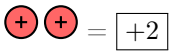
INTEGERS

A DEFINITION

A.1 COUNTING POSITIVE AND NEGATIVE NUMBERS

Ex 1:

 $= \boxed{+2}$

Answer:

- There are 2 positive units.
-  $= +2$

Ex 2:

 $= \boxed{-3}$

Answer:

- There are 3 negative units.
-  $= -3$

Ex 3:

 $= \boxed{-5}$

Answer:

- There are 5 negative units.
-  $= -5$

Ex 4:

 $= \boxed{+3}$

Answer:

- There are 3 positive units.
-  $= +3$

Ex 5:

 $= \boxed{-1}$

Answer:

- There is 1 negative unit.
-  $= -1$

A.2 WRITING INTEGERS FROM WORDS

Ex 6: Positive two is $\boxed{+2}$.

Answer:

- Positive two is $+2 =$ 

Ex 7: Negative three is $\boxed{-3}$.

Answer:

- Negative three is $-3 =$ 

Ex 8: Negative four is $\boxed{-4}$.

Answer:

- Negative four is $-4 =$ 

Ex 9: Positive five is $\boxed{+5}$.

Answer:

- Positive five is $+5 =$ 

Ex 10: Negative two is $\boxed{-2}$.

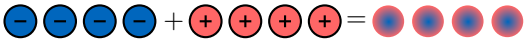
Answer:

- Negative two is $-2 =$ 

A.3 FINDING THE OPOSITE

Ex 11: The opposite of -4 is $\boxed{+4}$.

Answer:

- 
- $(-4) + (+4) = 0$
- The opposite of -4 is $+4$.

Ex 12: The opposite of -3 is $\boxed{+3}$.

Answer:

- 
- $(-3) + (+3) = 0$
- The opposite of -3 is $+3$.

Ex 13: The opposite of $+5$ is $\boxed{-5}$.

Answer:

- 
- $(+5) + (-5) = 0$
- The opposite of $+5$ is -5.

Ex 14: The opposite of $+1$ is $\boxed{-1}$.

Answer:

- 
- $(+1) + (-1) = 0$
- The opposite of $+1$ is -1.

Ex 15: The opposite of 0 is $\boxed{0}$.

Answer:

- 
- $0 + 0 = 0$
- The opposite of 0 is 0.

A.4 FINDING THE OPPOSITE FOR DECIMAL NUMBERS

Ex 16: The opposite of -4.1 is $\boxed{+4.1}$.

Answer:

- $(-4.1) + (+4.1) = 0$
- The opposite of $-4{,}1$ is $+4{,}1$.

Ex 17: The opposite of -0.5 is $\boxed{+0.5}$.

Answer:

- $(-0.5) + (+0.5) = 0$
- The opposite of $-0{,}5$ is $+0{,}5$.

Ex 18: The opposite of $+3.5$ is $\boxed{-3.5}$.

Answer:

- $(+3.5) + (-3.5) = 0$
- The opposite of $+3{,}5$ is $-3{,}5$.

Ex 19: The opposite of $+99.5$ is $\boxed{-99.5}$.

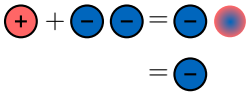
Answer:

- $99.5 = +99.5$
- $(+99.5) + (-99.5) = 0$
- The opposite of $+99{,}5$ is $-99{,}5$.

A.5 ADDING SMALL INTEGERS

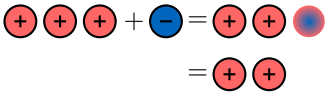
Ex 20: $(+1) + (-2) = \boxed{-1}$

Answer:

- 
- $(+1) + (-2) = -1$

Ex 21: $(+3) + (-1) = \boxed{+2}$

Answer:

- 
- $(+3) + (-1) = +2$

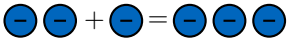
Ex 22: $(+2) + (-3) = \boxed{-1}$

Answer:

- 
- $(+2) + (-3) = -1$

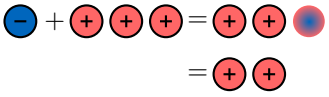
Ex 23: $(-2) + (-1) = \boxed{-3}$

Answer:

- 
- $(-2) + (-1) = -3$

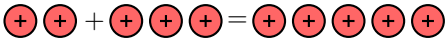
Ex 24: $(-1) + (+3) = \boxed{+2}$

Answer:

- 
- $(-1) + (+3) = +2$

Ex 25: $(+2) + (+3) = \boxed{+5}$

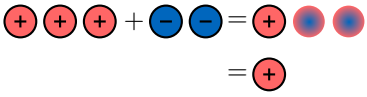
Answer:

- 
- $(+2) + (+3) = +5$

A.6 FINDING MISSING NUMBERS IN ADDITION

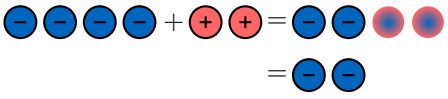
Ex 26: $(+3) + (\boxed{-2}) = +1$

Answer:

- 
- $(+3) + (-2) = +1$

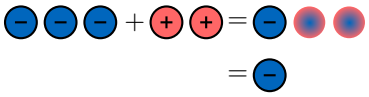
Ex 27: $(-4) + (\boxed{+2}) = -2$

Answer:

- 
- $(-4) + (+2) = -2$

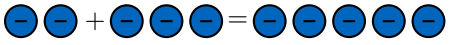
Ex 28: $(\boxed{-3}) + (+2) = -1$

Answer:

- 
- $(-3) + (+2) = -1$

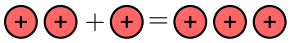
Ex 29: $(-2) + (\boxed{-3}) = -5$

Answer:

- 
- $(-2) + (-3) = -5$

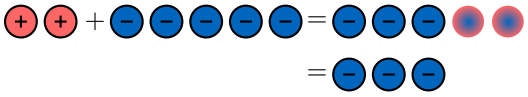
Ex 30: $(\boxed{+2}) + (+1) = +3$

Answer:

- 
- $(+2) + (+1) = +3$

Ex 31: $(+2) + (\boxed{-5}) = -3$

Answer:

- 
- $(+2) + (-5) = -3$

A.7 FINDING THE ABSOLUTE VALUE

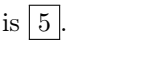
Ex 32: The absolute value of $+2$ is $\boxed{2}$.

Answer: The absolute value of $+2$ =  is 2.

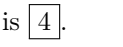
Ex 33: The absolute value of -3 is $\boxed{3}$.

Answer: The absolute value of -3 =  is 3.

Ex 34: The absolute value of $+5$ is $\boxed{5}$.

Answer: The absolute value of $+5$ =  is 5.

Ex 35: The absolute value of -4 is $\boxed{4}$.

Answer: The absolute value of -4 =  is 4.

Ex 36: The absolute value of -9 is $\boxed{9}$.

Answer: The absolute value of -9 =  is 9.

A.8 FINDING THE ABSOLUTE VALUE FOR DECIMAL NUMBERS

Ex 37: The absolute value of -2.1 is $\boxed{2.1}$.

Answer: The absolute value of -2.1 is 2.1

Ex 38: The absolute value of -5.4 is $\boxed{5.4}$.

Answer: The absolute value of -5.4 is 5.4.

Ex 39: The absolute value of 3.7 is $\boxed{3.7}$.

Answer: The absolute value of 3.7 is 3.7.

Ex 40: The absolute value of 0 is $\boxed{0}$.

Answer: The absolute value of 0 is 0.

B RULES OF ADDITION

B.1 ADDING INTEGERS

Ex 41: $(+6) + (-4) = \boxed{+2}$

Answer:

- When the signs are opposite, subtract the absolute values $(6 - 4 = 2)$, and take the sign of the larger number: $6 > 4$, so the result is $+$.
- $(+6) + (-4) = +2$
- 

Ex 42: $(+4) + (+7) = \boxed{+11}$

Answer:

- When the signs are the same, add the absolute values $(4 + 7 = 11)$, and keep the positive sign.
- $(+4) + (+7) = +11$
- 

Ex 43: $(-5) + (+8) = \boxed{+3}$

Answer:

- When the signs are opposite, subtract the absolute values $(8 - 5 = 3)$, and take the sign of the larger number: $8 > 5$, so the result is $+$.
- $(-5) + (+8) = +3$
- 

Ex 44: $(+6) + (-4) = \boxed{+2}$

Answer:

- When the signs are opposite, subtract the absolute values $(6 - 4 = 2)$, and take the sign of the larger number: $6 > 4$, so the result is $+$.
- $(+6) + (-4) = +2$
- 

Ex 45: $(-5) + (-4) = \boxed{-9}$

Answer:

- When the signs are the same, add the absolute values $(5 + 4 = 9)$, and keep the negative sign.
- $(-5) + (-4) = -9$
- 

B.2 ADDING INTEGERS WITHOUT EXPLICIT SIGNS

Ex 46: $6 + (-4) =$ $\boxed{+2}$

Answer:

- $6 + (-4) = (+6) + (-4)$
- When the signs are opposite, subtract the absolute values ($6 - 4 = 2$), and take the sign of the larger number: $6 > 4$, so the result is $+$.
- $(+6) + (-4) = +2$
- 

Ex 47: $-5 + 8 =$ $\boxed{+3}$

Answer:

- $-5 + 8 = (-5) + (+8)$
- When the signs are opposite, subtract the absolute values ($8 - 5 = 3$), and take the sign of the larger number: $8 > 5$, so the result is $+$.
- $(-5) + (+8) = +3$
- 

Ex 48: $-2 + (-3) =$ $\boxed{-5}$

Answer:

- $-2 + (-3) = (-2) + (-3)$
- When the signs are the same, add the absolute values ($2 + 3 = 5$), and keep the negative sign.
- $(-2) + (-3) = -5$
- 

Ex 49: $-6 + 0 =$ $\boxed{-6}$

Answer:

- $-6 + 0 = (-6) + 0$
- Adding zero to any number does not change the value, so the result is -6.
- $(-6) + 0 = -6$

B.3 ADDING SIGNED DECIMAL NUMBERS

Ex 50: $-5 + 8.1 =$ $\boxed{+3.1}$

Answer:

- $-5 + 8.1 = (-5) + (+8.1)$
- When the signs are opposite, subtract the absolute values ($8.1 - 5 = 3.1$), and take the sign of the larger number: $8.1 > 5$, so the result is $+$.
- $(-5) + (+8.1) = +3.1$

Ex 51: $-3 + (-2.5) =$ $\boxed{-5.5}$

Answer:

- $-3 + (-2.5) = (-3) + (-2.5)$
- When the signs are the same, add the absolute values ($3 + 2.5 = 5.5$), and keep the negative sign.
- $(-3) + (-2.5) = -5.5$

Ex 52: $-1.6 + (+2.6) =$ $\boxed{+1}$

Answer:

- $-1.6 + (+2.6) = (-1.6) + (+2.6)$
- When the signs are opposite, subtract the absolute values ($2.6 - 1.6 = 1.0$), and take the sign of the larger number: $2.6 > 1.6$, so the result is $+$.
- $(-1.6) + (+2.6) = +1.0$

Ex 53: $-3.5 + (+1.5) =$ $\boxed{-2}$

Answer:

- $-3.5 + (+1.5) = (-3.5) + (+1.5)$
- When the signs are opposite, subtract the absolute values ($3.5 - 1.5 = 2.0$), and take the sign of the larger number: $3.5 > 1.5$, so the result is $-$.
- $(-3.5) + (+1.5) = -2.0$

B.4 ADDING MULTIPLE INTEGERS

Ex 54: Calculate:

$$(+3) + (-7) + (-5) = \boxed{-9}$$

Answer:

$$\begin{aligned} (+3) + (-7) + (-5) &= (-4) + (-5) && ((+3)+(-7)=(-4)) \\ &= (-9) && ((-4)+(-5)=(-9)) \end{aligned}$$

Ex 55: Calculate:

$$(-2) + (-4) + (+7) = \boxed{+1}$$

Answer:

$$\begin{aligned} (-2) + (-4) + (+7) &= (-6) + (+7) && ((-2)+(-4)=(-6)) \\ &= +1 && ((-6)+(+7)=+1) \end{aligned}$$

Ex 56: Calculate:

$$(-2) + (+4) + (-2) = \boxed{0}$$

Answer:

$$\begin{aligned} (-2) + (+4) + (-2) &= (+2) + (-2) && ((-2)+(+4)=+2) \\ &= 0 && ((+2)+(-2)=0) \end{aligned}$$

Ex 57: Calculate:

$$(-10) + (+3) + (-7) = \boxed{-14}$$

Answer:

$$\begin{aligned} (-10) + (+3) + (-7) &= (-7) + (-7) && ((-10)+(+3)=-7) \\ &= -14 && ((-7)+(-7)=-14) \end{aligned}$$

B.5 ADDING INTEGERS IN REAL-WORLD PROBLEMS

Ex 58: During a hike, the hiker experiences altitude changes as follows. Positive numbers indicate climbing (gaining altitude), while negative numbers indicate descending (losing altitude):

- The hiker starts at an altitude of +300 meters.
- They climb +150 meters in the morning.
- In the afternoon, they descend by −200 meters.

What is the hiker's final altitude at the end of the day?

$\boxed{250}$ meters

Answer:

$$\begin{aligned}(+300)+(+150)+(-200) &= (+450)+(-200) && ((+300)+(+150)=+450)\\ &= +250 && ((+450)+(-200)=+250)\end{aligned}$$

Ex 59: A person keeps track of their bank account balance as follows. Positive numbers indicate deposits (money added), while negative numbers indicate withdrawals (money taken out):

- The person starts with +50 dollars in their account.
- They deposit +30 dollars.
- Later, they withdraw −40 dollars.

What is the person's final balance?

$\boxed{40}$ dollars

Answer:

$$\begin{aligned}(+50)+(+30)+(-40) &= (+80)+(-40) && ((+50)+(+30)=+80)\\ &= +40 && ((+80)+(-40)=+40)\end{aligned}$$

Ex 60: In a round of golf, each hole has a "par" score, and a player's score is based on how many strokes they take compared to par. A score of 0 means the player made par, a positive number means they took extra strokes (over par), and a negative number means they made fewer strokes (under par).

- Hole 1: Par 3, player scored −1 (under par)
- Hole 2: Par 4, player scored +2 (over par)
- Hole 3: Par 5, player scored 0 (made par)
- Hole 4: Par 3, player scored +1 (over par)
- Hole 5: Par 4, player scored −2 (under par)

What is the player's total score?

$\boxed{0}$

Answer:

$$\begin{aligned}&(-1)+(+2)+(0)+(+1)+(-2)\\ &\quad= (+1)+(0)+(+1)+(-2) && ((-1)+(+2)=(+1))\\ &\quad= (+1)+(+1)+(-2) && ((+1)+(0)=(+1))\\ &\quad= (+2)+(-2) && ((+1)+(+1)=(+2))\\ &\quad= 0 && ((+2)+(-2)=0)\end{aligned}$$

Ex 61: Throughout the day, the temperature in a city changes as follows. Positive numbers indicate a rise in temperature, while negative numbers indicate a drop in temperature:

- In the morning, the temperature started at +5°C and dropped by −3°C.
- In the afternoon, the temperature rose by +4°C.
- In the evening, the temperature dropped again by −6°C.
- At night, it dropped further by −1°C.

What is the temperature at the end of the day?

$\boxed{-1}°C$

Answer:

$$\begin{aligned}&(+5)+(-3)+(+4)+(-6)+(-1)\\ &\quad= (+2)+(+4)+(-6)+(-1) && ((+5)+(-3)=(+2))\\ &\quad= (+6)+(-6)+(-1) && ((+2)+(+4)=(+6))\\ &\quad= 0+(-1) && ((+6)+(-6)=0)\\ &\quad= -1 && (0+(-1)=-1)\end{aligned}$$

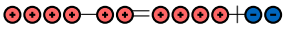
C SUBTRACTION

C.1 CONVERTING SUBTRACTION TO ADDITION

Ex 62: Convert the subtracting in addition:

$$(+4)-(+2)=(\boxed{+4})+(\boxed{-2})$$

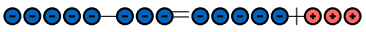
Answer:

- $(+4)-(+2)=(+4)+(-2)$
- 

Ex 63: Convert the subtraction into addition:

$$(-5)-(-3)=(\boxed{-5})+(\boxed{+3})$$

Answer:

- $(-5)-(-3)=(-5)+(+3)$
- 

Ex 64: Convert the subtraction into addition:

$$(+4)-(-2)=(\boxed{+4})+(\boxed{+2})$$

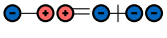
Answer:

- $(+4)-(-2)=(+4)+(+2)$
- 

Ex 65: Convert the subtraction into addition:

$$(-1)-(+2)=(\boxed{-1})+(\boxed{-2})$$

Answer:

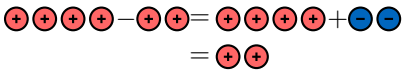
- $(-1)-(+2)=(-1)+(-2)$
- 

C.2 SUBTRACTING INTEGERS STEP BY STEP

Ex 66: Calculate:

$$(+4) - (+2) = (\boxed{+4}) + (\boxed{-2})$$
$$= \boxed{2}$$

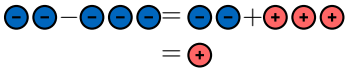
Answer:

- $(+4) - (+2) = (+4) + (-2)$
  $= (+2)$
- 

Ex 67: Calculate:

$$(-2) - (-3) = (\boxed{-2}) + (\boxed{+3})$$
$$= \boxed{+1}$$

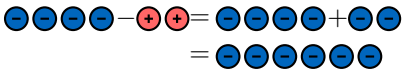
Answer:

- $(-2) - (-3) = (-2) + (+3)$
  $= (+1)$
- 

Ex 68: Calculate:

$$(-4) - (+2) = (\boxed{-4}) + (\boxed{-2})$$
$$= \boxed{-6}$$

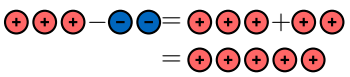
Answer:

- $(-4) - (+2) = (-4) + (-2)$
  $= (-6)$
- 

Ex 69: Calculate:

$$(+3) - (-2) = (\boxed{+3}) + (\boxed{+2})$$
$$= \boxed{+5}$$

Answer:

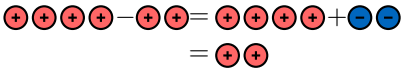
- $(+3) - (-2) = (+3) + (+2)$
  $= (+5)$
- 

C.3 SUBTRACTING INTEGERS

Ex 70: Calculate:

$$(+4) - (+2) = \boxed{2}$$

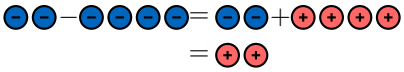
Answer:

- $(+4) - (+2) = (+4) + (-2)$
  $= (+2)$
- 

Ex 71: Calculate:

$$(-2) - (-4) = \boxed{+2}$$

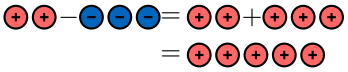
Answer:

- $(-2) - (-4) = (-2) + (+4)$
  $= (+2)$
- 

Ex 72: Calculate:

$$(+2) - (-3) = \boxed{+5}$$

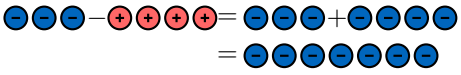
Answer:

- $(+2) - (-3) = (+2) + (+3)$
  $= (+5)$
- 

Ex 73: Calculate:

$$(-3) - (+4) = \boxed{-7}$$

Answer:

- $(-3) - (+4) = (-3) + (-4)$
  $= (-7)$
- 

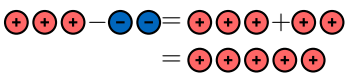
C.4 SUBTRACTING INTEGERS WITHOUT EXPLICIT SIGNS

Ex 74: Calculate:

$$3 - (-2) = \boxed{5}$$

Answer:

- $3 - (-2) = (+3) - (-2)$
  $= (+3) + (+2)$
  $= (+5)$

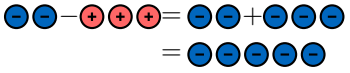


Ex 75: Calculate:

$$(-2) - 3 = \boxed{-5}$$

Answer:

- $(-2) - 3 = (-2) - (+3)$
  $= (-2) + (-3)$
  $= (-5)$

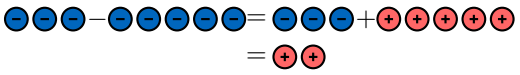


Ex 76: Calculate:

$$(-3) - (-5) = \boxed{+2}$$

Answer:

- $(-3) - (-5) = (-3) - (-5)$
  $= (-3) + (+5)$
  $= (+2)$

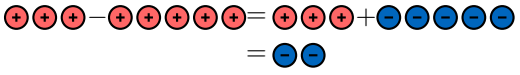


Ex 77: Calculate:

$$3 - 5 = \boxed{-2}$$

Answer:

- $3 - 5 = (+3) - (+5)$
  $= (+3) + (-5)$
  $= (-2)$



C.5 ADDING/SUBTRACTING MULTIPLE INTEGERS

Ex 78: Calculate:

$$(+3) - (-7) - (+5) = \boxed{+5}$$

Answer:

$$\begin{aligned}(+3) - (-7) - (-5) &= (+3) + (+7) + (-5) && \text{(subtraction to addition)}\\ &= (+10) + (-5) && ((+3)+(+7)=(+10))\\ &= (+5) && ((+10)+(-5)=(+5))\end{aligned}$$

Ex 79: Calculate:

$$(-2) - (-3) + (-2) = \boxed{-1}$$

Answer:

$$\begin{aligned}(-2) - (-3) + (-2) &= (-2) + (+3) + (-2) && \text{(subtraction to addition)}\\ &= (+1) + (-2) && ((-2)+(+3)=(+1))\\ &= (-1) && ((+1)+(-2)=(-1))\end{aligned}$$

Ex 80: Calculate:

$$(-5) - (-4) + (-3) = \boxed{-4}$$

Answer:

$$\begin{aligned}(-5) - (-4) + (-3) &= (-5) + (+4) + (-3) && \text{(subtraction to addition)}\\ &= (-1) + (-3) && ((-5)+(+4)=(-1))\\ &= (-4) && ((-1)+(-3)=(-4))\end{aligned}$$

Ex 81: Calculate:

$$(+6) - (-3) + (-4) = \boxed{+5}$$

Answer:

$$\begin{aligned}(+6) - (-3) + (-4) &= (+6) + (+3) + (-4) && \text{(subtraction to addition)}\\ &= (+9) + (-4) && ((+6)+(+3)=(+9))\\ &= (+5) && ((+9)+(-4)=(+5))\end{aligned}$$

C.6 SUBTRACTING INTEGERS IN REAL-WORLD PROBLEMS

Ex 82: In the morning, the temperature was -7°C, and by the evening, the temperature was -2°C.
Find the variation of temperature.

$$\boxed{5}\,°\text{C}$$

Answer:

$$\begin{aligned}\text{Temperature variation} &= \text{Final Temperature} - \text{Initial Temperature}\\ &= (-2) - (-7)\\ &= (-2) + (+7) && \text{(subtraction to addition)}\\ &= +5\ \text{C}^\circ\end{aligned}$$

Ex 83: In the morning, your bank account balance was -50 dollars, and by the evening, it was $+30$ dollars.
Find the change in your bank account balance.

$$\boxed{80}\ \text{dollars}$$

Answer:

$$\begin{aligned}\text{Change in Balance} &= \text{Final Balance} - \text{Initial Balance}\\ &= 30 - (-50)\\ &= 30 + 50 && \text{(subtraction to addition)}\\ &= 80\ \text{dollars}\end{aligned}$$

Ex 84: A lift starts at the 5th floor and descends to the -2nd floor (below ground level).
Find the change in the lift's position.

$$\boxed{-7}\ \text{floors}$$

Answer:

$$\begin{aligned}\text{Change in Position} &= \text{Final Floor} - \text{Initial Floor}\\ &= (-2) - 5\\ &= (-2) + (-5) && \text{(subtraction to addition)}\\ &= -7\ \text{floors}\end{aligned}$$

Ex 85: he GDP (Gross Domestic Product) growth rate of a country was -2% in 2024. It was $+3\%$ in 2025.
Find the variation in the GDP growth rate.

$\boxed{5}\%$

Answer:

GDP Variation = Final GDP − Initial GDP

Variation du PIB = Taux de croissance final du PIB − Taux de croissance initial du PIB

$= (+3) - (-2)$

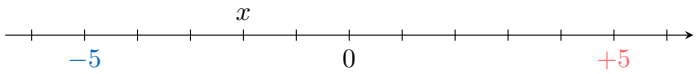
$= (+3) + (+2)$ (subtraction to addition)

$= +5\%$

D ON THE NUMBER LINE

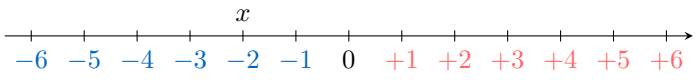
D.1 FINDING X ON THE NUMBER LINE

Ex 86: Find the value of x.



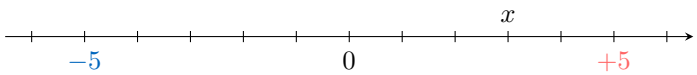
$x = \boxed{-2}$

Answer:



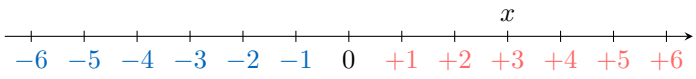
- $x = -2$.

Ex 87: Find the value of x.



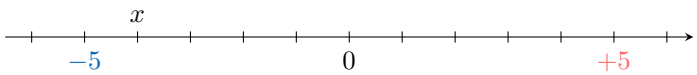
$x = \boxed{+3}$

Answer:



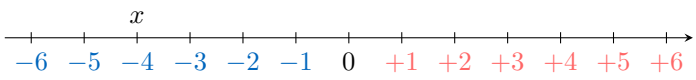
- $x = +3$.

Ex 88: Find the value of x.



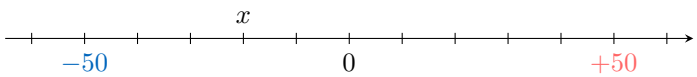
$x = \boxed{-4}$

Answer:



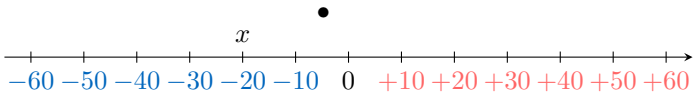
- $x = -4$.

Ex 89: Find the value of x.



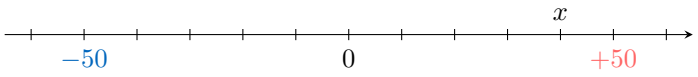
$x = \boxed{-20}$

Answer:



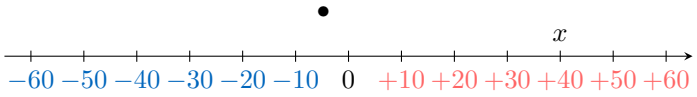
- $x = -20$.

Ex 90: Find the value of x.



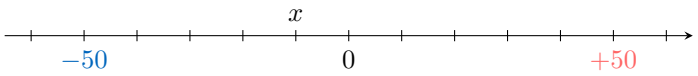
$x = \boxed{+40}$

Answer:



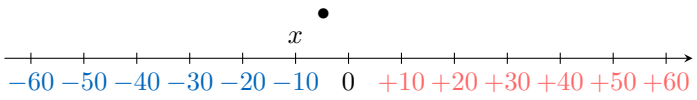
- $x = +40$.

Ex 91: Find the value of x.



$x = \boxed{-10}$

Answer:



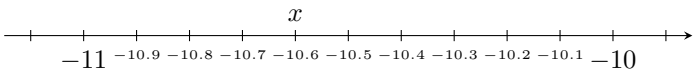
- $x = -10$.

D.2 FINDING DECIMAL NUMBERS ON THE NUMBER LINE

Ex 92: Find the value of x.

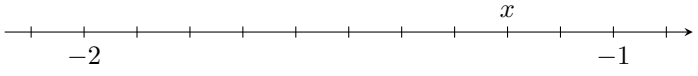
$x = \boxed{-10.6}$

Answer:



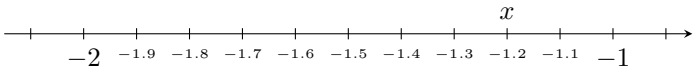
- $x = -10.6$.

Ex 93: Find the value of x.



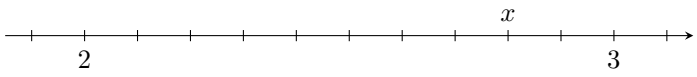
$x = \boxed{-1.2}$

Answer:



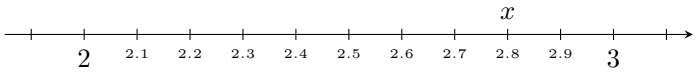
- $x = -1.2$.

Ex 94: Find the value of x.



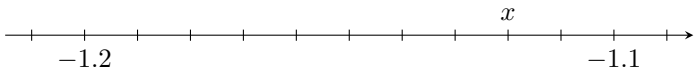
$x =$ 2.8

Answer:

- 

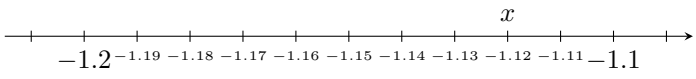
- $x = 2.8$.

Ex 95: Find the value of x.



$x =$ −1.12

Answer:

- 

- $x = -1.12$.

E ORDERING

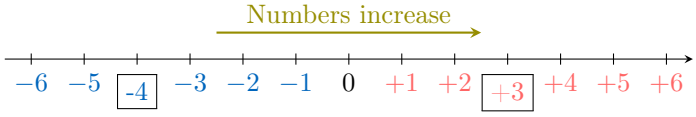
E.1 COMPARING SMALL INTEGERS

Ex 96: Compare the numbers: −4 < +3

Answer:

- +3 is positive, and −4 is negative. A positive number is always greater than a negative number:

$$-4 < +3$$

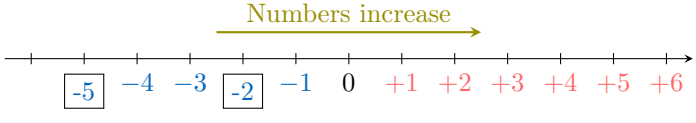
- 

Ex 97: Compare the numbers: −2 > −5

Answer:

- Both numbers are negative, but −2 is closer to zero than −5. Therefore, −2 is greater than −5:

$$-2 > -5$$

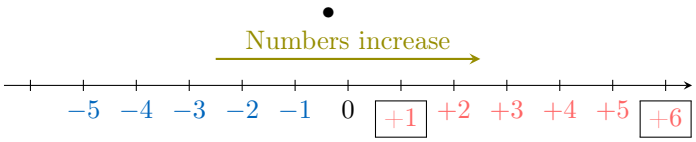
- 

Ex 98: Compare the numbers: +6 > +1

Answer:

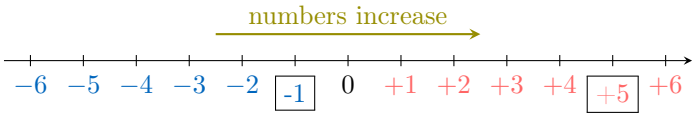
- Both numbers are positive, but +6 is farther from zero than +1. Therefore, +6 is greater than +1:

$$+6 > +1$$

- 

Ex 99: Compare: −1 < +5

Answer:

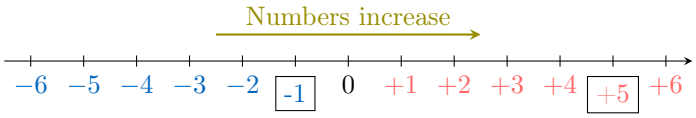
- As +5 is positive and −1 is negative, the positive number is greater than the negative number:−1 < +5
- 

Ex 100: Compare the numbers: −1 < +5

Answer:

- +5 is positive and −1 is negative. A positive number is always greater than a negative number:

$$-1 < +5$$

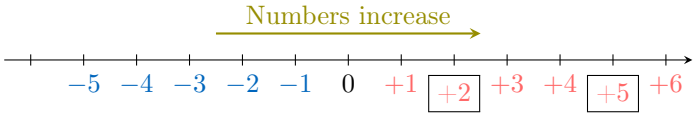
- 

Ex 101: Compare the numbers: +2 < +5

Answer:

- Both numbers are positive, but +5 is farther from zero than +2:

$$+2 < +5$$

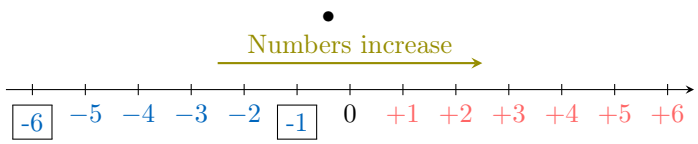
- 

Ex 102: Compare the numbers: −6 < −1

Answer:

- Both numbers are negative, but −1 is closer to zero than −6:

$$-6 < -1$$

- 

E.2 COMPARING INTEGERS

Ex 103: Compare the numbers: $-20 \boxed{<} 1$

Answer:

- 1 is positive and -20 is negative. A positive number is always greater than a negative number:

$$-20 < +1$$

Ex 104: Compare the numbers: $-99 \boxed{>} -100$

Answer:

- Both numbers are negative, but -99 is closer to zero than -100:

$$-99 > -100$$

Ex 105: Compare the numbers: $234 \boxed{>} -1200$

Answer:

- $+234$ is positive and -1200 is negative. A positive number is always greater than a negative number:

$$+234 > -1200$$

Ex 106: Compare the numbers: $-18 \boxed{<} -3$

Answer:

- Both numbers are negative, but -3 is closer to zero than -18:

$$-18 < -3$$

Ex 107: Compare the numbers: $230 \boxed{>} 200$

Answer:

- Both numbers are positive, but $+230$ is farther from zero than $+200$:

$$+230 > +200$$

Ex 108: Compare the numbers: $99 \boxed{>} -100$

Answer:

- $+99$ is positive and -100 is negative. A positive number is always greater than a negative number:

$$+99 > -100$$

E.3 COMPARING INTEGERS IN REAL-WORLD PROBLEMS

MCQ 109: During a golf tournament, the scores relative to par for five different holes were:

$$-2, -1, 0, +1, -3$$

Order these scores from the best performance (most under par) to the worst performance (above par).

Choose one answer:

☐ $0 < +1 < -1 < -2 < -3$

☒ $-3 < -2 < -1 < 0 < +1$

☐ $+1 > 0 > -1 > -2 > -3$

☐ $-1 < -2 < -3 < 0 < +1$

Answer:

- From the best performance to the worst: $-3 < -2 < -1 < 0 < +1$

MCQ 110: Given the depths of various lakes below sea level:

Lake	Depth below sea level
Lake Assal, Djibouti	-155 m
Death Valley, USA	-86 m
Caspian Sea, Central Asia	-28 m
Sea of Galilee, Israel	-214 m

Which lake is the deepest below sea level?

Choose one answer:

☐ Lake Assal, Djibouti

☐ Death Valley, USA

☐ Caspian Sea, Central Asia

☒ Sea of Galilee, Israel

Answer: The lake that is the deepest below sea level is the Sea of Galilee, Israel, with a depth of -214 m.

MCQ 111: The recorded temperatures in a particular week were:

$$-2.5°\text{C}, 1.2°\text{C}, -0.8°\text{C}, 0.5°\text{C}, -3.2°\text{C}$$

Order these temperatures from coldest to warmest.

Choose one answer:

☐ $1.2°\text{C} < 0.5°\text{C} < -0.8°\text{C} < -2.5°\text{C}$

☒ $-3.2°\text{C} < -2.5°\text{C} < -0.8°\text{C} < 0.5°\text{C} < 1.2°\text{C}$

☐ $1.2°\text{C} > 0.5°\text{C} > -0.8°\text{C} > -3.2°\text{C}$

☐ $-0.8°\text{C} < -2.5°\text{C} < -3.2°\text{C} < 0.5°\text{C} < 1.2°\text{C}$

Answer:

- From coldest to warmest: $-3.2°\text{C} < -2.5°\text{C} < -0.8°\text{C} < 0.5°\text{C} < 1.2°\text{C}$

MCQ 112: Given the years of significant events in Ancient Roman history:

Event	Year
Founding of Rome	-753
End of the Roman Republic	-27
Sacking of Rome by the Gauls	-390
Julius Caesar's assassination	-44

Which event happened the earliest?

Choose one answer:

☒ Founding of Rome

☐ End of the Roman Republic

☐ Sacking of Rome by the Gauls

☐ Julius Caesar's assassination

Answer: The event that happened the earliest was the Founding of Rome in -753.

F MULTIPLICATION

F.1 MULTIPLYING SMALL INTEGERS

Ex 113: Calculate:

$$(-2) \times (-7) = \boxed{+14}$$

Answer:

- Multiply the absolute values: $2 \times 7 = 14$.
- When you multiply two negative numbers, the result is positive: $(-) \times (-) = (+)$.
- Therefore, $(-2) \times (-7) = +14$.

Ex 114: Calculate:

$$(-4) \times (+6) = \boxed{-24}$$

Answer:

- Multiply the absolute values: $4 \times 6 = 24$.
- When you multiply a negative number by a positive number, the result is negative: $(-) \times (+) = (-)$.
- Therefore, $(-4) \times (+6) = -24$.

Ex 115: Calculate:

$$(+5) \times (-3) = \boxed{-15}$$

Answer:

- Multiply the absolute values: $5 \times 3 = 15$.
- When you multiply a positive number by a negative number, the result is negative: $(+) \times (-) = (-)$.
- Therefore, $(+5) \times (-3) = -15$.

Ex 116: Calculate:

$$(-6) \times (-2) = \boxed{+12}$$

Answer:

- Multiply the absolute values: $6 \times 2 = 12$.
- When you multiply two negative numbers, the result is positive: $(-) \times (-) = (+)$.
- Therefore, $(-6) \times (-2) = +12$.

Ex 117: Calculate:

$$(+3) \times (+5) = \boxed{+15}$$

Answer:

- Multiply the absolute values: $3 \times 5 = 15$.
- Multiplying two positive numbers gives a positive result: $(+) \times (+) = (+)$.
- Therefore, $(+3) \times (+5) = +15$.

Ex 118: Calculate:

$$(-1) \times (-1) = \boxed{+1}$$

Answer:

- Multiply the absolute values: $1 \times 1 = 1$.
- When you multiply two negative numbers, the result is positive: $(-) \times (-) = (+)$.
- Therefore, $(-1) \times (-1) = +1$.

F.2 MULTIPLYING INTEGERS WITHOUT EXPLICIT SIGNS

Ex 119: Calculate:

$$3 \times (-2) = \boxed{-6}$$

Answer:

$$\begin{aligned} 3 \times (-2) &= (+3) \times (-2) \\ &= -6 \qquad ((+) \times (-) = (-)) \end{aligned}$$

Ex 120: Calculate:

$$(-3) \times 8 = \boxed{-24}$$

Answer:

$$\begin{aligned} (-3) \times 8 &= (-3) \times (+8) \\ &= -24 \qquad ((-) \times (+) = (-)) \end{aligned}$$

Ex 121: Calculate:

$$(-5) \times (-8) = \boxed{+40}$$

Answer:

$$\begin{aligned} (-5) \times (-8) &= (-5) \times (-8) \\ &= +40 \qquad ((-) \times (-) = (+)) \end{aligned}$$

Ex 122: Calculate:

$$(-6) \times 9 = \boxed{-54}$$

Answer:

$$\begin{aligned} (-6) \times 9 &= (-6) \times (+9) \\ &= -54 \qquad ((-) \times (+) = (-)) \end{aligned}$$

F.3 CALCULATING POWERS OF NEGATIVE NUMBERS

Ex 123: Calculate:

$$(-2)^2 = \boxed{4}$$

Answer:

$$\begin{aligned} (-2)^2 &= (-2) \times (-2) \\ &= +4 \qquad ((-) \times (-) = (+)) \end{aligned}$$

Ex 124: Calculate:

$$(-4)^2 = \boxed{16}$$

Answer:

$$\begin{aligned}(-4)^2 &= (-4) \times (-4) \\ &= +16 \qquad ((-) \times (-) = (+))\end{aligned}$$

Ex 125: Calculate:

$$(-1)^3 = \boxed{-1}$$

Answer:

$$\begin{aligned}(-1)^3 &= (-1) \times (-1) \times (-1) \\ &= (+1) \times (-1) \qquad ((-1) \times (-1) = (+1)) \\ &= -1\end{aligned}$$

Ex 126: Calculate:

$$(-2)^3 = \boxed{-8}$$

Answer:

$$\begin{aligned}(-2)^3 &= (-2) \times (-2) \times (-2) \\ &= (+4) \times (-2) \qquad ((-2) \times (-2) = (+4)) \\ &= -8\end{aligned}$$

F.4 MULTIPLYING INTEGERS IN REAL-WORLD PROBLEMS

Ex 127: At midnight, the temperature was 0 degrees.
The temperature continued to change by -3 degrees every hour.
What was the temperature 4 hours later?

$$\boxed{-12}\,°C$$

Answer:

- The temperature changes by $-3°C$ every hour.
- In 4 hours, the total change in temperature is $4 \times -3 = -12°C$.
- Since the temperature started at $0°C$, 4 hours later, the temperature will be $0 + (-12) = -12°C$.

Ex 128: A diver starts at sea level (0 meters).
The diver descends 5 meters every minute.
How deep is the diver after 6 minutes?

$$\boxed{-30} \text{ meters}$$

Answer:

- The diver descends by 5 meters every minute.
- In 6 minutes, the total change in depth is $6 \times -5 = -30$ meters.
- Since the diver started at 0 meters, after 6 minutes, the diver will be $0 + (-30) = -30$ meters deep.

Ex 129: A hiker is at an altitude of 150 meters.
The hiker descends by 10 meters every minute.
What is the hiker's altitude after 7 minutes?

$$\boxed{80} \text{ meters}$$

Answer:

- The hiker descends by 10 meters every minute.
- In 7 minutes, the total descent is $7 \times -10 = -70$ meters.
- Starting at 150 meters, the hiker's altitude will be $150 + (-70) = 80$ meters after 7 minutes.

Ex 130: Your bank account has a balance of 90 euros.
You make a withdrawal of 40 euros every day for 5 days.
What is your account balance after 5 days?

$$\boxed{-110} \text{ euros}$$

Answer:

- You withdraw 40 euros every day.
- In 5 days, the total withdrawal is $5 \times -40 = -200$ euros.
- Starting with 90 euros, your balance will be $90 + (-200) = -110$ euros after 5 days.

G DIVISION

G.1 DIVIDING SMALL INTEGERS

Ex 131: Calculate:

$$(+8) \div (-2) = \boxed{-4}$$

Answer:

- Divide the absolute values: $8 \div 2 = 4$.
- When you divide a positive number by a negative number, the result is negative: $(+) \div (-) = (-)$.
- Therefore, $(+8) \div (-2) = -4$.

Ex 132: Calculate:

$$(-12) \div (-3) = \boxed{+4}$$

Answer:

- Divide the absolute values: $12 \div 3 = 4$.
- When you divide two negative numbers, the result is positive: $(-) \div (-) = (+)$.
- Therefore, $(-12) \div (-3) = +4$.

Ex 133: Calculate:

$$(-15) \div (+5) = \boxed{-3}$$

Answer:

- Divide the absolute values: $15 \div 5 = 3$.
- When you divide a negative number by a positive number, the result is negative: $(-) \div (+) = (-)$.
- Therefore, $(-15) \div (+5) = -3$.

Ex 134: Calculate:

$$(+20) \div (+4) = \boxed{+5}$$

Answer:

- Divide the absolute values: $20 \div 4 = 5$.
- When you divide two positive numbers, the result is positive: $(+) \div (+) = (+)$.
- Therefore, $(+20) \div (+4) = +5$.

G.2 DIVIDING INTEGERS WITHOUT EXPLICIT SIGNS

Ex 135: Calculate:

$$8 \div (-2) = \boxed{-4}$$

Answer:

$$\begin{aligned} 8 \div (-2) &= (+8) \div (-2) \\ &= -4 \qquad\qquad ((+) \div (-) = (-)) \end{aligned}$$

Ex 136: Calculate:

$$-12 \div (-3) = \boxed{+4}$$

Answer:

$$\begin{aligned} -12 \div (-3) &= (-12) \div (-3) \\ &= +4 \qquad\qquad ((-) \div (-) = (+)) \end{aligned}$$

Ex 137: Calculate:

$$-18 \div 6 = \boxed{-3}$$

Answer:

$$\begin{aligned} -18 \div 6 &= (-18) \div (+6) \\ &= -3 \qquad\qquad ((-) \div (+) = (-)) \end{aligned}$$

Ex 138: Calculate:

$$24 \div 4 = \boxed{6}$$

Answer:

$$\begin{aligned} 24 \div 4 &= (+24) \div (+4) \\ &= +6 \qquad\qquad ((+) \div (+) = (+)) \end{aligned}$$