INTERESTS

A DEFINITIONS

Discover: Understanding Interest: Simple vs. Compound Interest We've all heard of interest rates—whether on a mortgage, a credit card, or a loan. But what does it really mean?
Interest is essentially the "rent" you pay for borrowing money. It's the additional amount you pay to use someone else's money for a certain period of time.

- **Example of interest:** Imagine you borrow \$100 from someone today and promise to pay it back in one year. If you return exactly \$100 after one year, there's no interest involved. However, the lender might ask for more in return—they might want to be compensated for letting you use their money.
  They may request a percentage of the amount. For example, at a 10% interest rate per year, the interest you would pay is:

$$\begin{aligned}\text{Interest Paid} &= \text{Percentage of Original Amount}\\ &= \text{Interest Rate} \times \text{Original Amount}\\ &= 10\% \times 100\\ &= \frac{10}{100} \times 100\\ &= 10 \text{ dollars}\end{aligned}$$

  Therefore, after one year, you would owe:

$$\begin{aligned}\text{Amount at Year 1} &= \text{Original Amount} + \text{Interest Paid}\\ &= 100 + 10\\ &= 110 \text{ dollars}\end{aligned}$$

  In this case, you would pay back \$110 instead of \$100. The extra \$10 is the interest, which represents the cost of borrowing the money for a year.

- **Simple interest:** Suppose you borrow \$100 at a 10% interest rate per year. With simple interest, the interest is calculated only on the percentage of the original amount every year.
  - $\text{Amount at Year 1} = \text{Original Amount} + \text{Percentage of Original Amount}$
    $= 100 + \frac{10}{100} \times 100$
    $= 100 + 10$
    $= 110$ dollars
  - $\text{Amount at Year 2} = \text{Amount at Year 1} + \text{Percentage of Original Amount}$
    $= 110 + \frac{10}{100} \times 100$
    $= 110 + 10$
    $= 120$ dollars
  - $\text{Amount at Year 3} = \text{Amount at Year 2} + \text{Percentage of Original Amount}$
    $= 120 + \frac{10}{100} \times 100$
    $= 120 + 10$
    $= 130$ dollars
  - $\vdots$
  - $\text{Amount at Year 10} = \text{Amount at Year 9} + \text{Percentage of Original Amount}$
    $= 190 + \frac{10}{100} \times 100$
    $= 190 + 10$
    $= 200$ dollars

  After 10 years, you would owe \$200, which includes \$100 of the original amount and \$100 in interest (\$10 per year for 10 years).

- **Compound interest:** Unlike simple interest, compound interest is calculated on the original amount plus any accumulated interest. This means you pay interest on both the initial amount and the interest from previous periods.

- Amount after Year 1 = Original Amount + Percentage of Original Amount
$$= 100 + \frac{10}{100} \times 100$$
$$= 100 + 10$$
$$= 110 \text{ dollars}$$
- Amount after Year 2 = Amount after Year 1 + Percentage of Amount after Year 1
$$= 110 + \frac{10}{100} \times 110$$
$$= 110 + 11$$
$$= 121 \text{ dollars}$$
- Amount after Year 3 = Amount after Year 2 + Percentage of Amount after Year 2
$$= 121 + \frac{10}{100} \times 121$$
$$= 121 + 12.10$$
$$= 133.10 \text{ dollars}$$
- $\vdots$
- Amount after Year 10 = Amount after Year 9 + Percentage of Amount after Year 9
$$= 235.79 + \frac{10}{100} \times 235.79$$
$$= 235.79 + 23.58$$
$$= 259.37 \text{ dollars}$$

After 10 years, you would owe \$259.37, which includes \$100 of the original amount and \$159.37 in compounded interest.

- **Conclusion:** As you can see, with compound interest, the amount you owe grows faster than with simple interest because you are paying interest on the interest. The key takeaway is that **simple interest** is calculated on the original amount every year, while **compound interest** is calculated on the original amount plus any accumulated interest.

Definition **Principal**

The **principal** is the original amount of money that is either invested or loaned.

Definition **Interest**

Interest is the cost paid for borrowing money or the amount earned from lending or investing money.

B SIMPLE INTEREST

Discover: Suppose you borrow \$100 with an interest rate of 10% per year. With simple interest, the interest is calculated only on the initial amount each year.

- Total interest paid after 1 year = Percentage of Original Amount
$$= \text{Interest Rate} \times \text{Original Amount}$$
$$= 10\% \times 100$$
$$= \frac{10}{100} \times 100$$
$$= 10 \text{ dollars}$$
- Total interest paid after 2 years = 2 × Percentage of Original Amount
$$= 2 \times \text{Interest Rate} \times \text{Original Amount}$$
$$= 2 \times 10\% \times 100$$
$$= 2 \times \frac{10}{100} \times 100$$
$$= 20 \text{ dollars}$$
- Total interest paid after 3 years = 3 × Percentage of Original Amount
$$= 3 \times \text{Interest Rate} \times \text{Original Amount}$$
$$= 3 \times 10\% \times 100$$
$$= 3 \times \frac{10}{100} \times 100$$
$$= 30 \text{ dollars}$$

These observations lead to the simple interest formula:

$$\text{Simple Interest} = \text{Number of years} \times \text{Interest rate} \times \text{Principal (initial amount)}$$

Definition Simple Interest

The **simple interest** is calculated each year as a fixed percentage on the principal (original amount) of money borrowed or invested.

Proposition Simple Interest Formula

The simple interest, denoted by I, is calculated as:

$$I = t \times r \times P$$

where:

- P is the principal (original amount)
- r is the interest rate per year
- t is the time (in years)

The final amount, denoted by A, is:

$$\begin{aligned} A &= P + I \\ &= P + t \times r \times P \\ &= (1 + t \times r) \times P \end{aligned}$$

Ex: Find the simple interest on a principal of \$500 at a rate of 3% per year over 5 years.

Answer:

$$\begin{aligned} \text{Interest} &= 5 \times 3\% \text{ of } 500 \\ &= 5 \times \frac{3}{100} \times 500 \\ &= 75 \text{ dollars} \end{aligned}$$

C COMPOUND INTEREST

Discover: If you leave money in the bank for a period of time, the interest earned is automatically added to your account. After the interest is added, it also begins to earn interest in the next time period. This process is called compound interest.
Example of compound interest: \$1 000 is placed in an account that earns 10% interest per annum (p.a.), and the interest is allowed to compound over three years. This means the account is earning 10% p.a. in compound interest.
We can illustrate this in a table:

Year	Amount	Interest Earned
0	\$1 000	10% of \$1 000 = \$100
1	\$1 000 + \$100 = \$1 100	10% of \$1 100 = \$110
2	\$1 100 + \$110 = \$1 210	10% of \$1 210 = \$121
3	\$1 210 + \$121 = \$1 331	—

After 3 years, there will be a total of \$1 331 in the account, meaning we have earned \$331 in compound interest.
We can calculate the final amount using another method as well:

- $\begin{aligned} \text{Amount after 1 year} &= \text{Initial amount} + \text{Interest on the initial amount} \\ &= 1\,000 + 0.1 \times 1\,000 \\ &= (1 + 0.1) \times 1\,000 \quad \text{(factoring out } 1\,000\text{)} \\ &= 1.1 \times 1\,000 \end{aligned}$
- $\begin{aligned} \text{Amount after 2 years} &= \text{Amount after 1 year} + \text{Interest on the amount after 1 year} \\ &= 1.1 \times 1\,000 + 0.1 \times 1.1 \times 1\,000 \\ &= (1 + 0.1) \times 1.1 \times 1\,000 \quad \text{(factoring out } 1.1 \times 1\,000\text{)} \\ &= 1.1^2 \times 1\,000 \end{aligned}$
- $\begin{aligned} \text{Amount after 3 years} &= \text{Amount after 2 years} + \text{Interest on the amount after 2 years} \\ &= 1.1^2 \times 1\,000 + 0.1 \times 1.1^2 \times 1\,000 \\ &= (1 + 0.1) \times 1.1^2 \times 1\,000 \quad \text{(factoring out } 1.1^2 \times 1\,000\text{)} \\ &= 1.1^3 \times 1\,000 \end{aligned}$

These observations lead to the compound interest formula:

$$\text{Final amount} = (1 + \text{Interest rate})^{\text{Number of years}} \times \text{Initial amount}$$

Definition Compound Interest

Compound interest is interest that accumulates on both the principal sum and the previously accumulated interest.

Proposition Annual Compound Interest Formula

The final amount of an investment with interest compounded annually is:

$$A = P(1 + r)^t$$

where:

- P is the principal,
- r is the annual interest rate,
- t is the time (in years).

Ex: Find the final amount for compound interest on a principal of \$500 at a rate of 3% per year over 5 years.

Answer:

$$\begin{aligned} A &= P(1 + r)^t \\ &= 500 \times (1 + 0.03)^5 \\ &\approx \$580.81 \end{aligned}$$

The final amount is approximately \$580.81.