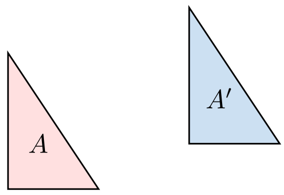
TRANSFORMATIONS

A TYPES OF TRANSFORMATIONS

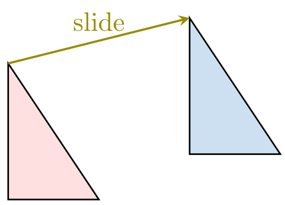
A.1 IDENTIFYING THE TRANSFORMATIONS

Ex 1: Identify the transformation that maps figure A (blue) onto figure A' (red).

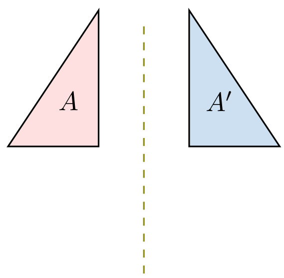


The transformation is an **translation**.

Answer: It is a **Translation**. The figure slides without turning or flipping.



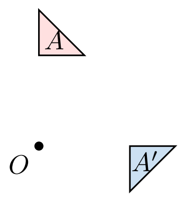
Ex 2: Identify the transformation that maps figure A (blue) onto figure A' (red).



The transformation is a **reflection**.

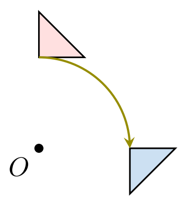
Answer: It is a **Reflection** (or Axial Symmetry). The figure is flipped across a line (the axis).

Ex 3: Identify the transformation that maps figure A (blue) onto figure A' (red).

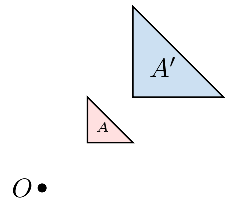


The transformation is a **rotation**.

Answer: It is a **Rotation**. The figure turns around the point O (by 90° clockwise).

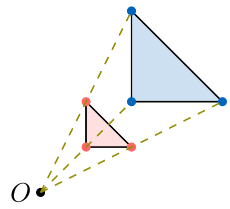


Ex 4: Identify the transformation that maps figure A (blue) onto figure A' (red).



The transformation is a **homothety**.

Answer: It is a **Homothety** (or Enlargement). The size of the figure changes, and points are aligned with the center O.



Ex 5: Which transformation matches the description?

1. "A mirror image across a line." $\rightarrow$ **reflection**.
2. "Turning around a fixed point." $\rightarrow$ **rotation**.
3. "Sliding in a straight line." $\rightarrow$ **translation**.
4. "Resizing (enlarging or reducing)." $\rightarrow$ **homothety**.

Answer:

1. Reflection
2. Rotation
3. Translation
4. Homothety

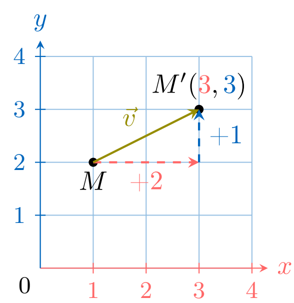
B TRANSLATION

B.1 DETERMINING THE IMAGE UNDER A TRANSLATION

Ex 6: Find the coordinates of the image of point $M(1, 2)$ under a translation by vector $\vec{v}\begin{pmatrix}2\\1\end{pmatrix}$.

$$M'(\boxed{3},\boxed{3})$$

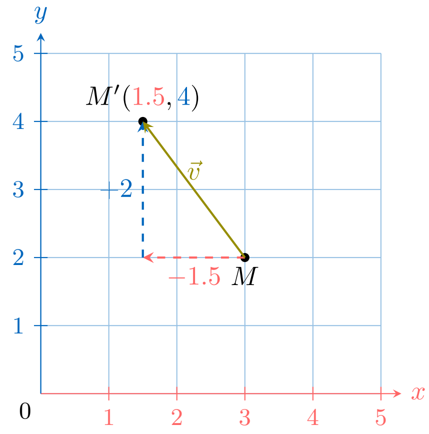
Answer: $M'(1+2, 2+1)$ so $M'(3, 3)$



Ex 7: Find the coordinates of the image of point $M(3, 2)$ under a translation by vector $\vec{v}\begin{pmatrix}-1.5\\2\end{pmatrix}$.

$$M'(\boxed{1.5}, \boxed{4})$$

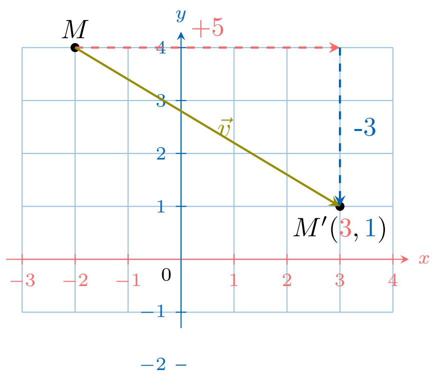
Answer: $M'(3 - 1.5, 2 + 2)$ so $M'(1.5, 4)$



Ex 8: Find the coordinates of the image of point $M(-2, 4)$ under a translation by vector $\vec{v}\begin{pmatrix}5\\-3\end{pmatrix}$.

$$M'(\boxed{3}, \boxed{1})$$

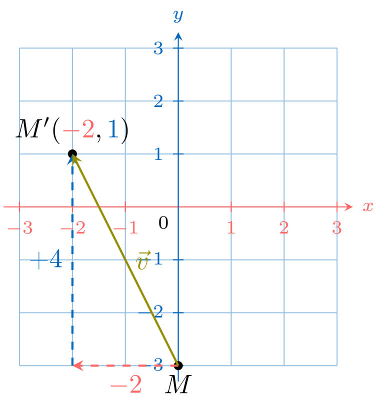
Answer: $M'(-2 + 5, 4 - 3)$ so $M'(3, 1)$



Ex 9: Find the coordinates of the image of point $M(0, -3)$ under a translation by vector $\vec{v}\begin{pmatrix}-2\\4\end{pmatrix}$.

$$M'(\boxed{-2}, \boxed{1})$$

Answer: $M'(0 - 2, -3 + 4)$ so $M'(-2, 1)$



B.2 DETERMINING THE ORIGINAL POINT UNDER A TRANSLATION

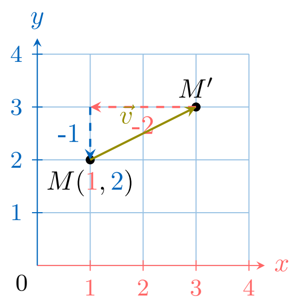
Ex 10: Find the coordinates of the point M whose image is $M'(3, 3)$ under a translation by vector $\vec{v}\begin{pmatrix}2\\1\end{pmatrix}$.

$$M(\boxed{1}, \boxed{2})$$

Answer:

$$\overrightarrow{MM'} = \vec{v}$$
$$\begin{pmatrix}3-x\\3-y\end{pmatrix} = \begin{pmatrix}2\\1\end{pmatrix}$$
$$3 - x = 2 \text{ and } 3 - y = 1$$
$$x = 3 - 2 \text{ and } y = 3 - 1$$
$$x = 1 \text{ and } y = 2$$

so $M(1, 2)$



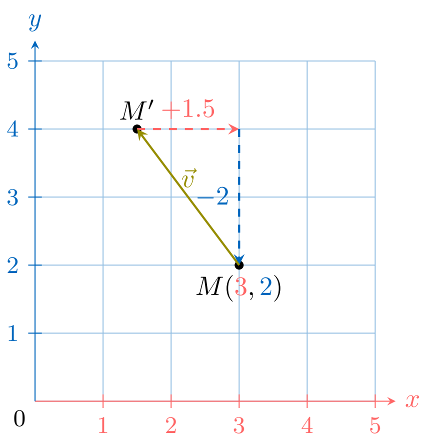
Ex 11: Find the coordinates of the point M whose image is $M'(1.5, 4)$ under a translation by vector $\vec{v}\begin{pmatrix}-1.5\\2\end{pmatrix}$.

$$M(\boxed{3}, \boxed{2})$$

Answer:

$$\overrightarrow{MM'} = \vec{v}$$
$$\begin{pmatrix}1.5-x\\4-y\end{pmatrix} = \begin{pmatrix}-1.5\\2\end{pmatrix}$$
$$1.5 - x = -1.5 \text{ and } 4 - y = 2$$
$$x = 1.5 + 1.5 = 3 \text{ and } y = 4 - 2 = 2$$

so $M(3, 2)$



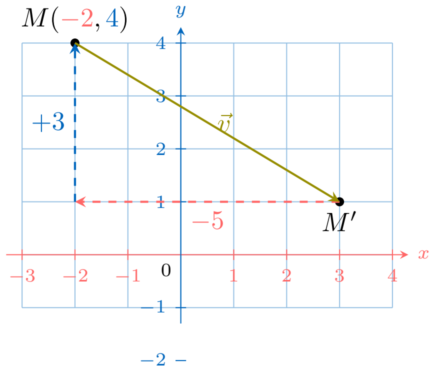
Ex 12: Find the coordinates of the point M whose image is $M'(3, 1)$ under a translation by vector $\vec{v}\begin{pmatrix}5\\-3\end{pmatrix}$.

$$M(\boxed{-2},\boxed{4})$$

Answer:

$$\overrightarrow{MM'} = \vec{v}$$
$$\begin{pmatrix}3-x\\1-y\end{pmatrix} = \begin{pmatrix}5\\-3\end{pmatrix}$$
$$3-x=5 \text{ and } 1-y=-3$$
$$x=3-5=-2 \text{ and } y=1+3=4$$

so $M(-2,4)$



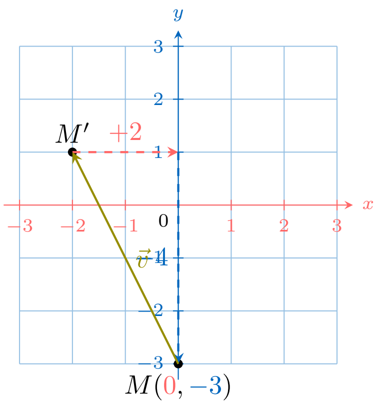
Ex 13: Find the coordinates of the point M whose image is $M'(-2,1)$ under a translation by vector $\vec{v}\begin{pmatrix}-2\\4\end{pmatrix}$.

$$M(\boxed{0},\boxed{-3})$$

Answer:

$$\overrightarrow{MM'} = \vec{v}$$
$$\begin{pmatrix}-2-x\\1-y\end{pmatrix} = \begin{pmatrix}-2\\4\end{pmatrix}$$
$$-2-x=-2 \text{ and } 1-y=4$$
$$x=0 \text{ and } y=1-4=-3$$

so $M(0,-3)$



B.3 DETERMINING THE IMAGE OF A LINEAR EQUATION UNDER A TRANSLATION

Ex 14: Find the equation of the image of the line $y=2x+1$ under a translation by the vector $\begin{pmatrix}1\\-1\end{pmatrix}$.

$$y=\boxed{2x-2}$$

Answer: Let (x,y) be a point on the original line and (x',y') be its image under the translation.The relationship between the coordinates is given by:

$$\begin{pmatrix}x'\\y'\end{pmatrix} = \begin{pmatrix}x\\y\end{pmatrix} + \begin{pmatrix}1\\-1\end{pmatrix} \implies \begin{cases}x'=x+1\\y'=y-1\end{cases}$$

We express the original coordinates (x,y) in terms of the image coordinates (x',y'):

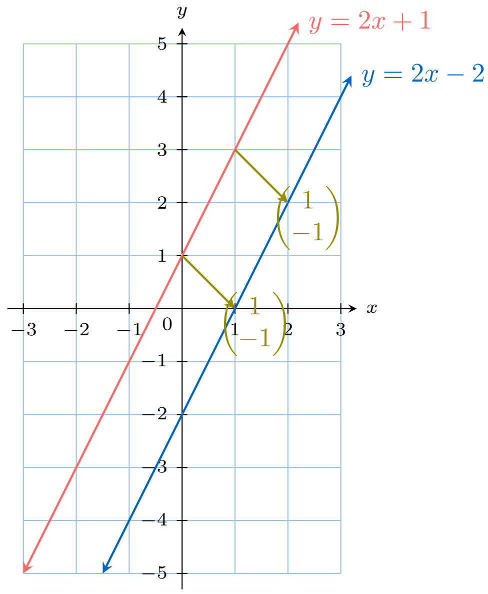
$$\begin{cases}x=x'-1\\y=y'+1\end{cases}$$

Substitute these expressions into the original equation $y=2x+1$:

$$(y'+1)=2(x'-1)+1$$
$$y'+1=2x'-2+1$$
$$y'=2x'-2$$

Replacing x' and y' with x and y, the equation of the image line is:

$$y=2x-2$$



Ex 15: Find the equation of the image of the line $y=-x+1$ under a translation by the vector $\begin{pmatrix}-2\\1\end{pmatrix}$.

$$y=\boxed{-x}$$

Answer: Let (x,y) be a point on the original line and (x',y') be its image.The transformation is defined by:

$$\begin{pmatrix}x'\\y'\end{pmatrix} = \begin{pmatrix}x\\y\end{pmatrix} + \begin{pmatrix}-2\\1\end{pmatrix} \implies \begin{cases}x'=x-2\\y'=y+1\end{cases}$$

Express x and y in terms of x' and y':

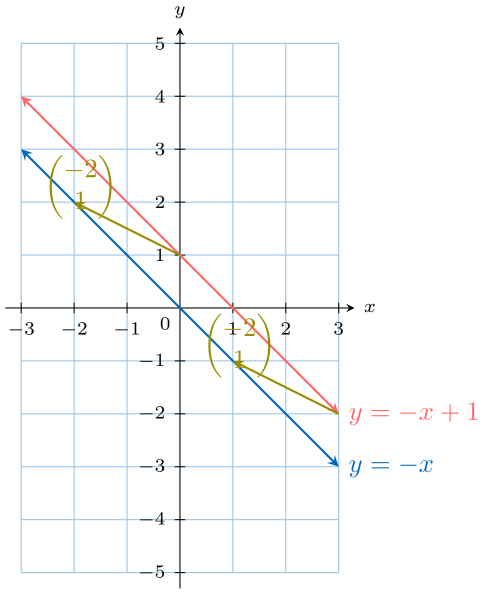
$$\begin{cases}x=x'+2\\y=y'-1\end{cases}$$

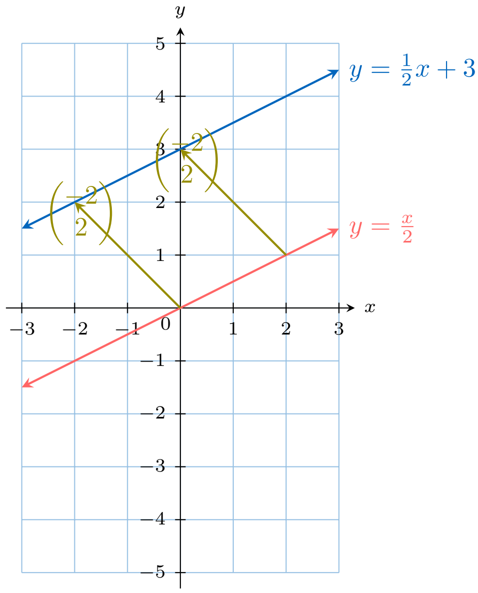
Substitute into the original equation $y=-x+1$:

$$(y'-1)=-(x'+2)+1$$
$$y'-1=-x'-2+1$$
$$y'=-x'$$

The image equation is:

$$y=-x$$





Ex 16: Find the equation of the image of the line $y = \frac{x}{2}$ under a translation by the vector $\begin{pmatrix}-2\\2\end{pmatrix}$.

$$y = \boxed{\frac{1}{2}x + 3}$$

Answer: Let (x, y) be a point on the original line and (x', y') be its image.The coordinate mapping is:

$$\begin{pmatrix}x'\\y'\end{pmatrix} = \begin{pmatrix}x\\y\end{pmatrix} + \begin{pmatrix}-2\\2\end{pmatrix} \implies \begin{cases}x' = x - 2\\y' = y + 2\end{cases}$$

Express x and y in terms of x' and y':

$$\begin{cases}x = x' + 2\\y = y' - 2\end{cases}$$

Substitute into the original equation $y = \frac{1}{2}x$:

$$(y' - 2) = \frac{1}{2}(x' + 2)$$
$$y' - 2 = \frac{1}{2}x' + 1$$
$$y' = \frac{1}{2}x' + 3$$

The image equation is:

$$y = \frac{1}{2}x + 3$$

C HOMOTHETY

C.1 DETERMINING THE IMAGE UNDER A HOMOTHETY

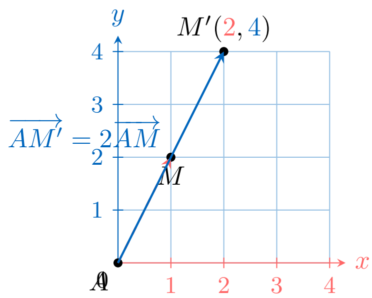
Ex 17: Find the coordinates of the image point M' of point $M(1, 2)$ under a homothety with center $A(0, 0)$ and scale factor 2.

$$M'(\boxed{2}, \boxed{4})$$

Answer:

$$\overrightarrow{AM'} = 2\overrightarrow{AM}$$
$$\begin{pmatrix}x' - 0\\y' - 0\end{pmatrix} = 2\begin{pmatrix}1 - 0\\2 - 0\end{pmatrix}$$
$$x' = 2(1) \text{ and } y' = 2(2)$$
$$x' = 2 \text{ and } y' = 4$$

so $M'(2, 4)$



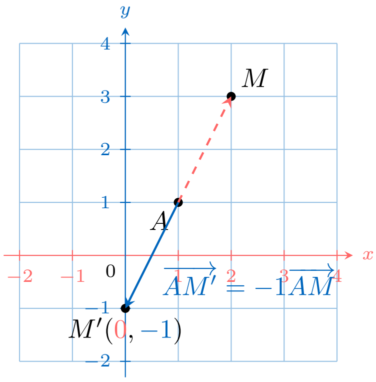
Ex 18: Find the coordinates of the image point M' of point $M(2, 3)$ under a homothety with center $A(1, 1)$ and scale factor -1.

$$M'(\boxed{0}, \boxed{-1})$$

Answer:

$$\overrightarrow{AM'} = -1\overrightarrow{AM}$$
$$\begin{pmatrix}x' - 1\\y' - 1\end{pmatrix} = -1\begin{pmatrix}2 - 1\\3 - 1\end{pmatrix}$$
$$x' - 1 = -1(1) \text{ and } y' - 1 = -1(2)$$
$$x' - 1 = -1 \text{ and } y' - 1 = -2$$
$$x' = 0 \text{ and } y' = -1$$

so $M'(0, -1)$



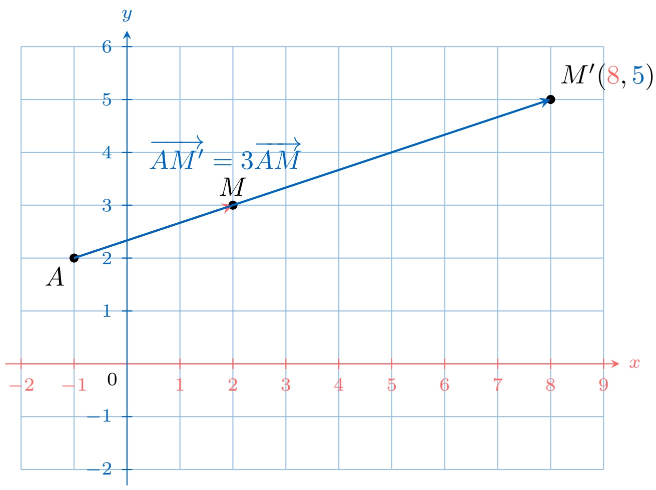
Ex 19: Find the coordinates of the image point M' of point $M(2,3)$ under a homothety with center $A(-1,2)$ and scale factor 3.

$M'(\boxed{8},\boxed{5})$

Answer:

$$\overrightarrow{AM'} = 3\overrightarrow{AM}$$
$$\begin{pmatrix} x'-(-1) \\ y'-2 \end{pmatrix} = 3\begin{pmatrix} 2-(-1) \\ 3-2 \end{pmatrix}$$
$$x'+1 = 3(3) \text{ and } y'-2 = 3(1)$$
$$x'+1 = 9 \text{ and } y'-2 = 3$$
$$x' = 8 \text{ and } y' = 5$$

so $M'(8,5)$



C.2 DETERMINING THE ORIGINAL POINT UNDER A HOMOTHETY

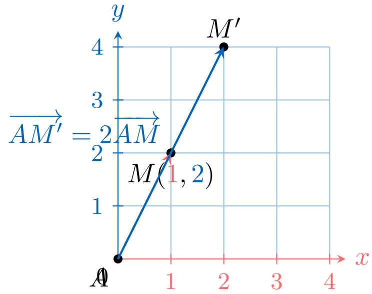
Ex 20: Find the coordinates of the point M whose image is $M'(2,4)$ under a homothety with center $A(0,0)$ and scale factor 2.

$M(\boxed{1},\boxed{2})$

Answer:

$$\overrightarrow{AM'} = 2\overrightarrow{AM}$$
$$\begin{pmatrix} 2-0 \\ 4-0 \end{pmatrix} = 2\begin{pmatrix} x-0 \\ y-0 \end{pmatrix}$$
$$2 = 2x \text{ and } 4 = 2y$$
$$x = 2/2 \text{ and } y = 4/2$$
$$x = 1 \text{ and } y = 2$$

so $M(1,2)$



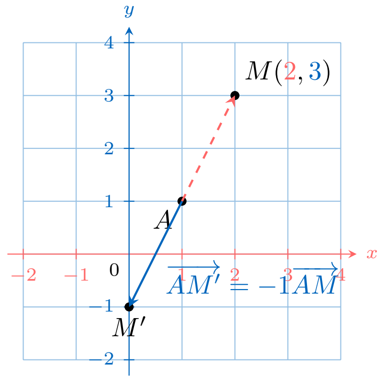
Ex 21: Find the coordinates of the point M whose image is $M'(0,-1)$ under a homothety with center $A(1,1)$ and scale factor -1.

$M(\boxed{2},\boxed{3})$

Answer:

$$\overrightarrow{AM'} = -1\overrightarrow{AM}$$
$$\begin{pmatrix} 0-1 \\ -1-1 \end{pmatrix} = -1\begin{pmatrix} x-1 \\ y-1 \end{pmatrix}$$
$$-1 = -1(x-1) \text{ and } -2 = -1(y-1)$$
$$-1 = -x+1 \text{ and } -2 = -y+1$$
$$x-1 = 1 \text{ and } y-1 = 2$$
$$x = 2 \text{ and } y = 3$$

so $M(2,3)$



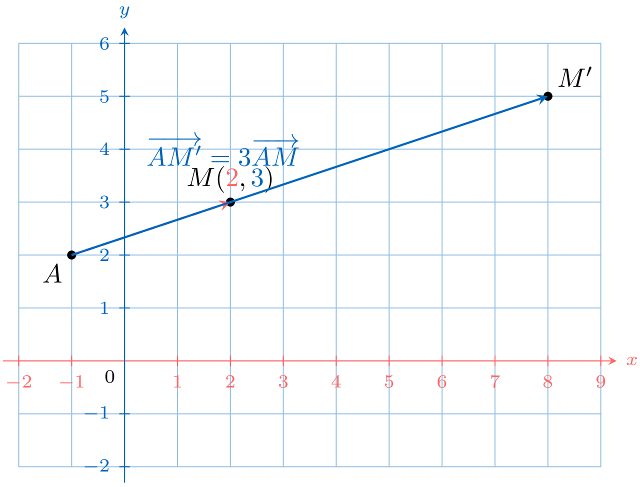
Ex 22: Find the coordinates of the point M whose image is $M'(8,5)$ under a homothety with center $A(-1,2)$ and scale factor 3.

$M(\boxed{2},\boxed{3})$

Answer:

$$\overrightarrow{AM'} = 3\overrightarrow{AM}$$
$$\begin{pmatrix} 8-(-1) \\ 5-2 \end{pmatrix} = 3\begin{pmatrix} x-(-1) \\ y-2 \end{pmatrix}$$
$$9 = 3(x+1) \text{ and } 3 = 3(y-2)$$
$$9/3 = x+1 \text{ and } 3/3 = y-2$$
$$x+1 = 3 \text{ and } y-2 = 1$$
$$x = 2 \text{ and } y = 3$$

so $M(2,3)$



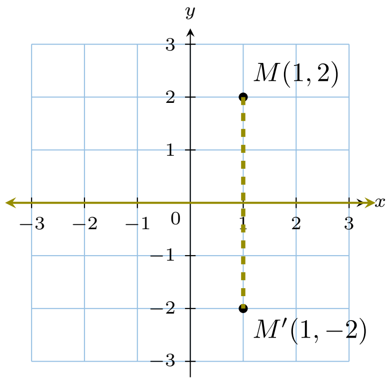
D SPECIFIC REFLECTIONS

D.1 DETERMINING THE IMAGE UNDER A REFLECTION

Ex 23: Find the coordinates of the image point M' of point $M(1,2)$ under reflection over the x-axis.

$$M'(\boxed{1},\boxed{-2})$$

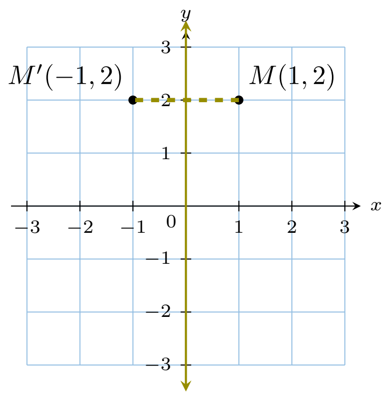
Answer: The image of $M(x,y)$ under reflection over the x-axis is $M'(x,-y)$.
So, for $M(1,2)$, $M'(1,-2)$.



Ex 24: Find the coordinates of the image point M' of point $M(1,2)$ under reflection over the y-axis.

$$M'(\boxed{-1},\boxed{2})$$

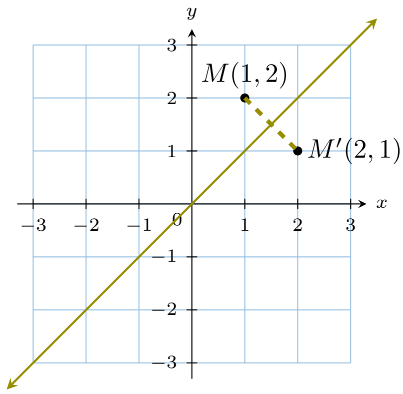
Answer: The image of $M(x,y)$ under reflection over the y-axis is $M'(-x,y)$.
So, for $M(1,2)$, $M'(-1,2)$.



Ex 25: Find the coordinates of the image point M' of point $M(1,2)$ under reflection over the line $y=x$.

$$M'(\boxed{2},\boxed{1})$$

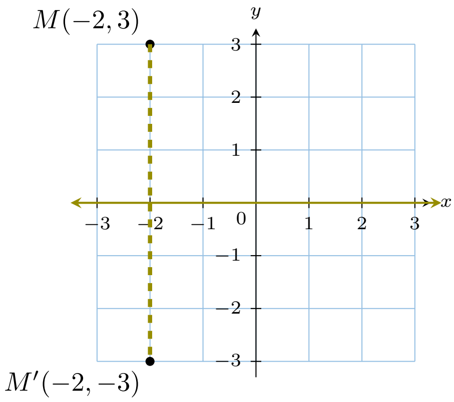
Answer: The image of $M(x,y)$ under reflection over the line $y=x$ is $M'(y,x)$.
So, for $M(1,2)$, $M'(2,1)$.



Ex 26: Find the coordinates of the image point M' of point $M(-2,3)$ under reflection over the x-axis.

$$M'(\boxed{-2},\boxed{-3})$$

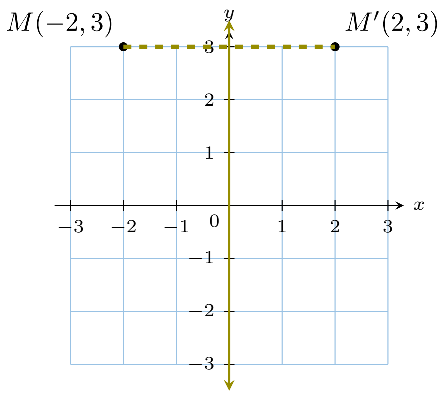
Answer: The image of $M(x,y)$ under reflection over the x-axis is $M'(x,-y)$.
So, for $M(-2,3)$, $M'(-2,-3)$.



Ex 27: Find the coordinates of the image point M' of point $M(-2,3)$ under reflection over the y-axis.

$$M'(\boxed{2},\boxed{3})$$

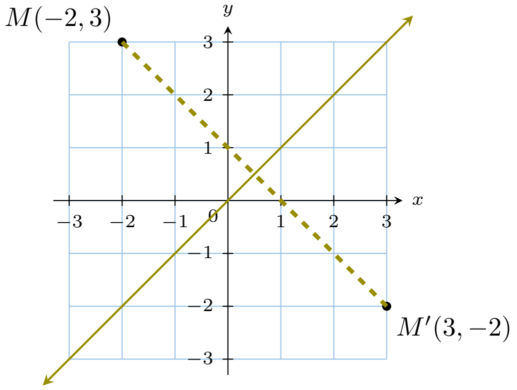
Answer: The image of $M(x,y)$ under reflection over the y-axis is $M'(-x,y)$.
So, for $M(-2,3)$, $M'(2,3)$.



Ex 28: Find the coordinates of the image point M' of point $M(-2,3)$ under reflection over the line $y=x$.

$$M'(\boxed{3},\boxed{-2})$$

Answer: The image of $M(x,y)$ under reflection over the line $y=x$ is $M'(y,x)$.
So, for $M(-2,3)$, $M'(3,-2)$.



D.2 DETERMINING THE IMAGE OF A LINEAR EQUATION UNDER A REFLECTION

Ex 29: Find the image equation of $y=2x+1$ under a reflection over the x-axis.

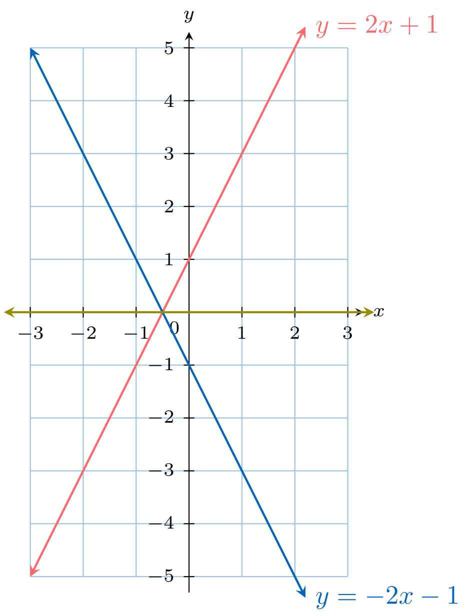
$$y=\boxed{-2x-1}$$

Answer: To find the image equation under reflection over the x-axis, replace y by $-y$ in the original equation:

$$-y=2x+1$$

Multiply both sides by -1:

$$y=-2x-1$$



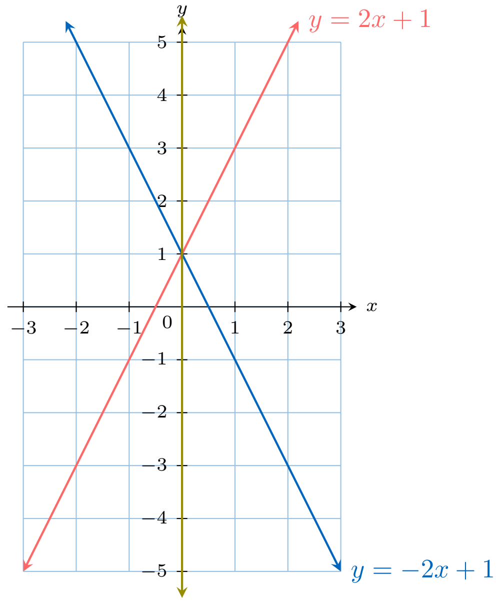
Ex 30: Find the image equation of $y=2x+1$ under a reflection over the y-axis.

$$y=\boxed{-2x+1}$$

Answer: To find the image equation under reflection over the y-axis, replace x by $-x$ in the original equation:

$$y=2(-x)+1$$

$$y=-2x+1$$



Ex 31: Find the image equation of $y=2x+1$ under a reflection over the line $y=x$.

$$y=\boxed{\frac{1}{2}x-\frac{1}{2}}$$

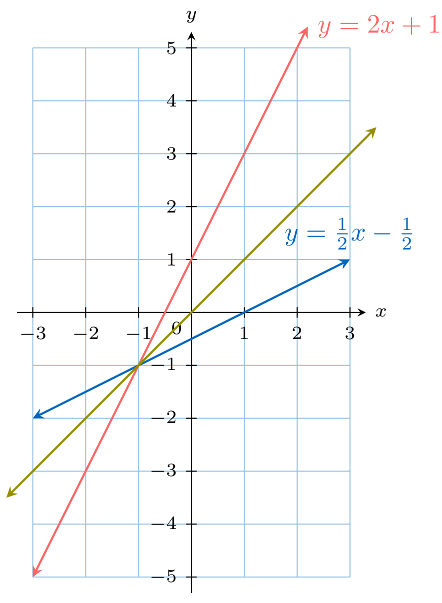
Answer: To find the image equation under reflection over the line $y=x$, replace x by y and y by x in the original equation:

$$x=2y+1$$

Solve for y:

$$x-1=2y$$

$$y=\frac{x-1}{2}=\frac{1}{2}x-\frac{1}{2}$$



Ex 32: Find the image equation of $y=\frac{x}{2}-1$ under a reflection over the x-axis.

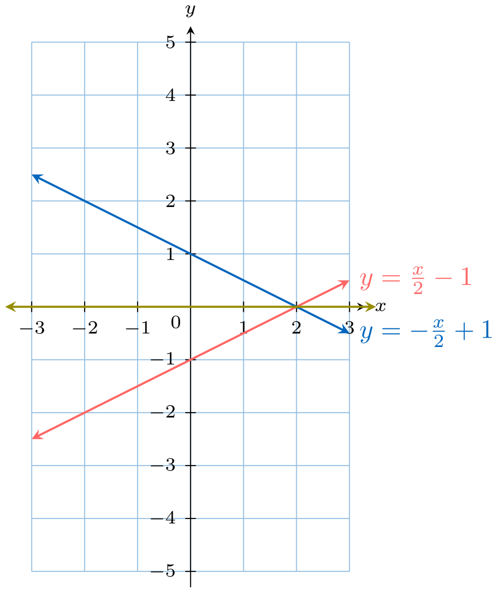
$$y = \boxed{-\frac{x}{2}+1}$$

Answer: To find the image equation under reflection over the x-axis, replace y by $-y$ in the original equation:

$$-y = \frac{x}{2} - 1$$

Multiply both sides by -1:

$$y = -\frac{x}{2} + 1$$



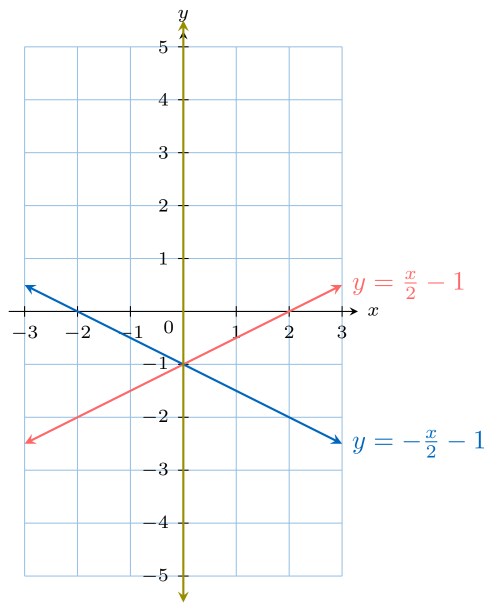
Ex 33: Find the image equation of $y = \frac{x}{2} - 1$ under a reflection over the y-axis.

$$y = \boxed{-\frac{x}{2}-1}$$

Answer: To find the image equation under reflection over the y-axis, replace x by $-x$ in the original equation:

$$y = \frac{-x}{2} - 1$$

$$y = -\frac{x}{2} - 1$$



Ex 34: Find the image equation of $y = \frac{x}{2} - 1$ under a reflection over the line $y = x$.

$$y = \boxed{2x+2}$$

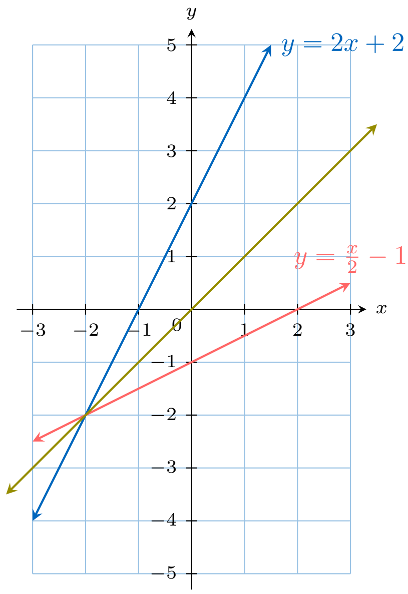
Answer: To find the image equation under reflection over the line $y = x$, replace x by y and y by x in the original equation:

$$x = \frac{y}{2} - 1$$

Solve for y:

$$x + 1 = \frac{y}{2}$$

$$y = 2(x+1) = 2x + 2$$



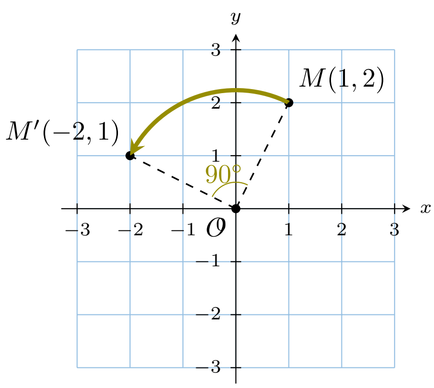
E SPECIFIC ROTATIONS

E.1 DETERMINING THE IMAGE UNDER A ROTATION

Ex 35: Find the coordinates of the image point M' of point $M(1,2)$ under a rotation of 90° (counterclockwise) around the origin.

$$M'(\boxed{-2},\boxed{1})$$

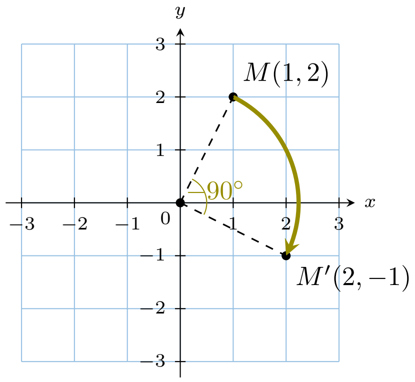
Answer: The image of $M(x,y)$ under a rotation of 90° (counterclockwise) around the origin is $M'(-y,x)$.
So, for $M(1,2)$, $M'(-2,1)$.



Ex 36: Find the coordinates of the image point M' of point $M(1,2)$ under a rotation of -90° (clockwise) around the origin.

$$M'(\boxed{2},\boxed{-1})$$

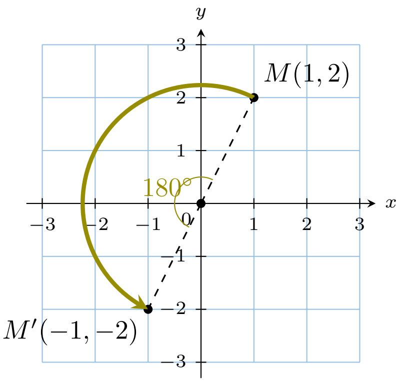
Answer: The image of $M(x, y)$ under a rotation of -90° (clockwise) around the origin is $M'(y, -x)$.
So, for $M(1, 2)$, $M'(2, -1)$.



Ex 37: Find the coordinates of the image point M' of point $M(1, 2)$ under a rotation of 180° around the origin.

$$M'(\boxed{-1},\boxed{-2})$$

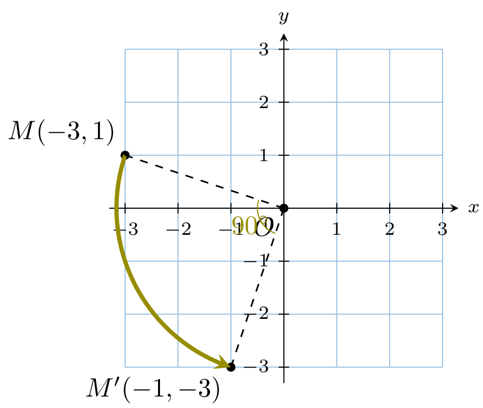
Answer: The image of $M(x, y)$ under a rotation of 180° around the origin is $M'(-x, -y)$.
So, for $M(1, 2)$, $M'(-1, -2)$.



Ex 38: Find the coordinates of the image point M' of point $M(-3, 1)$ under a rotation of 90° (counterclockwise) around the origin.

$$M'(\boxed{-1},\boxed{-3})$$

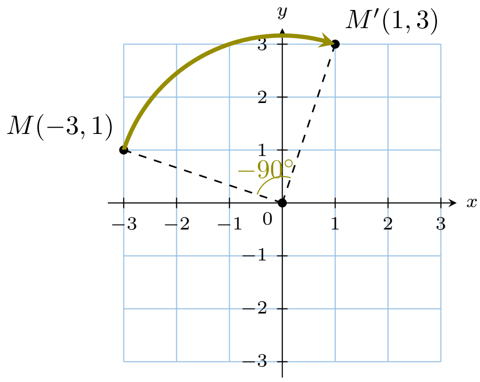
Answer: The image of $M(x, y)$ under a rotation of 90° (counterclockwise) around the origin is $M'(-y, x)$.
So, for $M(-3, 1)$, $M'(-1, -3)$.



Ex 39: Find the coordinates of the image point M' of point $M(-3, 1)$ under a rotation of -90° (clockwise) around the origin.

$$M'(\boxed{1},\boxed{3})$$

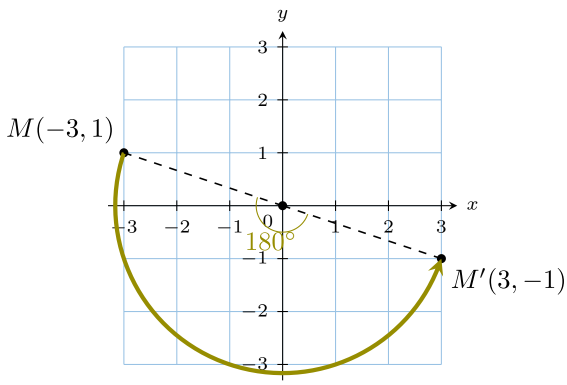
Answer: The image of $M(x, y)$ under a rotation of -90° (clockwise) around the origin is $M'(y, -x)$.
So, for $M(-3, 1)$, $M'(1, 3)$.



Ex 40: Find the coordinates of the image point M' of point $M(-3, 1)$ under a rotation of 180° around the origin.

$$M'(\boxed{3},\boxed{-1})$$

Answer: The image of $M(x, y)$ under a rotation of 180° around the origin is $M'(-x, -y)$.
So, for $M(-3, 1)$, $M'(3, -1)$.



E.2 DETERMINING THE IMAGE OF A LINEAR EQUATION UNDER A ROTATION

Ex 41: Find the image equation of $y = 2x + 1$ under a rotation of 90° (counterclockwise) around the origin.

$$y = \boxed{-\frac{1}{2}x - \frac{1}{2}}$$

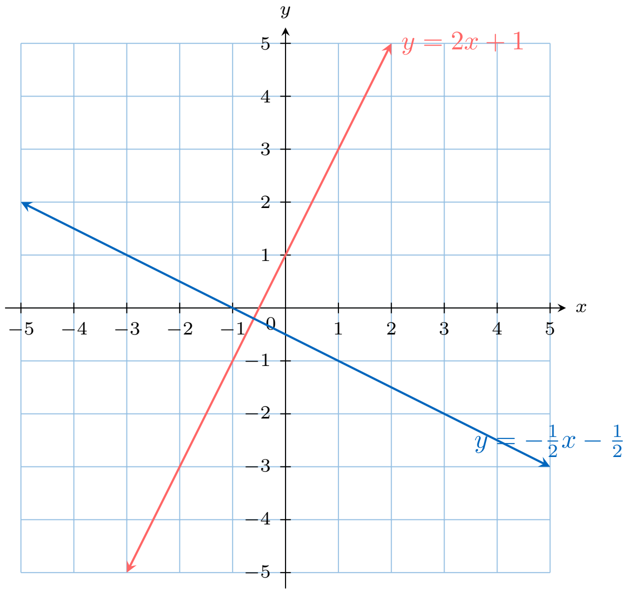
Answer: To find the image equation under a rotation of 90° (counterclockwise) around the origin, replace x by y and y by $-x$ in the original equation (since the inverse mapping is used for the equation):

$$-x = 2y + 1$$

Solve for y:

$$-x - 1 = 2y$$

$$y = \frac{-x - 1}{2} = -\frac{1}{2}x - \frac{1}{2}$$



Ex 42: Find the image equation of $y = 2x + 1$ under a rotation of -90° (clockwise) around the origin.

$$y = \boxed{-\frac{1}{2}x + \frac{1}{2}}$$

Answer: To find the image equation under a rotation of -90° (clockwise) around the origin, replace x by $-y$ and y by x in the original equation:

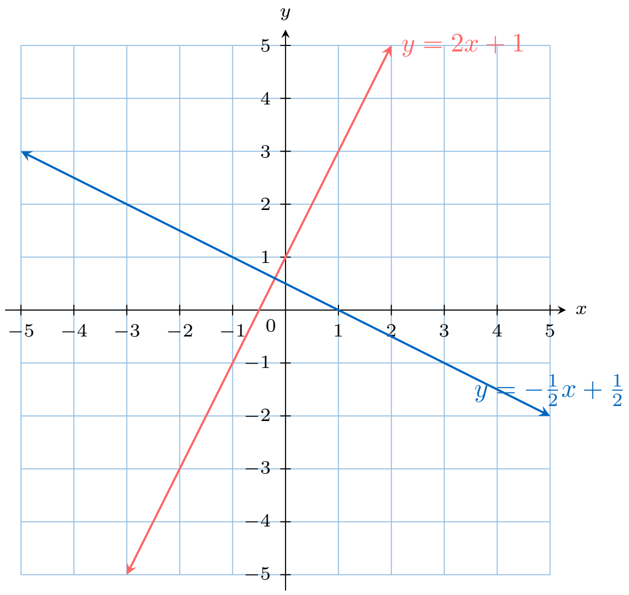
$$x = 2(-y) + 1$$

$$x = -2y + 1$$

Solve for y:

$$x - 1 = -2y$$

$$y = \frac{1 - x}{2} = -\frac{1}{2}x + \frac{1}{2}$$



Ex 43: Find the image equation of $y = 2x + 1$ under a rotation of 180° around the origin.

$$y = \boxed{2x - 1}$$

Answer: To find the image equation under a rotation of 180° around the origin, replace x by $-x$ and y by $-y$ in the original equation:

$$-y = 2(-x) + 1$$

$$-y = -2x + 1$$

Multiply both sides by -1:

$$y = 2x - 1$$