INTEGERS

A DEFINITION

> **Definition Integers**
>
> The **integers** are the set that contains the natural numbers $(1, 2, 3, \ldots)$, their opposites $(-1, -2, -3, \ldots)$, and 0.
>
> - **Positive numbers** $(+1, +2, \ldots)$ are written with a **positive sign** $(+)$. This sign is often omitted $(+2 = 2)$.
>
> $$+2 = \,\bigoplus \bigoplus$$
>
> - **Negative numbers** $(-1, -2, \ldots)$ are written with a **negative sign** $(-)$.
>
> $$-3 = \,\bigominus \bigominus \bigominus$$
>
> - **Zero** (0) is neither **positive** nor **negative**.
> - Two numbers are **opposites** if their sum is 0.
>
> $$\bigoplus \bigoplus \; + \; \bigominus \bigominus \; = \; \bullet \bullet$$
> $$(+2) \quad + \quad (-2) \quad = \quad 0$$
> $$-2 \text{ is the opposite of } +2.$$
>
> - To avoid confusion between a number's sign and an operation sign, we often use parentheses. For example, $+1 + -2$ can be written as $(+1) + (-2)$.

Ex: Calculate $(+1) + (-2)$.

Answer:

$$\bigoplus \; + \; \bigominus \bigominus \; = \; \bigominus \bullet$$
$$= \; \bigominus$$

- So, $(+1) + (-2) = -1$.

> **Definition Absolute Value**
>
> The **absolute value** of a number is the number without its sign.
>
> - The absolute value of $+2 = \bigoplus\bigoplus$ is 2.
> - The absolute value of $-3 = \bigominus\bigominus\bigominus$ is 3.

B RULES OF ADDITION

> **Method Rules of Addition**
>
> - When you add **two positive numbers**, add their absolute values. The sum is also a positive number:
>
> $$(+2) + (+4) = +6 \quad \text{as } 2 + 4 = 6.$$
>
> $$\bigoplus\bigoplus \; + \; \bigoplus\bigoplus\bigoplus\bigoplus \; = \; \bigoplus\bigoplus\bigoplus\bigoplus\bigoplus\bigoplus$$
>
> - When you add **two negative numbers**, add their absolute values. The sum is also a negative number:
>
> $$(-5) + (-3) = -8 \quad \text{as } 5 + 3 = 8.$$
>
> $$\bigominus\bigominus\bigominus\bigominus\bigominus \; + \; \bigominus\bigominus\bigominus \; = \; \bigominus\bigominus\bigominus\bigominus\bigominus\bigominus\bigominus\bigominus$$

- When you add a **positive number** and a **negative number**, subtract the smaller absolute value from the larger one and use the sign of the number with the larger absolute value.

$$(-2) + (+5) = +3 \quad \text{as } 5 - 2 = 3$$

$$\ominus \ominus \; + \; \oplus \oplus \oplus \oplus \oplus \; = \; \oplus \oplus \oplus \; \bullet \; \bullet$$

$$= \; \oplus \oplus \oplus$$

$$(+2) + (-6) = -4 \quad \text{as } 6 - 2 = 4$$

$$\oplus \oplus \; + \; \ominus \ominus \ominus \ominus \ominus \ominus \; = \; \ominus \ominus \ominus \ominus \; \bullet \; \bullet$$

$$= \; \ominus \ominus \ominus \ominus$$

Ex: Calculate $(-10) + (+3)$.

Answer:

- $(-10) + (+3) = -7$ as $10 - 3 = 7$.

$$\ominus\ominus\ominus\ominus\ominus\ominus\ominus\ominus\ominus\ominus \; + \; \oplus\oplus\oplus \; = \; \ominus\ominus\ominus\ominus\ominus\ominus\ominus \; \bullet\bullet\bullet$$

$$= \; \ominus\ominus\ominus\ominus\ominus\ominus\ominus$$

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C SUBTRACTION

> Definition **Subtraction**
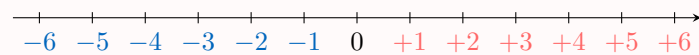> **Subtracting** a number means adding its opposite.

Ex: Calculate $(+4) - (-2)$.

Answer:

$$(+4) - (-2) = (+4) + (+2) \quad \text{(add the opposite)}$$
$$= +6 \quad \text{(same sign: add the absolute values)}$$

$$\oplus \oplus \oplus \oplus \; - \; \ominus \ominus \; = \; \oplus \oplus \oplus \oplus \; + \; \oplus \oplus$$

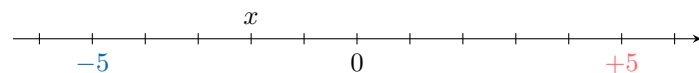$$= \; \oplus \oplus \oplus \oplus \oplus \oplus$$

D ON THE NUMBER LINE

> Definition **Number line**
> A **number line** is a straight line with markings at equal intervals to denote the numbers.
>
> $$-6 \quad -5 \quad -4 \quad -3 \quad -2 \quad -1 \quad 0 \quad +1 \quad +2 \quad +3 \quad +4 \quad +5 \quad +6$$

Ex: Find the value of x.

$$x$$
$$-5 \qquad\qquad 0 \qquad\qquad +5$$

Answer:

$$x$$
- $$-6 \quad -5 \quad -4 \quad -3 \quad -2 \quad -1 \quad 0 \quad +1 \quad +2 \quad +3 \quad +4 \quad +5 \quad +6$$
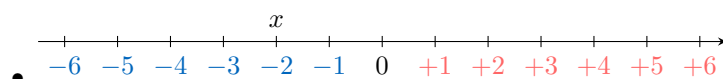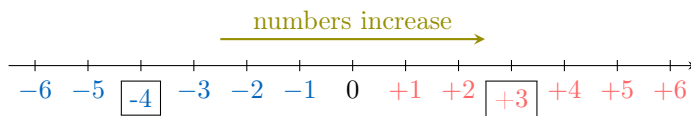
- So, $x = -2$.

E ORDERING

- When one number is **positive** and the other is **negative**, the positive number is **greater**.

- When both numbers are **negative**, the number closer to zero is **greater** (the number with the smaller absolute value is greater).

- When both numbers are **positive**, the number further from zero is **greater** (the number with the greater absolute value is greater).

numbers increase

$$-6 \quad -5 \quad -4 \quad -3 \quad -2 \quad -1 \quad 0 \quad +1 \quad +2 \quad +3 \quad +4 \quad +5 \quad +6$$
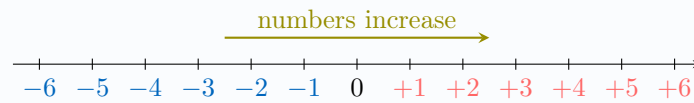
Ex: Compare -4 and $+3$.

Answer:

- As $+3$ is positive and -4 is negative, the positive number is greater than the negative number: $-4 < +3$.

numbers increase

$$-6 \quad -5 \quad \boxed{-4} \quad -3 \quad -2 \quad -1 \quad 0 \quad +1 \quad +2 \quad \boxed{+3} \quad +4 \quad +5 \quad +6$$

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F MULTIPLICATION

Definition **Multiplication**

The product of two signed integers is obtained by:

- multiplying their **absolute values**;

- then deciding the sign using the following rules:

  - $(+) \times (+) = (+)$: a **positive** times a **positive** gives a **positive**.
  - $(+) \times (-) = (-)$: a **positive** times a **negative** gives a **negative**.
  - $(-) \times (+) = (-)$: a **negative** times a **positive** gives a **negative**.
  - $(-) \times (-) = (+)$: a **negative** times a **negative** gives a **positive**.

Ex: Calculate $(+2) \times (-5)$.

Answer:

$$(+2) \times (-5) = -10 \quad (2 \times 5 = 10 \text{ and } (+) \times (-) = (-))$$

G DIVISION

Definition **Division**

The quotient of two integers (with a non-zero divisor) is obtained by:

- dividing their **absolute values**;

- then deciding the sign using the following rules:

  - $(+) \div (+) = (+)$: a **positive** divided by a **positive** gives a **positive**.
  - $(+) \div (-) = (-)$: a **positive** divided by a **negative** gives a **negative**.
  - $(-) \div (+) = (-)$: a **negative** divided by a **positive** gives a **negative**.
  - $(-) \div (-) = (+)$: a **negative** divided by a **negative** gives a **positive**.

Ex: Calculate $(+10) \div (-5)$.

Answer:

$$(+10) \div (-5) = -2 \quad (10 \div 5 = 2 \text{ and } (+) \div (-) = (-))$$