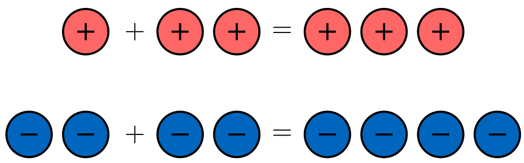
INTEGERS

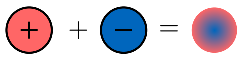
A DEFINITION

Discover: Imagine a world with two types of particles: **positives** (+) and **negatives (-)**. They interact in specific ways.

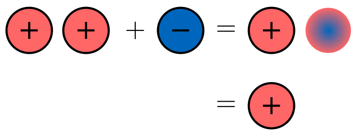
- When particles of the same type meet, they join forces.



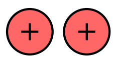
- When a **positive** and a **negative** particle meet, they cancel each other out, leaving nothing. This is called a **zero pair**.



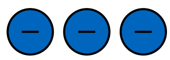
- Let's see what happens if **2 positives** meet **1 negative**. One zero pair is formed, leaving **1 positive**.



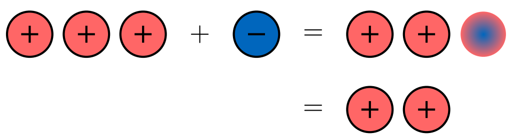
- To show which type of particle we have, we put a sign in front of the number:
  - The + sign for a group of **positives**.

    $+2 =$ 

  - The **-** sign for a group of **negatives**.

    $-3 =$ 

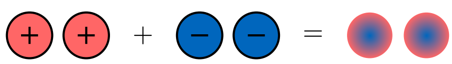
- Now, let's see what happens when **3 positives** meet **1 negative**.



$$(+3) + (-1) = +2$$

There are **2 positives** left.

- Finally, let's see what happens when **2 positives** meet **2 negatives**.



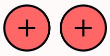
$$(+2) + (-2) = 0$$

There are 0 particles left.

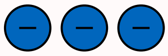
Definition Integers

The **integers** are the set that contains the natural numbers $(1, 2, 3, \dots)$, their opposites $(-1, -2, -3, \dots)$, and 0.

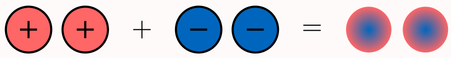
- **Positive numbers** $(+1, +2, \dots)$ are written with a **positive sign** $(+)$. This sign is often omitted $(+2 = 2)$.

  $+2 =$ 

- **Negative numbers** $(-1, -2, \dots)$ are written with a **negative sign** $(-)$.

  $-3 =$ 

- **Zero** (0) is neither **positive** nor **negative**.
- Two numbers are **opposites** if their sum is 0.

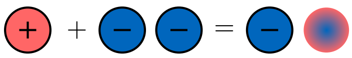
  $(+2) \quad + \quad (-2) \quad = \quad 0$

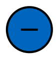
  -2 is the opposite of $+2$.

- To avoid confusion between a number's sign and an operation sign, we often use parentheses. For example, $+1 + -2$ can be written as $(+1) + (-2)$.

Ex: Calculate $(+1) + (-2)$.

Answer:



- 
- So, $(+1) + (-2) = -1$.

Definition Absolute Value

The **absolute value** of a number is the number without its sign.

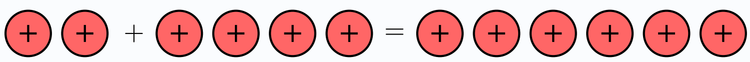
- The absolute value of $+2 =$  is 2.
- The absolute value of $-3 =$  is 3.

B RULES OF ADDITION

Method Rules of Addition

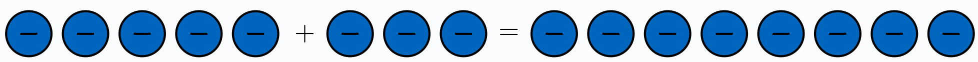
- When you add **two positive numbers**, add their absolute values. The sum is also a positive number:

  $$(+2) + (+4) = +6 \quad \text{as } 2 + 4 = 6.$$

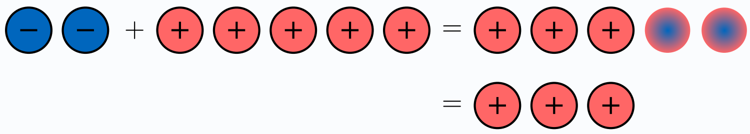
- When you add **two negative numbers**, add their absolute values. The sum is also a negative number:

  $$(-5) + (-3) = -8 \quad \text{as } 5 + 3 = 8.$$

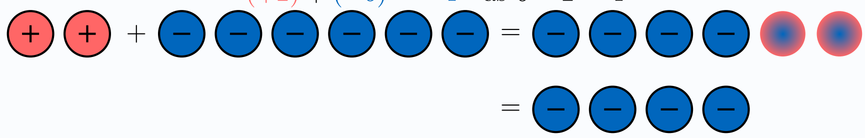
  

- When you add a **positive number** and a **negative number**, subtract the smaller absolute value from the larger one and use the sign of the number with the larger absolute value.

  $$(-2) + (+5) = +3 \quad \text{as } 5 - 2 = 3$$



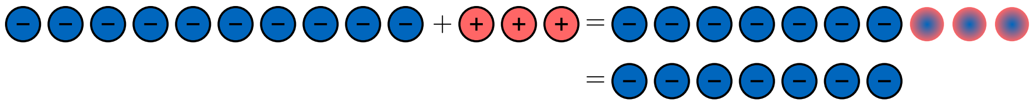
$(+2) + (-6) = -4$ as $6 - 2 = 4$



Ex: Calculate $(-10) + (+3)$.

Answer:

- $(-10) + (+3) = -7$ as $10 - 3 = 7$.



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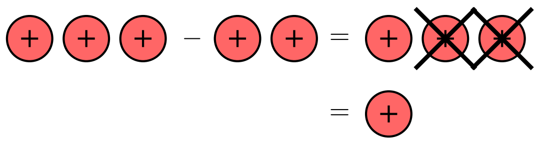
C SUBTRACTION

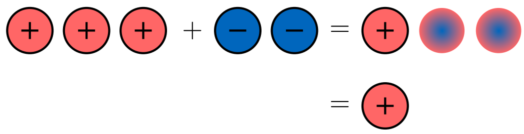
Discover:

- – For the subtraction, $(+3) - (+2)$:



we remove 2 positives from 3 positives, leaving us with 1 positive.

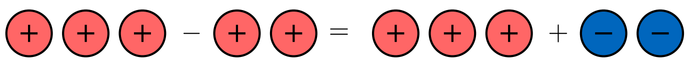
– For the addition, $(+3) + (-2)$:



we again remove 2 positives from 3 positives.

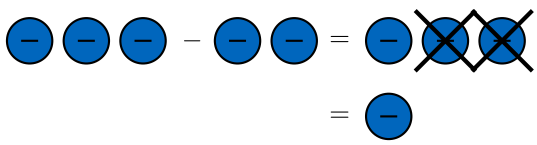
– Therefore, these two operations are equivalent:

$$(+3) - (+2) = (+3) + (-2)$$



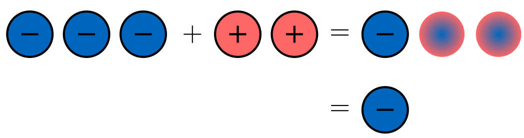
This shows that subtracting a positive number is the same as adding its opposite.

- – For the subtraction, $(-3) - (-2)$:



we remove 2 negatives from 3 negatives, leaving us with 1 negative.

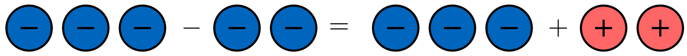
– For the addition, $(-3) + (+2)$:



we again remove 2 negatives from 3 negatives.

– Therefore, these two operations are equivalent:

$$(-3) - (-2) = (-3) + (+2)$$



This shows that subtracting a negative number is the same as adding its opposite.

- In conclusion, these examples show a fundamental rule in arithmetic: subtracting any number is equivalent to adding the number with its opposite sign.

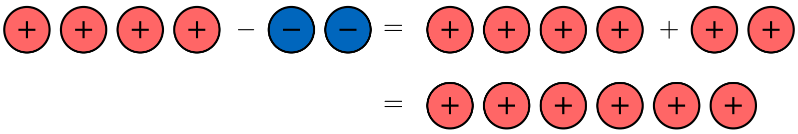
Definition Subtraction

Subtracting a number means adding its opposite.

Ex: Calculate $(+4) - (-2)$.

Answer:

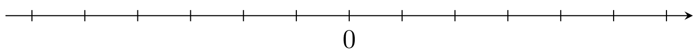
$$\begin{aligned}(+4) - (-2) &= (+4) + (+2) && \text{(add the opposite)}\\ &= +6 && \text{(same sign: add the absolute values)}\end{aligned}$$



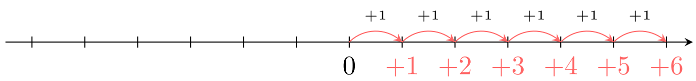
D ON THE NUMBER LINE

Discover:

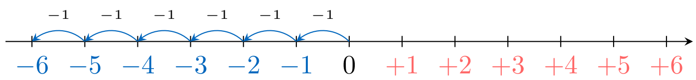
- To show both positive and negative numbers on a number line, we extend the number line in both directions from zero.



- For each move from left to right by 1, the number increases by 1: $0 + 1 = +1$, $+1 + 1 = +2, \ldots$

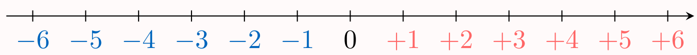


- For each move from right to left by 1, the number decreases by 1: $0 - 1 = -1$, $-1 - 1 = -2, \ldots$

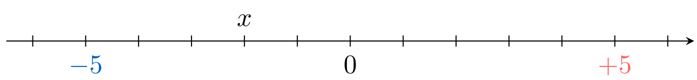


Definition Number line

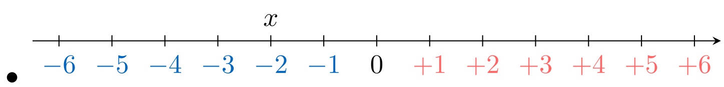
A **number line** is a straight line with markings at equal intervals to denote the numbers.



Ex: Find the value of x.



Answer:



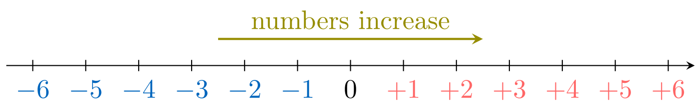
- So, $x = -2$.

E ORDERING

Discover: In the set of integers, the order is defined as:

$$\ldots < -3 < -2 < -1 < 0 < +1 < +2 < +3 < \ldots$$

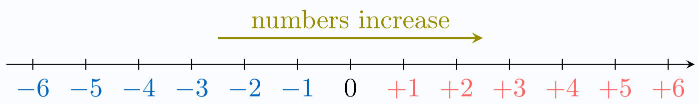
So, as you move along the number line from left to right, the numbers increase.



- As $+3$ is to the right of -5, $-5 < +3$. So, when one number is **positive** and the other is **negative**, the positive number is **greater**.
- As -2 is to the right of -4, $-4 < -2$. So, when both numbers are **negative**, the number closer to zero is **greater** (the number with the smaller absolute value is greater).
- As $+6$ is to the right of $+4$, $+4 < +6$. So, when both numbers are **positive**, the number further from zero is **greater** (the number with the greater absolute value is greater).

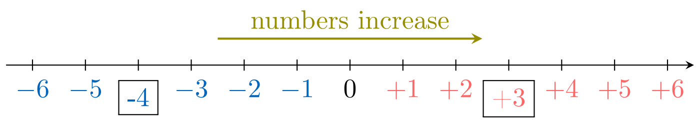
Method **Compare two numbers**

- When one number is **positive** and the other is **negative**, the positive number is **greater**.
- When both numbers are **negative**, the number closer to zero is **greater** (the number with the smaller absolute value is greater).
- When both numbers are **positive**, the number further from zero is **greater** (the number with the greater absolute value is greater).



Ex: Compare -4 and $+3$.

Answer:

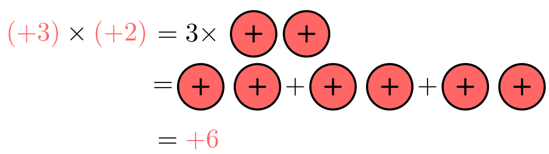
- As $+3$ is positive and -4 is negative, the positive number is greater than the negative number: $-4 < +3$.
- 

F MULTIPLICATION

Discover: Multiplication of two whole numbers can be seen as repeated addition: $3 \times 2 = 2 + 2 + 2 = 6$.
We now extend this idea to signed numbers:

- $(+) \times (+)$:

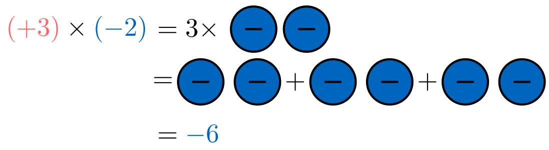
$$\begin{aligned}(+3) \times (+2) &= 3 \times (+2)\\ &= (+2) + (+2) + (+2)\\ &= +6\end{aligned}$$



So, a **positive** times a **positive** gives a **positive**.

- $(+) \times (-)$:

$$\begin{aligned}(+3) \times (-2) &= 3 \times (-2)\\ &= (-2) + (-2) + (-2)\\ &= -6\end{aligned}$$



So, a **positive** times a **negative** gives a **negative**.

- $(-) \times (+)$: Multiplication is commutative, so the order does not matter.

$$\begin{aligned}(-2) \times (+3) &= (+3) \times (-2)\\ &= -6\end{aligned}$$

So, a **negative** times a **positive** gives a **negative**.

- $(-) \times (-)$:We know that $0 \times (-2) = 0$. Also,$0 = (+3) + (-3)$, so:

$$\begin{aligned}((+3) + (-3)) \times (-2) &= 0\\ (+3) \times (-2) + (-3) \times (-2) &= 0\\ -6 + \big((-3) \times (-2)\big) &= 0\\ (-3) \times (-2) &= +6\end{aligned}$$

So, a **negative** times a **negative** gives a **positive**.

Definition Multiplication

The product of two signed integers is obtained by:

- multiplying their **absolute values**;
- then deciding the sign using the following rules:
  - $(+) \times (+) = (+)$: a **positive** times a **positive** gives a **positive**.
  - $(+) \times (-) = (-)$: a **positive** times a **negative** gives a **negative**.
  - $(-) \times (+) = (-)$: a **negative** times a **positive** gives a **negative**.
  - $(-) \times (-) = (+)$: a **negative** times a **negative** gives a **positive**.

Ex: Calculate $(+2) \times (-5)$.

Answer:

$$(+2) \times (-5) = -10 \quad (2 \times 5 = 10 \text{ and } (+) \times (-) = (-))$$

G DIVISION

Discover: Multiplication and division are inverse operations:

$$3 \times 2 = 6, \text{ so } 6 \div 3 = 2.$$

(Here we only divide by non-zero numbers.)
Now, let's look at division with negative numbers:

- $(+) \div (+)$:

$$(+3) \times (+2) = +6, \text{ so } (+6) \div (+3) = (+2).$$

So, a **positive** divided by a **positive** gives a **positive**.

- $(+) \div (-)$:

$$(-3) \times (-2) = +6, \text{ so } (+6) \div (-3) = (-2).$$

So, a **positive** divided by a **negative** gives a **negative**.

- $(-) \div (+)$:

$$(+3) \times (-2) = -6, \text{ so } (-6) \div (+3) = (-2).$$

  So, a **negative** divided by a **positive** gives a **negative**.

- $(-) \div (-)$:

$$(-3) \times (+2) = -6, \text{ so } (-6) \div (-3) = (+2).$$

  So, a **negative** divided by a **negative** gives a **positive**.

Definition **Division**

The quotient of two integers (with a non-zero divisor) is obtained by:

- dividing their **absolute values**;
- then deciding the sign using the following rules:
  - $(+) \div (+) = (+)$: a **positive** divided by a **positive** gives a **positive**.
  - $(+) \div (-) = (-)$: a **positive** divided by a **negative** gives a **negative**.
  - $(-) \div (+) = (-)$: a **negative** divided by a **positive** gives a **negative**.
  - $(-) \div (-) = (+)$: a **negative** divided by a **negative** gives a **positive**.

Ex: Calculate $(+10) \div (-5)$.

Answer:

$$(+10) \div (-5) = -2 \quad (10 \div 5 = 2 \text{ and } (+) \div (-) = (-))$$