}{0.03 \times 4\,500} \\ &= \frac{540}{135} \\ &= 4 \text{ years}\end{aligned}$$

**Ex 35:** Find the time required for an original investment of \$2 500 to earn \$375 in interest at an interest rate of 5% per year (you can use a calculator).

$\boxed{3}$ years

*Solution:* We can use the simple interest formula to find the time:

$$\text{Interest} = \text{Time} \times \text{Rate} \times \text{Principal}$$

Rearranging to solve for the time:

$$\text{Time} = \frac{\text{Interest}}{\text{Rate} \times \text{Principal}}$$

Substituting the values:

$$\begin{aligned}\text{Time} &= \frac{375}{0.05 \times 2\,500} \\ &= \frac{375}{125} \\ &= 3 \text{ years}\end{aligned}$$

**Ex 36:** Find the time required for an original investment of \$7 000 to earn \$840 in interest at an interest rate of 4% per year (you can use a calculator).

$\boxed{3}$ years

*Solution:* We can use the simple interest formula to find the time:

$$\text{Interest} = \text{Time} \times \text{Rate} \times \text{Principal}$$

Rearranging to solve for the time:

$$\text{Time} = \frac{\text{Interest}}{\text{Rate} \times \text{Principal}}$$

Substituting the values:

$$\begin{aligned}\text{Time} &= \frac{840}{0.04 \times 7\,000} \\ &= \frac{840}{280} \\ &= 3 \text{ years}\end{aligned}$$

## C COMPOUND INTEREST

### C.1 FINDING THE TOTAL AMOUNT USING A TABLE

**Ex 37:** \$1000 is placed in an account that earns 10% interest per annum (p.a.), and the interest is allowed to compound over three years. This means the account is earning 10% p.a. in compound interest.
Fill the compound interest table (you can use a calculator).

| Year | Amount | Compound interest |
|---|---|---|
| 0 | \$1000 | 10% of \$1000 = \$100 |
| 1 | \$1000 + \$100 = \$1100 | 10% of \$1100 = \$110 |
| 2 | \$ $\boxed{1210}$ | $\boxed{121}$ |
| 3 | \$ $\boxed{1331}$ | |

Find the amount at 3 years.

$\boxed{1331}$ dollars

*Solution:*

| Year | Amount | Compound interest |
|---|---|---|
| 0 | \$1000 | 10% of \$1000 = \$100 |
| 1 | \$1000 + \$100 = \$1100 | 10% of \$1100 = \$110 |
| 2 | \$1100 + \$110 = \$1210 | 10% of \$1210 = \$121 |
| 3 | \$1210 + \$121 = \$1331 | |

The amount at 3 years is 1331 dollars.

**Ex 38:** \$3 000 is placed in an account that earns 20% interest per annum (p.a.), and the interest is allowed to compound over three years. This means the account is earning 20% p.a. in compound interest.
Fill the compound interest table (you can use a calculator).

| Year | Amount | Compound interest |
|---|---|---|
| 0 | \$3 000 | 20% of \$3 000 = \$600 |
| 1 | \$3 000 + \$600 = \$3 600 | 20% of \$3 600 = \$720 |
| 2 | \$ $\boxed{4320}$ | $\boxed{864}$ |
| 3 | \$ $\boxed{5184}$ | |

Find the amount at 3 years.

$\boxed{5184}$ dollars

*Solution:*

| Year | Amount | Compound interest |
|---|---|---|
| 0 | \$3 000 | 20% de \$3 000 = \$600 |
| 1 | \$3 000 + \$600 = \$3 600 | 20% de \$3 600 = \$720 |
| 2 | \$3 600 + \$720 = \$4 320 | 20% de \$4 320 = \$864 |
| 3 | \$4 320 + \$864 = \$5 184 | |

The amount at 3 years is 5 184 dollars.

**Ex 39:** \$3 000 is placed in an account that earns 20% interest per annum (p.a.), and the interest is allowed to compound over three years. This means the account is earning 20% p.a. in compound interest.
Fill the compound interest table (you can use a calculator).

| Year | Amount | Compound interest |
|---|---|---|
| 0 | \$3 000 | $\boxed{600}$ |
| 1 | \$ $\boxed{3600}$ | $\boxed{720}$ |
| 2 | \$ $\boxed{4320}$ | |

Find the amount after 2 years.

$\boxed{4320}$ dollars

*Solution:*

| Year | Amount | Compound interest |
|---|---|---|
| 0 | \$3 000 | 20% of \$3 000 = \$600 |
| 1 | \$3 000 + \$600 = \$3 600 | 20% of \$3 600 = \$720 |
| 2 | \$3 600 + \$720 = \$4 320 | 20% of \$4 320 = \$864 |

The amount at 3 years is 4 320 dollars.

## C.2 FINDING THE TOTAL AMOUNT

**Ex 40:** Find the final amount on a principal of \$10 000 at a rate of 10% per year over 3 years compounded yearly (you can use a calculator).

$\boxed{13310}$ dollars

*Solution:*

- **Method 1: Amount over year**
  - Year 0: Initial amount = \$10 000
  - Year 1: \$10 000 + 10% of 10 000 = \$11 000
  - Year 2: \$11 000 + 10% of 11 000 = \$12 100
  - Year 3: \$12 100 + 10% of 12 100 = \$13 310

  So, the final amount after 3 years is \$13 310.
- **Method 2: Using the compound interest Formula**

$$\begin{aligned} A &= (1+r)^t P \\ &= (1+0.10)^3\, 10\,000 \quad \text{(substituting the values)} \\ &= 13\,310 \end{aligned}$$

  Thus, the final amount after 3 years is \$13 310.

**Ex 41:** Find the final amount on a principal of \$200 000 at a rate of 5% per year over 3 years compounded yearly (you can use a calculator).

$\boxed{231525}$ dollars

*Solution:*

- **Method 1: Amount over year**
  - Year 0: Initial amount = \$200 000
  - Year 1: \$200 000 + 5% of 200 000 = \$210 000
  - Year 2: \$210 000 + 5% of 210 000 = \$220 500
  - Year 3: \$220 500 + 5% of 220 500 = \$231 525

  So, the final amount after 3 years is \$231 525.
- **Method 2: Using the compound interest Formula**

$$\begin{aligned} A &= (1+r)^t P \\ &= (1+0.05)^3\, 200\,000 \quad \text{(substituting the values)} \\ &= 231\,525 \end{aligned}$$

  Thus, the final amount after 3 years is \$231 525.

**Ex 42:** Find the final amount on a principal of \$5 000 at a rate of 8% per year over 2 years compounded yearly (you can use a calculator).

$\boxed{5\,832}$ dollars

*Solution:*

- **Method 1: Amount over year**
  - Year 0: Initial amount = \$5 000
  - Year 1: \$5 000 + 8% of 5 000 = \$5 400
  - Year 2: \$5 400 + 8% of 5 400 = \$5 832

  So, the final amount after 2 years is \$5 832.
- **Method 2: Using the Compound Interest Formula**

$$\begin{aligned} A &= (1+r)^t P \\ &= (1+0.08)^2\, 5\,000 \quad \text{(substituting the values)} \\ &= 5\,832 \end{aligned}$$

  Thus, the final amount after 2 years is \$5 832.

**Ex 43:** Find the final amount on a principal of \$5 000 at a rate of 8% per year over 20 years compounded yearly (round at 2 decimal places).

$\boxed{23304.79}$ dollars

*Solution:* Using the compound interest formula,

$$\begin{aligned} A &= (1+r)^t P \\ &= (1+0.08)^{20}\, 5\,000 \quad \text{(substituting the values)} \\ &\approx 23\,304.79 \text{ dollars} \end{aligned}$$

Thus, the final amount after 20 years is \$23 304.79.

## C.3 FINDING THE BEST OPTION OF INVESTMENT

**Ex 44:** You have \$8000 to invest for 5 years and there are 2 possible options you have been offered:

- Option 1: Invest at 9% p.a. simple interest.
- Option 2: Invest at 8% p.a. compound interest.

You can use a calculator.

- Calculate the amount accumulated at the end of the 3 years for option 1 (round to the neareast integer)

$\boxed{11600}$ dollars

- Calculate the amount accumulated at the end of the 3 years for option 2 (round to the neareast integer)

$\boxed{11755}$ dollars

- Decide which option to take.
  $\boxed{\textbf{Option 2}}$

*Solution:*

- **Option 1: Simple Interest**For option 1, we substitute the values in the formula for simple interest:

$$\begin{aligned} A &= (1 + t \times r) \times P \\ &= (1 + 5 \times 0.09) \times 8\,000 \\ &= 11\,600 \text{ dollars} \end{aligned}$$

  Thus, the amount accumulated for option 1 after 5 years is \$11 600.

- **Option 2: Compound Interest**
  For option 2, we substitute the value in the formula for compound interest

$$\begin{aligned} A &= (1 + r)^t P \\ &= (1 + 0.08)^5 \times 8\,000 \\ &\approx 11\,755 \text{ dollars} \end{aligned}$$

  Thus, the amount accumulated for option 2 after 5 years is \$11 754.

- **Conclusion**
  Comparing the two options, we see that:
  - Option 1 (simple interest) gives \$11 600,
  - Option 2 (compound interest) gives \$11 755.

  Since option 2 gives a higher final amount, it would be better to choose option 2 with compound interest.

**Ex 45:** You have \$20 000 to invest for 5 years and there are 2 possible options you have been offered:

- Option 1: Invest at 7% p.a. simple interest.
- Option 2: Invest at 6% p.a. compound interest.

You can use a calculator.

- Calculate the amount accumulated at the end of 5 years for option 1 (round to the nearest integer):

  27000 dollars

- Calculate the amount accumulated at the end of 5 years for option 2 (round to the nearest integer):

  26764 dollars

- Decide which option to take.
  **Option 1**

*Solution:*

- **Option 1: Simple Interest**
  For option 1, we substitute the values in the formula for simple interest:

$$\begin{aligned} A &= (1 + t \times r) \times P \\ &= (1 + 5 \times 0.07) \times 20\,000 \\ &= (1 + 0.35) \times 20\,000 \\ &= 1.35 \times 20\,000 \\ &= 27\,000 \text{ dollars} \end{aligned}$$

  Thus, the amount accumulated for option 1 after 5 years is \$27 000.

- **Option 2: Compound Interest**
  For option 2, we substitute the values in the formula for compound interest:

$$\begin{aligned} A &= (1 + r)^t \times P \\ &= (1 + 0.06)^5 \times 20\,000 \\ &= (1.06)^5 \times 20\,000 \\ &\approx 26\,764 \text{ dollars} \end{aligned}$$

  Thus, the amount accumulated for option 2 after 5 years is \$26 744.

- **Conclusion**
  Comparing the two options:
  - Option 1 (simple interest) gives \$27 000,
  - Option 2 (compound interest) gives \$26 764.

  Since option 1 gives a higher final amount, it is better to choose option 1 with simple interest.

**Ex 46:** You have \$50 000 to invest for 30 years and there are 2 possible options you have been offered:

- Option 1: Invest at 10% p.a. simple interest.
- Option 2: Invest at 9% p.a. compound interest.

You can use a calculator.

- Calculate the amount accumulated at the end of the 30 years for option 1 (round to the nearest integer):

  200000 dollars

- Calculate the amount accumulated at the end of the 30 years for option 2 (round to the nearest integer):

  663384 dollars

- Decide which option to take.
  **Option 2**

*Solution:*

- **Option 1: Simple Interest**
  For option 1, we substitute the values in the formula for simple interest:

$$\begin{aligned} A &= (1 + t \times r) \times P \\ &= (1 + 30 \times 0.10) \times 50\,000 \\ &= (1 + 3.0) \times 50\,000 \\ &= 4.0 \times 50\,000 \\ &= 200\,000 \text{ dollars} \end{aligned}$$

  Thus, the amount accumulated for option 1 after 30 years is \$200 000.

- **Option 2: Compound Interest**
  For option 2, we substitute the values in the formula for compound interest:

$$\begin{aligned} A &= (1 + r)^t \times P \\ &= (1 + 0.09)^{30} \times 50\,000 \\ &= (1.09)^{30} \times 50\,000 \\ &\approx 663\,384 \text{ dollars} \end{aligned}$$

  Thus, the amount accumulated for option 2 after 30 years is \$663 384.

- **Conclusion**
  Comparing the two options:
  - Option 1 (simple interest) gives \$200 000,
  - Option 2 (compound interest) gives \$663 384.

  Since option 2 gives a much higher final amount, it is better to choose option 2 with compound interest.

# D COMPOUND INTEREST BY PERIOD

## D.1 FINDING THE FINAL AMOUNT

**Ex 47:** Find the final amount for compound interest on a principal of \$5 000 at a rate of 2% over 10 years, compounded monthly (you can use a calculator).

6106 dollars (round to the nearest integer)

*Solution:*

- Method 1: Using the formula

$$\begin{aligned} A &= \left(1+\frac{r}{c_y}\right)^{c_y t} P \\ &= \left(1+\frac{0.02}{12}\right)^{12\times 10} \times 5\,000 \\ &\approx \$6\,106 \end{aligned}$$

  The final amount is \$6 106.

- Method 2: Using the TVM Solver
  $n = 12\times 10 = 120$, $I\% = 2$, $PV = -5\,000$, $C/Y = 12$, $(PMT = 0,\quad P/Y = 12)$
  The final amount is $FV \approx 6\,106$.

**Ex 48:** Find the final amount for compound interest on a principal of \$10 000 at a rate of 3.5% over 8 years, compounded quarterly (you can use a calculator).

13215 dollars (round to the nearest integer)

*Solution:*

- Method 1: Using the formula

$$\begin{aligned} A &= \left(1+\frac{r}{c_y}\right)^{c_y t} P \\ &= \left(1+\frac{0.035}{4}\right)^{4\times 8} \times 10\,000 \\ &\approx \$13\,215 \end{aligned}$$

  The final amount is \$13 215.

- Method 2: Using the TVM Solver
  $n = 4\times 8 = 32$, $I\% = 3.5$, $PV = -10\,000$, $C/Y = 4$, $(PMT = 0,\quad P/Y = 4)$
  The final amount is $FV \approx 113\,215$.

**Ex 49:** Find the final amount for compound interest on a principal of \$15 000 at a rate of 4% over 5 years, compounded semi-annually (you can use a calculator).

18249 dollars (round to the nearest integer)

*Solution:*

- Method 1: Using the formula

$$\begin{aligned} A &= \left(1+\frac{r}{c_y}\right)^{c_y t} P \\ &= \left(1+\frac{0.04}{2}\right)^{2\times 5} \times 15\,000 \\ &\approx \$18\,249 \end{aligned}$$

  The final amount is \$18 249.

- Method 2: Using the TVM Solver
  $n = 2\times 5 = 10$, $I\% = 4$, $PV = -15\,000$, $C/Y = 2$, $(PMT = 0,\quad P/Y = 2)$
  The final amount is $FV \approx 18\,249$.

**Ex 50:** Find the final amount for compound interest on a principal of \$7 500 at a rate of 5% over 12 years, compounded annually (you can use a calculator).

13469 dollars (round to the nearest integer)

*Solution:*

- Method 1: Using the formula

$$\begin{aligned} A &= \left(1+\frac{r}{c_y}\right)^{c_y t} P \\ &= \left(1+\frac{0.05}{1}\right)^{1\times 12} \times 7\,500 \\ &\approx \$13\,469 \end{aligned}$$

  The final amount is \$13 469.

- Method 2: Using the TVM Solver
  $n = 12$, $I\% = 5$, $PV = -7\,500$, $C/Y = 1$, $(PMT = 0,\quad P/Y = 1)$
  The final amount is $FV \approx 13\,509$.

## D.2 FINDING THE INTEREST

**Ex 51:** Khesya invested \$6 000 in an account paying 5.4% per annum interest, compounded half-yearly for 5 years (you can use a calculator).

- Find the future value of the investment.

  7832 dollars (round to the nearest integer)

- How much interest did Khesya earn?

  1832 dollars

*Solution:*

- - Method 1: Using the formula

$$\begin{aligned} A &= \left(1+\frac{r}{c_y}\right)^{c_y t} P \\ &= \left(1+\frac{0.054}{2}\right)^{2\times 5} \times 6\,000 \\ &\approx \$7\,832 \end{aligned}$$

    The future value is \$7 832.

- Method 2: Using the TVM Solver
  $n = 2 \times 5 = 10$, $I\% = 5.4$, $PV = -6\,000$, $C/Y = 2$, $(PMT = 0,\ P/Y = 2)$
  The future value is $FV \approx 7\,832$.

- The interest earned is:

$$\begin{aligned}\text{Interest} &= A - P\\ &= 7\,832 - 6\,000\\ &= \$1\,832\end{aligned}$$

**Ex 52:** Amir deposited \$4 500 in a savings account paying 3.8% per annum interest, compounded quarterly for 6 years (you can use a calculator).

- Find the future value of the deposit.

  [5646] dollars (round to the nearest integer)

- How much interest did Amir earn?

  [1146] dollars

*Solution:*

- - Method 1: Using the formula

$$\begin{aligned}A &= \left(1 + \frac{r}{c_y}\right)^{c_y t} P\\ &= \left(1 + \frac{0.038}{4}\right)^{4\times 6} \times 4\,500\\ &\approx \$5\,646\end{aligned}$$

  The future value is \$5 646.
  - Method 2: Using the TVM Solver
    $n = 4 \times 6 = 24$, $I\% = 3.8$, $PV = -4\,500$, $C/Y = 4$, $(PMT = 0,\ P/Y = 4)$
    The future value is $FV \approx 5\,646$.

- The interest earned is:

$$\begin{aligned}\text{Interest} &= A - P\\ &= 5\,646 - 4\,500\\ &= \$1\,146\end{aligned}$$

**Ex 53:** Emma deposited \$5 000 in a savings account paying 4.2% per annum interest, compounded monthly for 8 years (you can use a calculator).

- Find the future value of the deposit.

  [6993] dollars (round to the nearest integer)

- How much interest did Emma earn?

  [1993] dollars

*Solution:*

- - Method 1: Using the formula

$$\begin{aligned}A &= \left(1 + \frac{r}{c_y}\right)^{c_y t} P\\ &= \left(1 + \frac{0.042}{12}\right)^{12\times 8} \times 5\,000\\ &\approx \$7\,136\end{aligned}$$

  The future value is \$7 136.
  - Method 2: Using the TVM Solver
    $n = 12 \times 8 = 96$, $I\% = 4.2$, $PV = -5\,000$, $C/Y = 12$, $(PMT = 0,\ P/Y = 12)$
    The future value is $FV \approx 7\,136$.

- The interest earned is:

$$\begin{aligned}\text{Interest} &= A - P\\ &= 6\,993 - 5\,000\\ &= \$1\,993\end{aligned}$$

## D.3 FINDING THE PRINCIPAL

**Ex 54:** Liam wants to open an investment account for his daughter's college fund. The account pays 4.8% p.a. compounded monthly. If Liam wants to have \$30 000 in the account when his daughter turns 18, how much does he need to deposit now (you can use a calculator) ?

[12666] dollars (round to the nearest integer)

*Solution:*

- Method 1: Using the formula

$$\begin{aligned}A &= \left(1 + \frac{r}{c_y}\right)^{c_y t} P\\ P &= \frac{A}{\left(1 + \frac{r}{c_y}\right)^{c_y t}}\\ &= \frac{30\,000}{\left(1 + \frac{0.048}{12}\right)^{12\times 18}}\\ &\approx \$12\,666\end{aligned}$$

  The initial amount is \$12 666.

- Method 2: Using the TVM Solver
  We input the following values:
  - $FV = 30\,000$
  - $I\% = 4.8$
  - $n = 12 \times 18 = 216$
  - $C/Y = 12$
  - $PV =?$ (what we want to find)

  The TVM Solver calculates the present value $PV \approx 12\,666$.

**Ex 55:** Oliver wants to save for a down payment on a house. The account pays 5.2% p.a. compounded monthly. If Oliver wants to have \$50 000 in the account in 10 years, how much does he need to deposit now (you can use a calculator)?

[29759] dollars (round to the nearest integer)

*Solution:*

- Method 1: Using the formula

$$\begin{aligned}A &= \left(1 + \frac{r}{c_y}\right)^{c_y t} P\\ P &= \frac{A}{\left(1 + \frac{r}{c_y}\right)^{c_y t}}\\ &= \frac{50\,000}{\left(1 + \frac{0.052}{12}\right)^{12\times 10}}\\ &\approx \$29\,759\end{aligned}$$

  The initial amount is \$29 759.

- Method 2: Using the TVM Solver
  We input the following values:
  - $FV = 50\,000$
  - $I\% = 5.2$
  - $n = 12 \times 10 = 120$
  - $C/Y = 12$
  - $PV = ?$ (what we want to find)

  The TVM Solver calculates the present value $PV \approx 29\,759$.

**Ex 56:** Sophia wants to save for a car purchase. The account pays 6.1% p.a. compounded monthly. If Sophia wants to have \$40 000 in the account in 7 years, how much does she need to deposit now (you can use a calculator)?

26127 dollars (round to the nearest integer)

*Solution:*

- Method 1: Using the formula

$$\begin{aligned} A &= \left(1+\frac{r}{c_y}\right)^{c_y t} P \\ P &= \frac{A}{\left(1+\frac{r}{c_y}\right)^{c_y t}} \\ &= \frac{40\,000}{\left(1+\frac{0.061}{12}\right)^{12\times 7}} \\ &\approx \$26\,127 \end{aligned}$$

  The initial amount is \$26 127.

- Method 2: Using the TVM Solver
  We input the following values:
  - $FV = 40\,000$
  - $I\% = 6.1$
  - $n = 12 \times 7 = 84$
  - $C/Y = 12$
  - $PV = ?$ (what we want to find)

  The TVM Solver calculates the present value $PV \approx 26\,127$.

**Ex 57:** Ethan wants to invest in a retirement fund. The account pays 5.9% p.a. compounded monthly. If Ethan wants to have \$100 000 in the account after 15 years, how much does he need to deposit now (you can use a calculator)?

41361 dollars (round to the nearest integer)

*Solution:*

- Method 1: Using the formula

$$\begin{aligned} A &= \left(1+\frac{r}{c_y}\right)^{c_y t} P \\ P &= \frac{A}{\left(1+\frac{r}{c_y}\right)^{c_y t}} \\ &= \frac{100\,000}{\left(1+\frac{0.059}{12}\right)^{12\times 15}} \\ &\approx \$41\,361 \end{aligned}$$

  The initial amount is \$41 361.

- Method 2: Using the TVM Solver
  We input the following values:
  - $FV = 100\,000$
  - $I\% = 5.9$
  - $n = 12 \times 15 = 180$
  - $C/Y = 12$
  - $PV = ?$ (what we want to find)

  The TVM Solver calculates the present value $PV \approx 41\,361$.

## D.4 FINDING TIME

**Ex 58:** You currently have \$5 000 saved in an account that pays 3.8% p.a., compounded monthly. You want to buy a house that costs \$100 000.

- How long would it take you to save \$100 000 using the TVM solver on your calculator?

  948 months (round to the nearest month)

- Do you think it will be practical to wait this long before buying a house?
  **No**

*Solution:*

- We will use the TVM Solver to find how long it will take to save \$100 000.
  - $PV = -5\,000$ (the initial savings, entered as negative).
  - $FV = 100\,000$ (the future amount you want to save).
  - $I\% = 3.8$ (the annual interest rate).
  - $PMT = 0$ (since there are no additional deposits).
  - $C/Y = 12$ (monthly compounding).
  - $P/Y = 12$ (monthly payments, but in this case, it's just savings).
  - $n = ?$ (the number of months, which we need to solve for).

  Enter these values into your calculator's TVM Solver to find how long it will take. The result should be approximately 948 months (rounded to the nearest month).
  Thus, it will take you around 948 months to save \$100 000 with no additional deposits.

- Given that it would take approximately 948 months (nearly 79 years) to save \$100, 000 with no additional deposits, it likely wouldn't be practical to wait this long before buying a house.

**Ex 59:** You currently have \$20 000 saved in an account that pays 5.2% p.a., compounded monthly. You want to buy a car that costs \$23 000.

- How long would it take you to save \$23 000 using the TVM solver on your calculator?

  32 months (round to the nearest month)

- Do you think it will be practical to wait this long before buying a car?
  **Yes**

*Solution:*

- We will use the TVM Solver to find how long it will take to save $23 000.
  - $PV = -20\,000$ (the initial savings, entered as negative).
  - $FV = 23\,000$ (the future amount you want to save).
  - $I\% = 5.2$ (the annual interest rate).
  - $PMT = 0$ (since there are no additional deposits).
  - $C/Y = 12$ (monthly compounding).
  - $P/Y = 12$ (monthly payments, but in this case, it's just savings).
  - $n = ?$ (the number of months, which we need to solve for).

  Enter these values into your calculator's TVM Solver to find how long it will take. The result should be approximately 32 months (rounded to the nearest month).
  Thus, it will take you around 32 months to save $23 000 with no additional deposits.
- Given that it would take approximately 32 months (a little over 2.5 years) to save $23, 000, it seems reasonable to wait this long before buying a car.

**Ex 60:** You currently have $10 000 saved in an account that pays 4.2% p.a., compounded monthly. You want to buy a car that costs $35 000.

- How long would it take you to save $35 000 using the TVM solver on your calculator?

  [359] months (round to the nearest month)

- Do you think it will be practical to wait this long before buying a car?
  **[No]**

*Solution:*

- We will use the TVM Solver to find how long it will take to save $35 000.
  - $PV = -10\,000$ (the initial savings, entered as negative).
  - $FV = 35\,000$ (the future amount you want to save).
  - $I\% = 4.2$ (the annual interest rate).
  - $PMT = 0$ (since there are no additional deposits).
  - $C/Y = 12$ (monthly compounding).
  - $P/Y = 12$ (monthly payments, but in this case, it's just savings).
  - $n = ?$ (the number of months, which we need to solve for).

  Enter these values into your calculator's TVM Solver to find how long it will take. The result should be approximately 359 months (rounded to the nearest month).
  Thus, it will take you around 359 months to save $35 000 with no additional deposits.
- Given that it would take approximately 359 months (nearly 30 years) to save $35, 000 with no additional deposits, it likely wouldn't be practical to wait this long before buying a car.

# E VARIABLE RATE INVESTMENTS

## E.1 FINDING THE TOTAL AMOUNT

**Ex 61:** Louis invested $200 in a variable rate investment account for 8 years. The interest rates were:

- for the first six years: 3% compounded yearly.
- for the last two years: 2% compounded yearly.

You can use a calculator or the TVM solver on your calculator.

- Find the final amount after 6 years.

  [238] dollars (round to the nearest integer)

- Find the final amount after 8 years.

  [248] dollars (round to the nearest integer)

*Solution:* We can solve this problem using two methods: the compound interest formula and the TVM solver.

- Method 1: Using the compound interest formula
  We break the calculation into two periods:
  - First, calculate the amount after the first 6 years with a 3% interest rate compounded yearly ($c_y = 1$):

$$\begin{aligned} A &= \left(1+\frac{r}{c_y}\right)^{c_y t} P \\ &= \left(1+\frac{0.03}{1}\right)^{1\times 6} 200 \\ &\approx \$238 \end{aligned}$$

  - Then, for the next 2 years, the principal is the amount accumulated after the first 6 years, which is $238. Now, calculate the amount after the last 2 years with a 2% interest rate compounded yearly ($c_y = 1$):

$$\begin{aligned} A &= \left(1+\frac{r}{c_y}\right)^{c_y t} P \\ &= \left(1+\frac{0.02}{1}\right)^{1\times 2} 238 \\ &\approx \$248 \end{aligned}$$

  Thus, the final amount after 8 years is approximately $248.

- Method 2: Using the TVM Solver
  We will break the calculation into two periods and use the TVM solver for each period.
  - Step 1: First 6 years at 3%
    Using the TVM Solver:
    * $PV = -200$ (initial investment, entered as negative).
    * $FV = ?$ (the future value after 6 years, which we want to find).
    * $I\% = 3$ (interest rate for the first 6 years).
    * $n = 6$ (number of years).
    * $C/Y = 1$ (compounded annually).

The TVM solver gives $FV \approx 238$ after the first 6 years.

  - Step 2: Last 2 years at 2%
    Now, we use the accumulated amount of \$238 as the new principal for the last 2 years.
    Using the TVM Solver again:
    * $PV = -238$ (the principal after 6 years).
    * $FV = ?$ (the future value after 2 more years).
    * $I\% = 2$ (interest rate for the last 2 years).
    * $n = 2$ (number of years).
    * $C/Y = 1$ (compounded annually).

    The TVM solver gives $FV \approx 248$ after the 2 additional years.
    Thus, the final amount after 8 years is approximately \$248.

**Ex 62:** Sarah invested \$300 in a variable rate investment account for 10 years. The interest rates were:

- for the first seven years: 4% compounded monthly.
- for the last three years: 3% compounded monthly.

You can use a calculator or the TVM solver on your calculator.

- Find the final amount after 7 years.

  $\boxed{397}$ dollars (round to the nearest integer)

- Find the final amount after 10 years.

  $\boxed{434}$ dollars (round to the nearest integer)

*Solution:* We can solve this problem using two methods: the compound interest formula and the TVM solver.

- Method 1: Using the compound interest formula
  We break the calculation into two periods:
  - First, calculate the amount after the first 7 years with a 4% interest rate compounded monthly:

$$A_7 = \left(1 + \frac{0.04}{12}\right)^{12\times 7} \times 300 \approx \$397$$

  - Then, for the next 3 years, the principal is the amount accumulated after the first 7 years, which is \$397. Now, calculate the amount after the last 3 years with a 3% interest rate compounded monthly:

$$A_{10} = \left(1 + \frac{0.03}{12}\right)^{12\times 3} \times 397 \approx \$434$$

    Thus, the final amount after 10 years is approximately \$434.

- Method 2: Using the TVM Solver
  We will break the calculation into two periods and use the TVM solver for each period.
  - Step 1: First 7 years at 4%
    Using the TVM Solver:
    * $PV = -300$ (initial investment, entered as negative).
    * $FV = ?$ (the future value after 7 years, which we want to find).
    * $I\% = 4$ (interest rate for the first 7 years).
    * $n = 12 \times 7 = 84$ (number of months).
    * $C/Y = 12$ (compounded monthly).

    The TVM solver gives $FV \approx 397$ after the first 7 years.
  - Step 2: Last 3 years at 3%
    Now, we use the accumulated amount of \$397 as the new principal for the last 3 years.
    Using the TVM Solver again:
    * $PV = -397$ (the principal after 7 years).
    * $FV = ?$ (the future value after 3 more years).
    * $I\% = 3$ (interest rate for the last 3 years).
    * $n = 12 \times 3 = 36$ (number of months).
    * $C/Y = 12$ (compounded monthly).

    The TVM solver gives $FV \approx 434$ after the 3 additional years.
    Thus, the final amount after 10 years is approximately \$434.

**Ex 63:** Emma invested \$1 000 in a variable rate investment account for 9 years. The interest rates were:

- for the first five years: 3.5% compounded monthly.
- for the last four years: 2.8% compounded monthly.

You can use a calculator or the TVM solver on your calculator.

- Find the final amount after 5 years.

  $\boxed{1191}$ dollars (round to the nearest integer)

- Find the final amount after 9 years.

  $\boxed{1332}$ dollars (round to the nearest integer)

*Solution:* We can solve this problem using two methods: the compound interest formula and the TVM solver.

- Method 1: Using the compound interest formula
  We break the calculation into two periods:
  - First, calculate the amount after the first 5 years with a 3.5% interest rate compounded monthly:

$$A_5 = \left(1 + \frac{0.035}{12}\right)^{12\times 5} \times 1\,000 \approx \$1\,191$$

  - Then, for the next 4 years, the principal is the amount accumulated after the first 5 years, which is \$1 191. Now, calculate the amount after the last 4 years with a 2.8% interest rate compounded monthly:

$$A_9 = \left(1 + \frac{0.028}{12}\right)^{12\times 4} \times 1\,191 \approx \$1\,332$$

    Thus, the final amount after 9 years is approximately \$1 332.

- Method 2: Using the TVM Solver
  We will break the calculation into two periods and use the TVM solver for each period.
  - Step 1: First 5 years at 3.5%
    Using the TVM Solver:

* $PV = -1\,000$ (initial investment, entered as negative).
* $FV = ?$ (the future value after 5 years, which we want to find).
* $I\% = 3.5$ (interest rate for the first 5 years).
* $n = 12 \times 5 = 60$ (number of months).
* $C/Y = 12$ (compounded monthly).

The TVM solver gives $FV \approx 1\,191$ after the first 5 years.

– Step 2: Last 4 years at 2.8%
Now, we use the accumulated amount of \$1 191 as the new principal for the last 4 years.
Using the TVM Solver again:

* $PV = -1\,191$ (the principal after 5 years).
* $FV = ?$ (the future value after 4 more years).
* $I\% = 2.8$ (interest rate for the last 4 years).
* $n = 12 \times 4 = 48$ (number of months).
* $C/Y = 12$ (compounded monthly).

The TVM solver gives $FV \approx 1\,332$ after the 4 additional years.
Thus, the final amount after 9 years is approximately \$1 332.

**Ex 64:** Sophie invested \$1 500 in a variable rate investment account for 9 years. The interest rates were:

- for the first six years: 4.1% compounded monthly.
- for the last three years: 3.5% compounded monthly.

You can use a calculator or the TVM solver on your calculator.

- Find the final amount after 6 years.

$\boxed{1,918}$ dollars (round to the nearest integer)

- Find the final amount after 9 years.

$\boxed{2,130}$ dollars (round to the nearest integer)

*Solution:* We can solve this problem using two methods: the compound interest formula and the TVM solver.

- Method 1: Using the compound interest formula
We break the calculation into two periods:

  – First, calculate the amount after the first 6 years with a 4.1% interest rate compounded monthly:

$$A_6 = \left(1 + \frac{0.041}{12}\right)^{12\times 6} \times 1\,500 \approx \$1\,918$$

  – Then, for the next 3 years, the principal is the amount accumulated after the first 6 years, which is \$1 918. Now, calculate the amount after the last 3 years with a 3.5% interest rate compounded monthly:

$$A_9 = \left(1 + \frac{0.035}{12}\right)^{12\times 3} \times 1\,918 \approx \$2\,130$$

Thus, the final amount after 9 years is approximately \$2 130.

- Method 2: Using the TVM Solver
We will break the calculation into two periods and use the TVM solver for each period.

  – Step 1: First 6 years at 4.1%
Using the TVM Solver:

* $PV = -1\,500$ (initial investment, entered as negative).
* $FV = ?$ (the future value after 6 years, which we want to find).
* $I\% = 4.1$ (interest rate for the first 6 years).
* $n = 12 \times 6 = 72$ (number of months).
* $C/Y = 12$ (compounded monthly).

The TVM solver gives $FV \approx 1\,918$ after the first 6 years.

  – Step 2: Last 3 years at 3.5%
Now, we use the accumulated amount of \$1 918 as the new principal for the last 3 years.
Using the TVM Solver again:

* $PV = -1\,918$ (the principal after 6 years).
* $FV = ?$ (the future value after 3 more years).
* $I\% = 3.5$ (interest rate for the last 3 years).
* $n = 12 \times 3 = 36$ (number of months).
* $C/Y = 12$ (compounded monthly).

The TVM solver gives $FV \approx 2\,130$ after the 3 additional years.
Thus, the final amount after 9 years is approximately \$2 130.

## F PERIODIC PAYMENT

### F.1 FINDING PERIODIC PAYMENTS IN LOANS

**Ex 65:** Suppose Maria takes out a loan of \$15 000 to buy a car. The loan has an interest rate of 5.5% per annum, compounded monthly, and she agrees to repay the loan over 5 years.
Calculate the periodic (monthly) payment Maria needs to make using the TVM solver on your calculator.

$\boxed{287}$ dollars (round to the nearest integer)

*Solution:* We will use the TVM Solver to find the monthly payment.

- $PV = -15\,000$ (the loan amount is an outgoing, so it's negative).
- $FV = 0$ (since there is no remaining balance at the end of the loan).
- $I\% = 5.5$ (the annual interest rate).
- $n = 12 \times 5 = 60$ (monthly payments over 5 years).
- $C/Y = 12$ (monthly compounding).
- $P/Y = 12$ (monthly payment).
- $PMT = ?$ (the value we are solving for).

Enter these values into your calculator's TVM Solver to find the monthly payment. The result should be approximately \$286.52.Therefore, Maria will need to make monthly payments of approximately \$287 to repay her car loan over 5 years.

**Ex 66:** Suppose John takes out a loan of \$25 000 to buy a motorcycle. The loan has an interest rate of 4.8% per annum, compounded monthly, and he agrees to repay the loan over 6 years.
Calculate the periodic (monthly) payment John needs to make using the TVM solver on your calculator.

$\boxed{400}$ dollars (round to the nearest integer)

*Solution:* We will use the TVM Solver to find the monthly payment.

- $PV = -25\,000$ (the loan amount is an outgoing, so it's negative).
- $FV = 0$ (since there is no remaining balance at the end of the loan).
- $I\% = 4.8$ (the annual interest rate).
- $n = 12 \times 6 = 72$ (monthly payments over 6 years).
- $C/Y = 12$ (monthly compounding).
- $P/Y = 12$ (monthly payment).
- $PMT = ?$ (the value we are solving for).

Enter these values into your calculator's TVM Solver to find the monthly payment. The result should be approximately \$400.31.Therefore, John will need to make monthly payments of approximately \$400 to repay his motorcycle loan over 6 years.

**Ex 67:** Suppose Su takes out a loan of \$18 000 to buy a car. The loan has an interest rate of 5.2% per annum, compounded monthly, and she agrees to repay the loan over 4 years.
Calculate the periodic (monthly) payment Su needs to make using the TVM solver on your calculator.

$\boxed{416}$ dollars (round to the nearest integer)

*Solution:* We will use the TVM Solver to find the monthly payment.

- $PV = -18\,000$ (the loan amount is an outgoing, so it's negative).
- $FV = 0$ (since there is no remaining balance at the end of the loan).
- $I\% = 5.2$ (the annual interest rate).
- $n = 12 \times 4 = 48$ (monthly payments over 4 years).
- $C/Y = 12$ (monthly compounding).
- $P/Y = 12$ (monthly payment).
- $PMT = ?$ (the value we are solving for).

Enter these values into your calculator's TVM Solver to find the monthly payment. The result should be approximately \$416.16.Therefore, Su will need to make monthly payments of approximately \$416 to repay her car loan over 4 years.

**Ex 68:** Suppose Amir takes out a loan of \$200 000 to buy a house. The loan has an interest rate of 3.9% per annum, compounded monthly, and he agrees to repay the loan over 20 years.
Calculate the periodic (monthly) payment Amir needs to make using the TVM solver on your calculator.

$\boxed{1201}$ dollars (round to the nearest integer)

*Solution:* We will use the TVM Solver to find the monthly payment.

- $PV = -200\,000$ (the loan amount is an outgoing, so it's negative).
- $FV = 0$ (since there is no remaining balance at the end of the loan).
- $I\% = 3.9$ (the annual interest rate).
- $n = 12 \times 20 = 240$ (monthly payments over 20 years).
- $C/Y = 12$ (monthly compounding).
- $P/Y = 12$ (monthly payment).
- $PMT = ?$ (the value we are solving for).

Enter these values into your calculator's TVM Solver to find the monthly payment. The result should be approximately \$1 201.45.Therefore, Amir will need to make monthly payments of approximately \$1 201 to repay his home loan over 20 years.