HYPOTHESIS TESTING

A PROCEDURE OF STATISTICAL TESTING

A.1 LOGIC OF STATISTICAL TESTING

A.1.1 FORMULATING NULL AND ALTERNATIVE HYPOTHESES

Ex 1: A machine is programmed to fill juice bottles with an average of 500 ml. A quality control manager suspects that the calibration is off and that the mean volume is no longer 500 ml (it could be more or less). Write the null and alternative hypotheses.

Ex 2: A school claims that the average score of their students on a standardized math test is 75. The principal introduces a new teaching method and believes it has increased the average score. Write the null and alternative hypotheses.

Ex 3: A bank states that the average waiting time for customers in line is 15 minutes. They install a new fast-track system and claim that the average waiting time has been reduced. Write the null and alternative hypotheses.

Ex 4: A packet of chips is advertised to contain 150g. A consumer protection agency receives complaints that the packets are being underfilled (contain less than advertised). Write the null and alternative hypotheses to test this complaint.

A.2 P-VALUE AND SIGNIFICANCE LEVEL

A.2.1 INTERPRETING THE P-VALUE

Ex 5: A marketing team analyzes whether a new advertising campaign increased sales.

- H_0: The campaign had no effect on sales.
- H_1: The campaign increased sales.

The statistical test results in a very small p**-value of** 0.0004.
Is this result significant at the 0.1% level ($\alpha = 0.001$)? What does this imply for the marketing team?

Ex 6: Researchers are testing whether there is a difference in the effectiveness of two pain relief drugs, Drug A and Drug B.

- H_0: There is no difference in effectiveness between the two drugs.
- H_1: There is a difference in effectiveness between the two drugs.

The statistical test yields a p**-value of** 0.023.
State the conclusion at the 5% significance level ($\alpha = 0.05$). What does this mean for the researchers?

Ex 7: A school principal introduces a new teaching method to improve student test scores.

- H_0: The mean test score remains $\mu = 72$ (The method has no effect).
- H_1: The mean test score is $\mu > 72$ (The method improves scores).

The statistical test yields a p**-value of** 0.041.
State the conclusion at the 5% significance level ($\alpha = 0.05$). Is the new method considered effective?

Ex 8: An ecologist is testing whether the acidity level (pH) of a lake has changed from its normal level of 7.0.

- $H_0 : \mu = 7.0$.
- $H_1 : \mu \neq 7.0$.

The test produces a p**-value of** 0.08.Explain the decision that would be made if the ecologist uses a 10% significance level versus a 5% significance level.

A.3 5-STEP PROCEDURE

A.3.1 IDENTIFYING THE STEPS OF A HYPOTHESIS TEST

MCQ 9: After completing the calculations, a scientist writes: "There is sufficient evidence to suggest that the new medication lowers blood pressure."
Which step of the 5-step procedure is this?

- ☐ State the Hypotheses
- ☐ State the Test and Level
- ☐ Calculate Statistics
- ☐ Compare
- ☐ Conclude

MCQ 10: A student begins their investigation by writing: "Let μ be the average height of the plants. We set $H_0 : \mu = 15$ and $H_1 : \mu > 15$."
Which step of the 5-step procedure is this?

- ☐ State the Hypotheses
- ☐ State the Test and Level
- ☐ Calculate Statistics
- ☐ Compare
- ☐ Conclude

MCQ 11: A report states: "We will perform a one-sample t-test at the 5% significance level ($\alpha = 0.05$)."
Which step of the 5-step procedure is this?

- ☐ State the Hypotheses
- ☐ State the Test and Level
- ☐ Calculate Statistics
- ☐ Compare
- ☐ Conclude

MCQ 12: Using a calculator, a researcher finds that $t \approx 2.45$ and p-value ≈ 0.014.
Which step of the 5-step procedure is this?

- ☐ State the Hypotheses
- ☐ State the Test and Level
- ☐ Calculate Statistics
- ☐ Compare
- ☐ Conclude

MCQ 13: A mathematician writes: "Since $0.03 < 0.05$, we reject the null hypothesis."
Which step of the 5-step procedure is this?

- ☐ State the Hypotheses
- ☐ State the Test and Level
- ☐ Calculate Statistics
- ☐ Compare
- ☐ Conclude

A.4 TYPE I AND TYPE II ERRORS

A.4.1 INTERPRETING TYPE I AND TYPE II ERRORS IN CONTEXT

Ex 14: Context: One-Sample t-test
A factory produces bags of flour with a target mean weight of 1 kg. A quality control manager performs a **one-sample t-test** to check if the machine is calibrated correctly.

- $H_0 : \mu = 1$ (The machine is working correctly).
- $H_1 : \mu \neq 1$ (The machine is malfunctioning).

1. Describe a Type I error in this specific context. What is the consequence for the factory?
2. Describe a Type II error in this specific context. What is the consequence for the customers?

Ex 15: Context: Two-Sample t-test
A pharmaceutical company compares a new drug against a placebo using a **two-sample t-test**.

- $H_0 : \mu_{\text{drug}} = \mu_{\text{placebo}}$ (The drug has no effect).
- $H_1 : \mu_{\text{drug}} > \mu_{\text{placebo}}$ (The drug is effective).

1. Describe the Type I error. Why might regulatory bodies (like the FDA) want to minimize this error (α)?
2. Describe the Type II error. Why might the company want to minimize this error?

Ex 16: Context: χ^2 Test for Independence
A marketing team uses a **Chi-squared test** to see if "Age Group" and "Brand Preference" are associated.

- H_0: Age and Brand Preference are independent.
- H_1: Age and Brand Preference are not independent.

1. Describe a Type I error. What bad business decision might result from this?
2. Describe a Type II error. What opportunity is missed here?

Ex 17: Context: χ^2 Goodness of Fit Test
A casino wants to check if a die is fair using a **Goodness of Fit test**.

- H_0: The die is fair (Uniform distribution).
- H_1: The die is biased.

Describe the consequences of Type I and Type II errors for the casino owner.

B t-TEST

B.1 ONE-SAMPLE T-TEST

B.1.1 CONDUCTING ONE-SAMPLE T-TESTS

Ex 18: A bottling company claims that their bottles contain an average of 500 ml of water. A consumer group suspects the average is different. They measure a random sample of 40 bottles and find a mean of $\bar{x} = 498$ ml with a standard deviation of $s = 5$ ml.
Test the company's claim at the 5% significance level.

Ex 19: The national average score on a mathematics exam is known to be 72. A teacher applies a new teaching method to her class of 25 students. The class average score is $\bar{x} = 76$ with a standard deviation of $s = 12$.
Test at the 1% significance level whether the new teaching method has **increased** the mean score.

Ex 20: An environmental agency claims that a specific car model emits an average of 120 g of CO2 per km. A researcher suspects the actual emission is lower due to new fuel regulations. A sample of 10 cars yields a mean of $\bar{x} = 118$ g/km with a standard deviation of $s = 3$ g/km.
Perform a hypothesis test at the 5% level.

Ex 21: A farmer claims his potatoes weigh an average of 200g. A supermarket weighs a random sample of 8 potatoes from a delivery:

$$195, 202, 190, 198, 205, 188, 192, 199$$

Determine if there is evidence at the 10% significance level that the mean weight is different from 200g.

B.2 TWO-SAMPLE T-TEST (INDEPENDENT)

B.2.1 CONDUCTING TWO-SAMPLE T-TESTS

Ex 22: A farmer wants to test if a new fertilizer increases the yield of his tomato plants. He applies the new fertilizer to a group of 10 plants (Group A) and uses the standard fertilizer on another group of 10 plants (Group B).

- Group A (New): 5.2, 5.5, 5.8, 6.1, 5.9, 6.3, 5.7, 6.0, 5.4, 5.6 kg
- Group B (Standard): 4.8, 5.0, 4.9, 5.2, 5.1, 5.3, 4.7, 5.0, 4.9, 5.1 kg

Test at the 1% significance level whether the new fertilizer produces a **higher** mean yield.

Ex 23: A coffee shop chain wants to compare the average daily sales of two different branches, Branch X and Branch Y.

- Branch X ($n_1 = 30$ days): Mean $\bar{x}_1 = \$1500$, Standard deviation $s_1 = \$200$.
- Branch Y ($n_2 = 30$ days): Mean $\bar{x}_2 = \$1420$, Standard deviation $s_2 = \$180$.

Test at the 5% significance level whether there is a significant difference in the daily sales between the two branches.

Ex 24: An athletic coach compares the 100m sprint times of two groups of athletes.

- Group 1 (Training Plan A): $\{11.2, 11.4, 11.1, 11.5, 11.3, 11.0\}$
- Group 2 (Training Plan B): $\{11.6, 11.8, 11.5, 11.9, 11.7\}$

Assuming population variances are equal, test at the 5% significance level whether athletes on Plan A are **faster** (have a lower mean time) than those on Plan B.

Ex 25: A psychologist investigates if the average reaction time differs between two age groups.

- Group 1 (20-30 years): $n_1 = 15, \bar{x}_1 = 0.35s, s_1 = 0.05s$
- Group 2 (50-60 years): $n_2 = 15, \bar{x}_2 = 0.40s, s_2 = 0.06s$

Test at the 1% level if there is a difference in reaction times.

B.3 PAIRED T-TEST

B.3.1 CONDUCTING PAIRED T-TESTS

Ex 26: A company organizes a training workshop to improve employees' typing speed. The typing speeds (in words per minute) of 8 employees are recorded before and after the workshop.

Employee	A	B	C	D	E	F	G	H
Before	45	52	60	58	40	65	70	48
After	50	55	62	64	42	68	69	55

Test at the 1% significance level whether the workshop has increased typing speed.

Ex 27: A nutritionist wants to test if a specific diet changes cholesterol levels. The cholesterol levels of 6 patients are measured before starting the diet and after 4 weeks.

Patient	1	2	3	4	5	6
Before	210	235	208	190	245	220
After	205	228	210	192	240	215

Test at the 5% significance level whether there is a change in cholesterol levels.

Ex 28: A teacher tests whether listening to music affects concentration. 10 students perform a concentration task twice: once in silence and once while listening to music. The scores (out of 20) are:

Student	1	2	3	4	5	6	7	8	9	10
Silence	15	18	12	14	19	16	10	13	17	15
Music	14	16	13	11	18	14	9	12	15	14

Test at the 5% significance level whether scores are **lower** when listening to music.

Ex 29: A biologist measures the growth of 7 plants after applying a growth hormone. The height (in cm) is measured at the start and after 2 weeks.

Plant	1	2	3	4	5	6	7
Week 0	12.5	10.2	11.8	13.0	9.5	12.1	10.8
Week 2	13.1	10.8	12.0	13.2	10.1	12.9	11.2

Test at the 1% significance level whether there has been significant growth.

C CHI-SQUARED TEST (χ^2)

C.1 CHI-SQUARED TEST FOR INDEPENDENCE

C.1.1 CONDUCTING CHI-SQUARED TESTS FOR INDEPENDENCE

Ex 30: A market researcher surveys 150 smartphone users to see if brand preference is independent of gender.

	Brand A	Brand B	Brand C
Male	25	30	15
Female	35	25	20

Test at the 5% significance level whether gender and brand preference are independent.

Ex 31: A medical study investigates if smoking habits are associated with exercise levels.

	Smoker	Non-Smoker
High Exercise	10	50
Low Exercise	40	20

Perform a χ^2 test for independence at the 1% level.

Ex 32: A sociologist categorizes 200 people by their highest education level and their job satisfaction.

	Satisfied	Neutral	Dissatisfied
High School	20	30	10
University	40	40	20
Post-Grad	25	10	5

Test at the 5% level if education level and job satisfaction are independent.

Ex 33: A school offers three extra-curricular activities: Sports, Music, and Drama. The participation by grade level is recorded.

	Sports	Music	Drama
Grade 9	50	20	30
Grade 10	40	30	30

Test at the 5% level whether the choice of activity is independent of the grade level.

C.2 χ^2 GOODNESS OF FIT TEST

C.2.1 CONDUCTING CHI-SQUARED GOODNESS OF FIT TESTS

Ex 34: A 12-sided die is rolled 120 times. The observed frequencies of the results are:

Outcome	1	2	3	4	5	6	7	8	9	10	11	1
Freq	8	9	11	10	12	11	9	10	12	9	10	9

Test at the 5% level whether the die is fair.

Ex 35: A geneticist claims that the offspring of a certain plant cross should follow a ratio of 9:3:3:1 for four different phenotypes.
She observes 160 plants with the following distribution:

Type A	Type B	Type C	Type D
82	35	33	10

Test the geneticist's claim at the 5% significance level.

Ex 36: The number of goals scored by a team in 50 matches is recorded:

Goals	0	1	2	3	4
Freq	10	15	15	8	2

Test at the 10% level whether the number of goals follows a Binomial distribution $B(4, 0.35)$.

Ex 37: The weights of 100 apples are categorized into bins.

Weight (g)	< 100	$100 - 120$	$120 - 140$	> 140
Freq	10	20	45	25

The sample mean is $\bar{x} = 128$ and sample standard deviation is $s = 18$.
Test at the 5% level if the weights follow a Normal distribution.
Hint: Calculate expected frequencies using Normal CDF with sample statistics. Remember to adjust degrees of freedom.