TRANSFORMATIONS

A TYPES OF TRANSFORMATIONS

Transformations are rules that take every point of a figure to another point in the plane.
They can move (translate), flip (reflect), turn (rotate), or resize (enlarge/reduce) a shape.

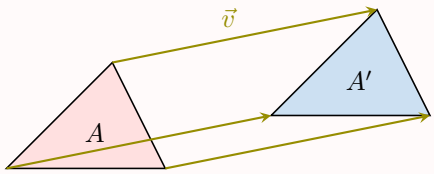
Definition **Object and Image**

When a transformation is applied to a shape, the original shape is called the **object**.
The resulting shape after the transformation is called the **image**.
Often, if a point is called A in the object, its image is written A' ("A prime").

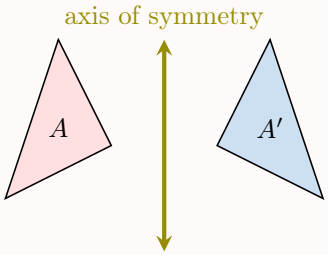
Definition **Types of Transformations**

There are several types of transformations, including:

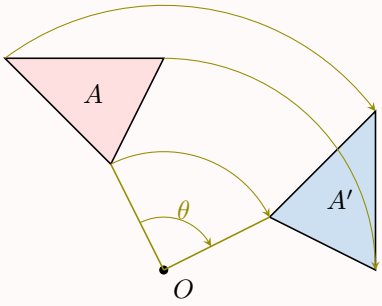
- **Translation:** Slides every point of a shape the same distance in the same direction.
  It does not change the shape or size of the figure and does not change the orientation (it is a rigid motion).



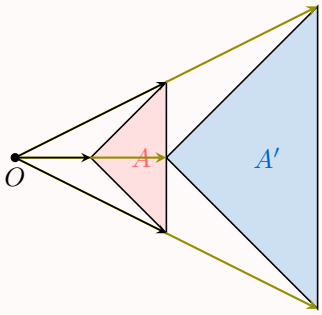
- **Reflection:** Flips a shape across a line (like a mirror), creating a mirror image.
  This line is called the *axis (line) of reflection*.



- **Rotation:** Turns a shape around a fixed point (the *centre of rotation*) by a certain angle.
  A positive angle is usually taken to mean a counterclockwise rotation. Distances and angles are preserved.



- **Homothety (enlargement/reduction):** Resizes a shape from a centre point by a constant scale factor, so that the image is *similar* to the original (angles are preserved, lengths are multiplied by the same factor).

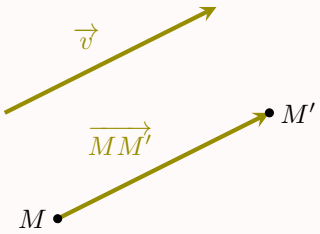


B TRANSLATION

A **translation** moves a figure from one place to another. Every point on the figure moves the same distance in the same direction.

Definition Translation

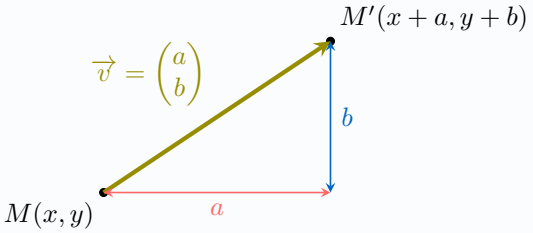
A translation by the vector $\vec{v}$ maps a point M to its image M' such that $\overrightarrow{MM'} = \vec{v}$.
All points of the plane are shifted by the same vector $\vec{v}$.



Proposition Coordinates of the Image Point

In a coordinate system, if the point M has coordinates (x, y) and the translation vector is $\vec{v} = \begin{pmatrix} a \\ b \end{pmatrix}$, then the image point M' has coordinates

$$M'(x + a,\ y + b).$$



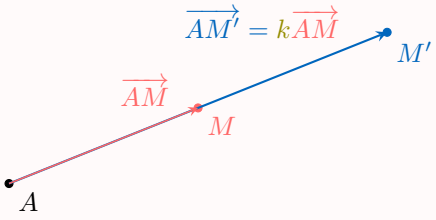
C HOMOTHETY

Definition Homothety

A **homothety** with center A and scale factor k maps a point M to a point M' on the line AM such that

$$\overrightarrow{AM'} = k\,\overrightarrow{AM}.$$

If $|k| > 1$, the figure is enlarged; if $0 < |k| < 1$, the figure is reduced.



$$\frac{AM'}{AM} = |k|$$

Proposition Coordinates of the Image Point

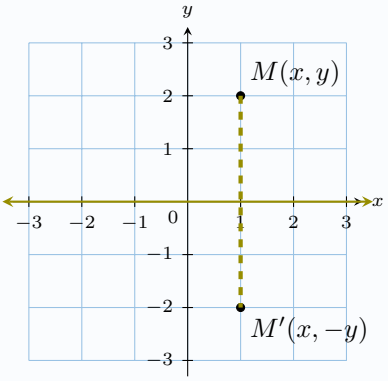
In a coordinate system, if the center A has coordinates (a, b), the point M has coordinates (x, y), and the scale factor is k, then the image point M' has coordinates

$$M'\big(a + k(x - a),\ b + k(y - b)\big).$$

D SPECIFIC REFLECTIONS

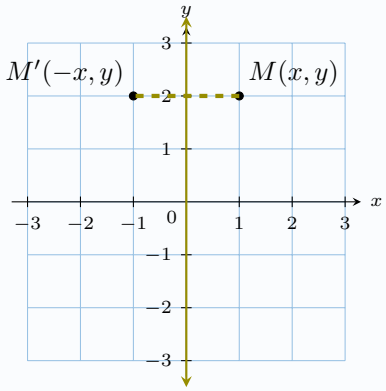
Proposition Reflection over the x-axis

The image of the point $M(x, y)$ under the reflection over the x-axis is $M'(x, -y)$.
This reflection keeps the x-coordinate and changes the sign of the y-coordinate.



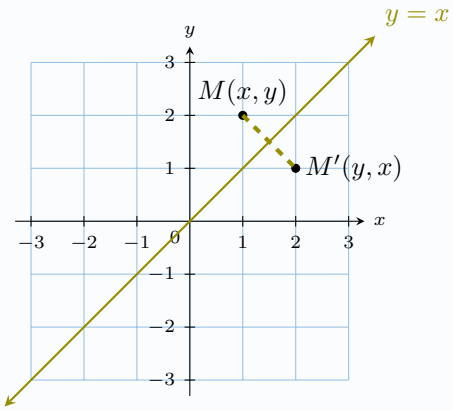
Proposition **Reflection over the y-axis**

The image of the point $M(x, y)$ under the reflection over the y-axis is $M'(-x, y)$.
This reflection keeps the y-coordinate and changes the sign of the x-coordinate.



Proposition **Reflection over the line $y = x$**

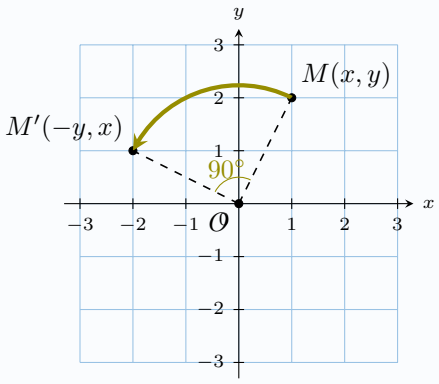
The image of the point $M(x, y)$ under the reflection $M_{y=x}$ over the line $y = x$ is $M'(y, x)$.
The coordinates are swapped.



E SPECIFIC ROTATIONS

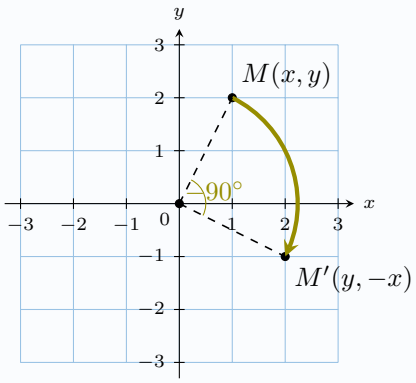
Proposition **Rotation of 90°**

The image of the point $M(x, y)$ under the rotation of 90° (counterclockwise) around the origin is $M'(-y, x)$.
In coordinates, a 90° anticlockwise rotation sends (x, y) to $(-y, x)$.



Proposition Rotation of -90°

The image of the point $M(x, y)$ under the rotation of $-90°$ (clockwise) around the origin is $M'(y, -x)$.
In coordinates, a 90° clockwise rotation sends (x, y) to $(y, -x)$.



Proposition Rotation of 180°

The image of the point $M(x, y)$ under the rotation of 180° around the origin is $M'(-x, -y)$.
In coordinates, a half-turn about the origin sends (x, y) to $(-x, -y)$.