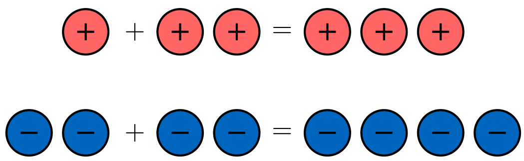
INTEGERS

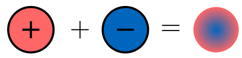
A DEFINITION

Discover: On a distant planet, two tribes are at war: the **positives** and the **negatives**.

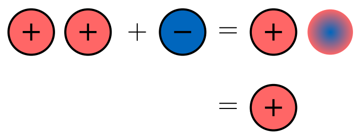
- When troops from the same tribe meet, they unite.



- When a **positive** and a **negative** meet, they cancel each other out.

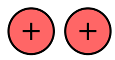


- Let's see what happens if **2 positives** meet **1 negative**,

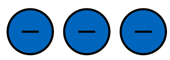


  There remains **1 positive**.

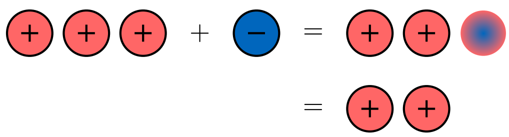
- To show which tribe the number belongs to, we put a sign in front of the number:
  - The $+$ sign for the tribe of **positives**.

$+2 =$ 

  - The **-** sign for the tribe of **negatives**.

$-3 =$ 

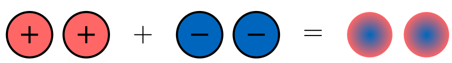
- Now, let's see what happens when **3 positives** meet **1 negative**.



$$(+3) + (-1) = +2$$

  There remains **2 positives**.

- Finally, let's see what happens when **2 positives** meet **2 negatives**.

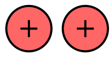


$$(+2) + (-2) = 0$$

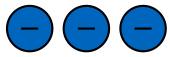
  There remains 0.

Definition Positive and Negative Numbers

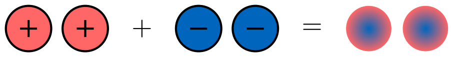
- **Positive numbers** are $+1, +2, \ldots$. We write them with a **positive sign** $(+)$ before the number:

  $+2 =$ 

- **Negative numbers** are $-1, -2, \ldots$. We write them with a **negative sign** $(-)$ before the number:

  $-3 =$ 

- **Positive numbers** are the opposite of **negative numbers**:

  $$(+2) \quad + \quad (-2) \quad = \quad 0$$

  -2 is the opposite of $+2$.

- Integer numbers are **positive numbers**, **negative numbers**, and zero :

  $$\ldots, -3, -2, -1, 0, +1, +2, +3, \ldots$$

- Positive numbers can be written **with** or **without** a positive sign $(+)$ in front of the number:

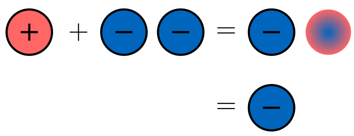
  $1 = +1 =$ 

- To avoid confusion between the sign of the number and the sign of the operation, we can use parentheses. For example, $+1 + -2$ becomes $(+1) + (-2)$.

- 0 is neither positive nor negative.

Ex: Calculate $(+1) + (-2)$.

Answer:



- So, $(+1) + (-2) = -1$.

Definition Absolute Value

The **absolute value** of a number is the number without its sign.

- The absolute value of $+2 =$  is 2.
- The absolute value of $-3 =$  is 3.

B RULES OF ADDITION

Method Rules of Addition

- When you add **two positive numbers**, add their absolute values. The sum is also a positive number.

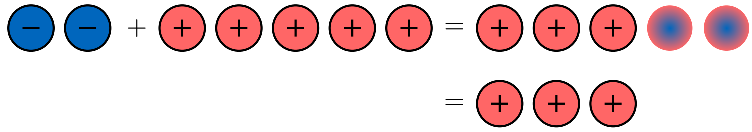
  $$(+2) + (+7) = +9 \quad \text{as } 2 + 7 = 9$$

- When you add **two negative numbers**, add their absolute values. The sum is also a negative number.

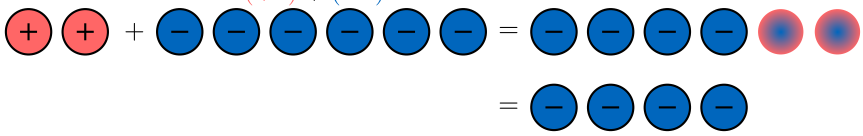
  $$(-5) + (-10) = -15 \quad \text{as } 5 + 10 = 15$$

- When you add a **positive number** and a **negative number**, subtract the smaller absolute value from the larger one and use the sign of the number with the larger absolute value.

  $$(-2) + (+5) = +3 \quad \text{as } 5 - 2 = 3$$



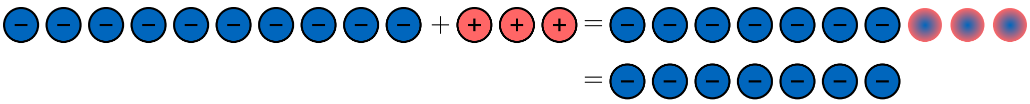
$(+2) + (-6) = -4$ as $6 - 2 = 4$



Ex: Calculate $(-10) + (+3)$

Answer:

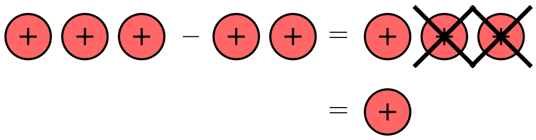
- $(-10) + (+3) = -7$ as $10 - 3 = 7$



C SUBTRACTION

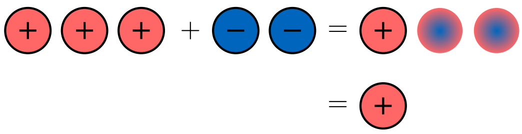
Discover:

- – For the subtraction, $(+3) - (+2)$:

  we remove 2 positives from 3 positives, leaving us with 1 positive.

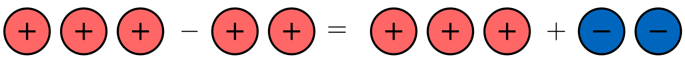
  – For the addition, $(+3) + (-2)$:

  we remove again 2 positives from 3 positives.

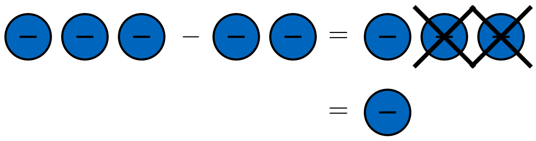
  – Therefore, these two operations are equivalent:

  $$(+3) - (+2) = (+3) + (-2)$$

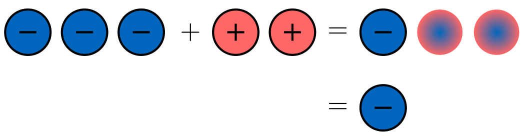
  This shows that subtracting a positive number is the same as adding its opposite.

- – For the subtraction, $(-3) - (-2)$:

  we remove 2 negatives from 3 negatives, leaving us with 1 negative.

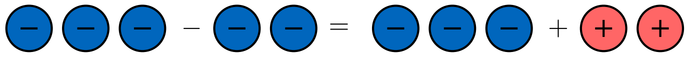
  – For the addition, $(-3) + (+2)$:

  we remove again 2 negatives from 3 negatives.

  – Therefore, these two operations are equivalent:

$$(-3) - (-2) = (-3) + (+2)$$



This shows that subtracting a negative number is the same as adding its opposite.

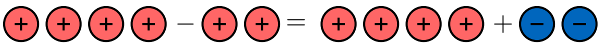
- In conclusion, these examples show a fundamental principle of arithmetic: subtracting any number is equivalent to adding the number with its opposite sign.

Definition Subtraction

Subtracting a number is adding its opposite.

Ex: Convert the subtraction into addition: $(+4) - (+2)$

Answer:

- $(+4) - (+2) = (+4) + (-2)$
- 

Ex: Calculate $(+4) - (-2)$

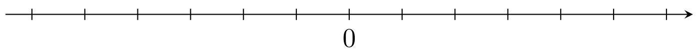
Answer:

$$\begin{aligned} (+4) - (-2) &= (+4) + (+2) && \text{(add the opposite)} \\ &= +6 && \text{(same sign: add the absolute values)} \end{aligned}$$

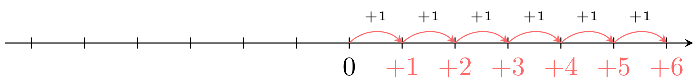
D ON THE NUMBER LINE

Discover:

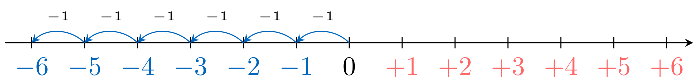
- To show both positive and negative numbers on a number line, we extend the number line in both directions from zero.



- For each move from left to right by 1, the number increases by 1: $0 + 1 = +1$, $+1 + 1 = +2, \ldots$

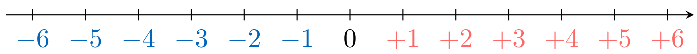


- For each move from right to left by 1, the number decreases by 1: $0 - 1 = -1$, $-1 - 1 = -2, \ldots$

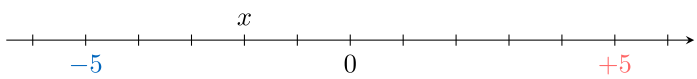


Definition Number line

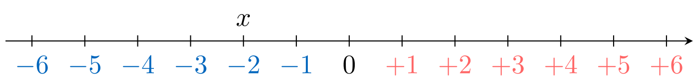
A **number line** is a straight line with markings at equal intervals to denote the numbers.



Ex: Find the value of x.



Answer:

- 

- So, $x = -2$.

E ORDERING

Discover: In the set of numbers, the order is defined as:

$$\ldots < -3 < -2 < -1 < 0 < +1 < +2 < +3 < \ldots$$

So, as you move along the number line from left to right, the numbers increase.

numbers increase

-6 -5 -4 -3 -2 -1 0 $+1$ $+2$ $+3$ $+4$ $+5$ $+6$

- As $+3$ is to the right of -5, $-5 < +3$. So, when one number is **positive** and the other is **negative**, the positive number is **greater**.
- As -2 is to the right of -4, $-4 < -2$. So, when both numbers are **negative**, the number closer to zero is **greater** (the number with the smaller absolute value is greater).
- As $+6$ is to the right of $+4$, $+4 < +6$. So, when both numbers are **positive**, the number further from zero is **greater** (the number with the greater absolute value is greater).

Method Compare two numbers

- When one number is **positive** and the other is **negative**, the positive number is **greater**.
- When both numbers are **negative**, the number closer to zero is **greater** (the number with the smaller absolute value is greater).
- When both numbers are **positive**, the number further from zero is **greater** (the number with the greater absolute value is greater).

numbers increase

-6 -5 -4 -3 -2 -1 0 $+1$ $+2$ $+3$ $+4$ $+5$ $+6$

Ex: Compare -4 and $+3$

Answer:

- As $+3$ is positive and -4 is negative, the positive number is greater than the negative number: $-4 < +3$
- numbers increase

  -6 -5 $\boxed{-4}$ -3 -2 -1 0 $+1$ $+2$ $\boxed{+3}$ $+4$ $+5$ $+6$

F MULTIPLICATION

Discover: Multiplication of two whole numbers is repeated addition: $3 \times 2 = 2 + 2 + 2 = 6$.

- $(+) \times (+)$:

$$\begin{aligned}(+3) \times (+2) &= 3 \times (+2)\\ &= (+2) + (+2) + (+2)\\ &= +6\end{aligned}$$

$$\begin{aligned}(+3) \times (+2) &= 3\times\ + \ +\\ &= + \ + + + \ + + + \ +\\ &= +6\end{aligned}$$

So, a **positive** times a **positive** gives a **positive**.

- $(+) \times (-)$:

$$\begin{aligned}(+3) \times (-2) &= 3 \times (-2)\\ &= (-2) + (-2) + (-2)\\ &= -6\end{aligned}$$

$$\begin{aligned}(+3) \times (-2) &= 3 \times \ominus\ominus\\ &= \ominus\ominus + \ominus\ominus + \ominus\ominus\\ &= -6\end{aligned}$$

So, a **positive** times a **negative** gives a **negative**.

- $(-) \times (+)$: Multiplication is commutative, so the order doesn't matter.

$$\begin{aligned}(-2) \times (+3) &= (+3) \times (-2)\\ &= -6\end{aligned}$$

So, a **negative** times a **positive** gives a **negative**.

- $(-) \times (-)$:

$$\begin{aligned}0 \times (-2) &= 0\\ ((+3) + (-3)) \times (-2) &= 0\\ ((+3) \times (-2)) + ((-3) \times (-2)) &= 0\\ (-6) + ((-3) \times (-2)) &= 0\\ (-3) \times (-2) &= +6\end{aligned}$$

So, a **negative** times a **negative** gives a **positive**.

Definition **Multiplication**

- $(+) \times (+) = (+)$: a **positive** times a **positive** gives a **positive**.
- $(+) \times (-) = (-)$: a **positive** times a **negative** gives a **negative**.
- $(-) \times (+) = (-)$: a **negative** times a **positive** gives a **negative**.
- $(-) \times (-) = (+)$: a **negative** times a **negative** gives a **positive**.

Ex: Calculate $(+2) \times (-5)$

Answer: $(+2) \times (-5) = -10$ as $(+) \times (-) = (-)$

G DIVISION

Discover: Multiplication and division are inverse operations:

$$3 \times 2 = 12, \text{ so } 6 \div 3 = 2$$

Now, let's look at division with negative numbers:

- $(+) \div (+)$:

$$(+3) \times (+2) = +6, \text{ so } (+6) \div (+3) = (+2)$$

So, a **positive** divided by a **positive** gives a **positive**.

- $(+) \div (-)$:

$$(-3) \times (-2) = +6, \text{ so } (+6) \div (-3) = (-2)$$

So, a **positive** divided by a **negative** gives a **negative**.

- $(-) \div (+)$:

$$(+3) \times (-2) = -6, \text{ so } (-6) \div (+3) = (-2)$$

So, a **negative** divided by a **positive** gives a **negative**.

- $(-) \div (-)$:

$$(-3) \times (+2) = -6, \text{ so } (-6) \div (-3) = (+2)$$

So, a **negative** divided by a **negative** gives a **positive**.

Definition **Division**

- $(+) \div (+) = (+)$:a **positive** divided by a **positive** gives a **positive**.
- $(+) \div (-) = (-)$: a **positive** divided by a **negative** gives a **negative**.
- $(-) \div (+) = (-)$: a **negative** divided by a **positive** gives a **negative**.
- $(-) \div (-) = (+)$: a **negative** divided by a **negative** gives a **positive**.

Ex: Calculate $(+10) \div (-5)$

Answer: $(+10) \div (-5) = -2$ as $(+) \div (-) = (-)$