FUNCTIONS

A FUNDAMENTAL CONCEPTS OF FUNCTIONS

A.1 WHAT IS A FUNCTION?

A.1.1 WRITING FUNCTIONS: LEVEL 1

Ex 1: Consider the following calculation program:

1. Choose a number.
2. Subtract 5 from the chosen number.

Let x be the number chosen initially. Determine the function f that corresponds to the result obtained with this program.

$$f(x) = \square$$

Ex 2: Consider the following calculation program:

1. Choose a number.
2. Multiply the chosen number by three.

Let x be the number chosen initially. Determine the function f that corresponds to the result obtained with this program.

$$f(x) = \square$$

Ex 3: Consider the following calculation program:

1. Choose a number.
2. Multiply the chosen number by five.
3. Subtract 2 from the result obtained.

Let x be the number chosen initially. Determine the function f that corresponds to the result obtained with this program.

$$f(x) = \square$$

Ex 4: Consider the following calculation program:

1. Choose a number.
2. Multiply the chosen number by -2.
3. Add 5 to the result obtained.

Let x be the number chosen initially. Determine the function f that corresponds to the result obtained with this program.

$$f(x) = \square$$

A.1.2 WRITING FUNCTIONS: LEVEL 2

Ex 5: Consider the following calculation program:

1. Choose a number.
2. Multiply the chosen number by itself.
3. Subtract 1 from the result obtained.

Let x be the number chosen initially. Determine the function f that corresponds to the result obtained with this program.

$$f(x) = \square$$

Ex 6: Consider the following calculation program:

1. Choose a number.
2. Square the chosen number.
3. Multiply the result by 2.

Let x be the number chosen initially. Determine the function f that corresponds to the result obtained with this program.

$$f(x) = \square$$

Ex 7: Consider the following calculation program:

1. Choose a number.
2. Subtract 1 from the chosen number.
3. Multiply the result by the original number chosen.

Let x be the number chosen initially. Determine the function f that corresponds to the result obtained with this program.

$$f(x) = \square$$

A.1.3 CALCULATING $f(x)$

Ex 8: For $f(x) = x + 3$,

$$f(4) = \square$$

Ex 9: For $f(x) = 2x - 1$,

$$f(5) = \square$$

Ex 10: For $f(x) = 3x + 2$,

$$f(2) = \square$$

Ex 11: For $f(x) = x^2 - 1$,

$$f(3) = \square$$

Ex 12: For $f(x) = 5x - 3$,

$$f(1) = \square$$

Ex 13: For $f(x) = \frac{x}{2} + 4$,

$$f(6) = \square$$

Ex 14: For $f(x) = x - 5$,

$$f(10) = \square$$

Ex 15: For $f(x) = 2x - 5$,

$$f(-2) = \square$$

Ex 16: For $f(x) = -x + 4$,

$$f(-3) = \square$$

Ex 17: For $f(x) = 3x - 7$,

$$f(-1) = \square$$

Ex 18: For $f(x) = x^2 - 2x$,

$$f(-2) = \square$$

Ex 19: For $f(x) = 2x + 3$,

$$f(-3) = \square$$

Ex 20: For $f(x) = \frac{x}{2} - 4$,

$$f(8) = \square$$

Ex 21: For $f(x) = \frac{3x-5}{2}$,

$$f(-1) = \square$$

Ex 22: For $f(x) = \frac{x-6}{2} - 3$,

$$f(10) = \square$$

A.1.4 CALCULATING $f(x)$

Ex 23: For $f : x \mapsto x + 3$,

$$f(4) = \square$$

Ex 24: For $f : x \mapsto x^2 - 1$,

$$f(2) = \square$$

Ex 25: For $f : x \mapsto (x - 1)(x - 2)$,

$$f(0) = \square$$

Ex 26: For $f : x \mapsto x^3$,

$$f(-1) = \square$$

A.1.5 EVALUATING FUNCTIONS WITH ALGEBRAIC EXPRESSIONS

Ex 27: For the function $f(x) = 2x + 3$, expand and simplify the expression for $f(x + 1)$.

$$f(x + 1) = \square$$

Ex 28: For the function $f(x) = x^2 - 1$, expand and simplify the expression for $f(x - 1)$.

$$f(x - 1) = \square$$

Ex 29: For the function $f(x) = 10 - 3x$, expand and simplify the expression for $f(x + 2)$.

$$f(x + 2) = \square$$

Ex 30: For the function $f(x) = x^2 - 1$, expand and simplify the expression for $f(x^2 + 1)$.

$$f(x^2 + 1) = \square$$

A.1.6 SUBSTITUTING VALUES AND EXPRESSIONS INTO A FUNCTION

Ex 31: For $f : x \mapsto 1 - 3x$, find in simplest form:

1. $f(-2) = \square$
2. $f(3) = \square$
3. $f(x + 1) = \square$
4. $f(x^2) = \square$

Ex 32: For $f : x \mapsto x^2$, find in simplest form:

1. $f(3) = \square$
2. $f(-1) = \square$
3. $f(-x) = \square$
4. $f(x + 1) = \square$
5. $f(x + 2) = \square$
6. $f(2x) = \square$

Ex 33: For $g : x \mapsto x^2 - 2x + 1$, find in simplest form:

1. $g(3) = \square$
2. $g(-1) = \square$
3. $g(-x) = \square$
4. $g(x + 1) = \square$
5. $g(x + 2) = \square$
6. $g(2x) = \square$

A.1.7 SOLVING LINEAR EQUATIONS FOR $f(x) = y$

Ex 34: Let $f(x) = 3x + 12$. Find all x such that $f(x) = 0$. Justify your answer.

Ex 35: Let $f(x) = 2x - 18$. Find all x such that $f(x) = 0$. Justify your answer.

Ex 36: Let $f(x) = 2x + 20$. Find all x such that $f(x) = 10$. Justify your answer.

Ex 37: Let $f(x) = -6x + 7$. Find all x such that $f(x) = 2$. Justify your answer.

A.1.8 FINDING PREIMAGES

Ex 38: Let $f : x \mapsto \dfrac{4x+1}{x-2}$. Find the value of x for which $f(x) = 3$. Justify your answer.

Ex 39: Let $f : x \mapsto \sqrt{2x+5}$. Find the value of x such that $f(x) = 3$. Justify your answer.

Ex 40: Let $f : x \mapsto x^2 - 6x + 8$. Find all x such that $f(x) = 0$. Justify your answer.

Ex 41: Let $f(x) = x^2 - 2x + 5$. Find all real numbers x such that $f(x) = 1$. Justify your answer.

A.1.9 ANALYZING LINEAR MODELS IN CONTEXT

Ex 42: The value of a laptop t years after purchase is given by $V(t) = 1800 - 300t$ dollars.

1. Find $V(3)$

   State what this value means
   - ☐ The original purchase price is \$900.
   - ☐ The laptop depreciates by \$900 per year.
   - ☐ The value of the laptop after 3 years is \$900.

2. Find t when $V(t) = 600$.

Explain what this represents.
☐ After 4 years, the laptop is worth \$600.
☐ The depreciation rate is \$4 per year.
☐ The original price was \$600 after 4 years.

3. Find the original purchase price of the laptop.

Ex 43: The height of a plant t weeks after planting is given by $H(t) = 5 + 2t$ cm.

1. Find $H(4)$

State what this value means
☐ The initial height is 13 cm.
☐ The plant grows by 13 cm per week.
☐ The height of the plant after 4 weeks is 13 cm.

2. Find t when $H(t) = 15$.

Explain what this represents.
☐ After 5 weeks, the plant is 15 cm tall.
☐ The growth rate is 5 cm per week.
☐ The initial height was 15 cm after 5 weeks.

3. Find the initial height of the plant.

Ex 44: The temperature of water t minutes after starting to heat it is given by $T(t) = 25 + 15t$ degrees Celsius.

1. Find $T(3)$

State what this value means
☐ The temperature of the water after 3 minutes is 70°C.
☐ The initial temperature is 70°C.
☐ The water heats up by 70 degrees per minute.

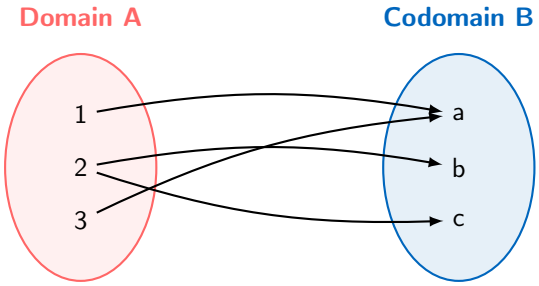
2. Find t when $T(t) = 100$.

Explain what this represents.
☐ The water is at 100°C after 100 minutes.
☐ The heating rate is 5 degrees per minute.
☐ After 5 minutes, the water reaches boiling point at 100°C.

3. Find the initial temperature of the water.

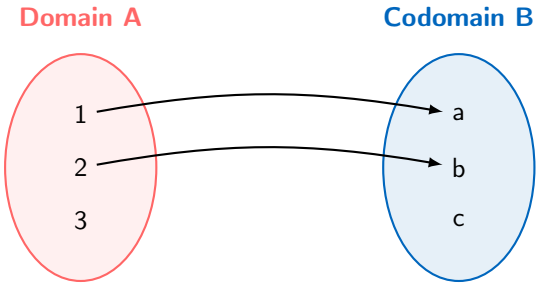
A.1.10 IDENTIFYING FUNCTIONS FROM MAPPINGS

Ex 45: A rule f maps elements from the set $A = \{1, 2, 3\}$ to the set $B = \{a, b, c\}$. The mappings are shown in the diagram below.



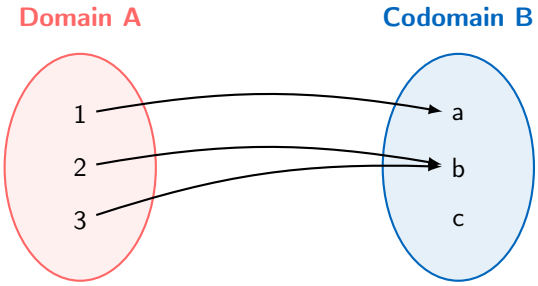
Is f a function? Explain your reasoning.

Ex 46: A rule g maps elements from the set $A = \{1, 2, 3\}$ to the set $B = \{a, b, c\}$. The mappings are shown in the diagram below.



Is g a function? Explain your reasoning.

Ex 47: A rule h maps elements from the set $A = \{1, 2, 3\}$ to the set $B = \{a, b, c\}$. The mappings are shown in the diagram below.



Is h a function? Explain your reasoning.

A.1.11 DOMAIN, CODOMAIN, AND NOTATION

Ex 48: Consider the function defined as $f : \mathbb{Z} \longrightarrow \mathbb{Z}$ $\quad x \longmapsto x - 5$.

1. What is the domain of f?
2. What is the codomain of f?
3. What is the image of $x = 7$?
4. What is the preimage of $y = -3$?

Ex 49: A function g has the domain $\mathbb{N} = \{1, 2, 3, ...\}$ and codomain $\mathbb{N}$. The rule is "divide the input by 2".

1. Write the function using formal notation.
2. Explain why this rule does not define a valid function $g : \mathbb{N} \to \mathbb{N}$.

Ex 50: Consider the function defined as $f : \mathbb{R} \longrightarrow \mathbb{R}$ $\quad x \longmapsto x^2 + 1$.

1. What is the domain of f?
2. What is the codomain of f?
3. What is the image of $x = -3$?
4. What are the preimage(s) of $y = 5$?

Ex 51: A rule h is defined by $h : \mathbb{Z} \longrightarrow \mathbb{R}$ $\quad x \longmapsto \sqrt{x}$.

1. State the domain and codomain of h.
2. Explain why this rule does not define a valid function.

Ex 52: Let $A = \{-2, -1, 0, 1, 2\}$ and $B = \{0, 1, 2, 3, 4\}$. Consider the function $k : A \longrightarrow B$.

$$x \longmapsto x^2$$

1. What is the domain of k?
2. What is the codomain of k?
3. Find the image for each element in the domain.

A.2 NATURAL DOMAIN AND RANGE

A.2.1 FINDING THE NATURAL DOMAIN: LEVEL 1

MCQ 53: Find the domain of the function $f : x \mapsto x^2$.

- ☐ $\mathbb{R}$
- ☐ $\{x \in \mathbb{R} \mid x \neq 0\}$
- ☐ $[0, +\infty)$
- ☐ $(-\infty, 0)$

MCQ 54: Find the domain of the function $f : x \mapsto \dfrac{1}{x}$.

- ☐ $\mathbb{R}$
- ☐ $\{x \in \mathbb{R} \mid x \neq 0\}$
- ☐ $[0, +\infty)$
- ☐ $(-\infty, 0)$

MCQ 55: Find the domain of the function $f : x \mapsto \sqrt{x}$.

- ☐ $\mathbb{R}$
- ☐ $\{x \in \mathbb{R} \mid x \neq 0\}$
- ☐ $[0, +\infty)$
- ☐ $(-\infty, 0)$

A.2.2 FINDING THE NATURAL DOMAIN: LEVEL 2

MCQ 56: Find the domain of the function $f : x \mapsto \sqrt{2x - 4}$.

- ☐ $\mathbb{R}$
- ☐ $\{x \in \mathbb{R} \mid x \neq 4\}$
- ☐ $[2, +\infty)$
- ☐ $(-\infty, 4]$

MCQ 57: Find the domain of the function $f : x \mapsto \dfrac{x}{x - 3}$.

- ☐ $\mathbb{R}$
- ☐ $\{x \in \mathbb{R} \mid x \neq 3 \text{ and } x \neq 0\}$
- ☐ $[3, +\infty)$
- ☐ $(-\infty, 3)$
- ☐ $\{x \in \mathbb{R} \mid x \neq 3\}$

MCQ 58: Find the domain of the function $f : x \mapsto \dfrac{1}{x^2 - 9}$.

- ☐ $\mathbb{R}$
- ☐ $(-3, 3)$
- ☐ $[0, +\infty)$
- ☐ $\{x \in \mathbb{R} \mid x \neq -3 \text{ and } x \neq 3\}$
- ☐ $x > 3$

MCQ 59: Find the domain of the function $f : x \mapsto \sqrt{6 - 2x}$.

- ☐ $\mathbb{R}$
- ☐ $(-\infty, 3]$
- ☐ $[3, +\infty)$
- ☐ $(-\infty, 6]$

A.2.3 FINDING THE NATURAL DOMAIN: LEVEL 3

Ex 60: Find the natural domain of the function $f(x) = \dfrac{5}{x + 3}$. Express your answer in interval notation.

Ex 61: Find the natural domain of the function $g(x) = \sqrt{x-4}$. Express your answer in interval notation.

Ex 62: Find the natural domain of the function $h(x) = \dfrac{1}{\sqrt{x-5}}$. Express your answer in interval notation.

Ex 63: Find the natural domain of the function $k(x) = \sqrt{16-x^2}$. Express your answer in interval notation.

A.2.4 FINDING THE RANGE

Ex 64: Find the range of the function $f : \mathbb{R} \longrightarrow \mathbb{R}$ $x \longmapsto |x| - 2$.

Express your answer in interval notation.

Ex 65: Find the range of the function $f : \mathbb{R} \longrightarrow \mathbb{R}$ $x \longmapsto (x-2)^2 + 3$.

Express your answer in interval notation.

Ex 66: Find the range of the function $g : [0, \infty) \longrightarrow \mathbb{R}$ $x \longmapsto 5 - \sqrt{x}$.

Express your answer in interval notation.

A.3 TABLES OF VALUES

A.3.1 FILLING TABLES OF VALUES

Ex 67: For $f(x) = -2x + 1$, fill in the table:

x	-2	-1	0	1	2
$f(x)$					

Ex 68: For $f(x) = x^2 - 3x + 1$, fill in the table:

x	-2	-1	0	1	2
$f(x)$					

Ex 69: For the rational function $f(x) = \dfrac{2x}{x+1}$, fill in the table of values.

x	-2	0	1	2
$f(x)$				

Ex 70: For the absolute value function $g(x) = |x-2|$, fill in the table of values:

x	-1	0	1	2	3
$g(x)$					

A.3.2 FINDING THE FUNCTION FROM A TABLE

Ex 71: The table below gives some values for the function $h(x) = ax + b$. Find the values of a and b and complete the table.

x	0	1	2	5
$h(x)$	-3		1	

Ex 72: The table below gives some values for the function $f(x) = ax^2 + c$. Find the values of a and c and complete the table.

x	-1	0	2	3
$f(x)$		-1	11	

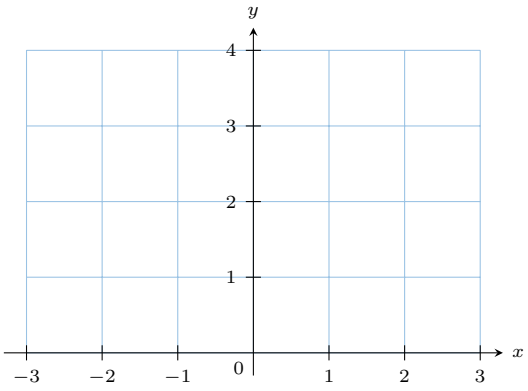
A.4 GRAPHS

A.4.1 PLOTTING LINE GRAPHS

Ex 73: Here is a table of values for the function $f(x) = x^2$:

x	-2	-1	-0.5	0	0.5	1	2
$f(x)$	4	1	0.25	0	0.25	1	4

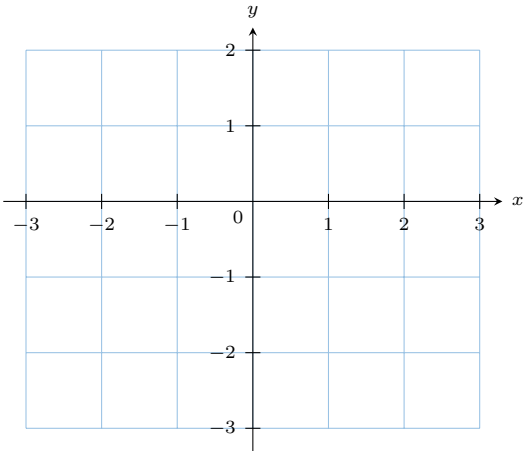
Plot the line graph of f.



Ex 74: Here is a table of values for the function $f(x) = 0.5x - 1$:

x	-3	-2	-1	0	1	2	3
$f(x)$	-2.5	-2	-1.5	-1	-0.5	0	0.5

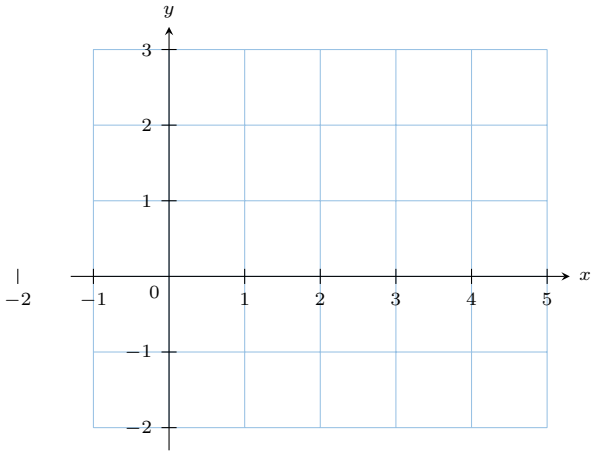
Plot the line graph of f.



Ex 75: Here is a table of values for the function $f(x) = -|x-2| + 2$:

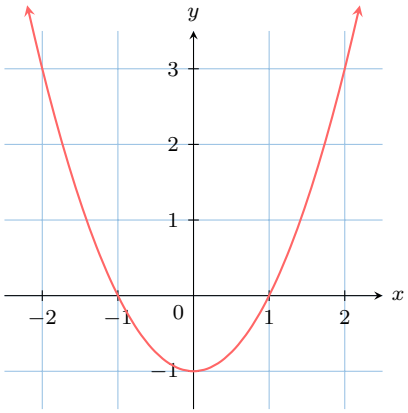
x	-1	0	1	2	3	4	5
$f(x)$	-1	0	1	2	1	0	-1

Plot the graph of f.



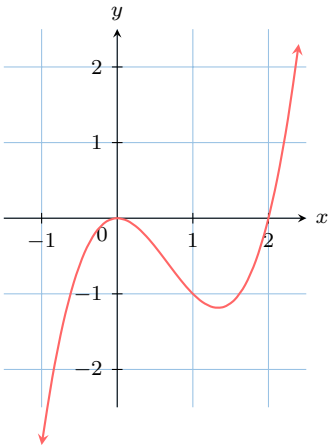
A.4.2 FINDING $f(x)$

Ex 76: The graph of $y = f(x)$ is:



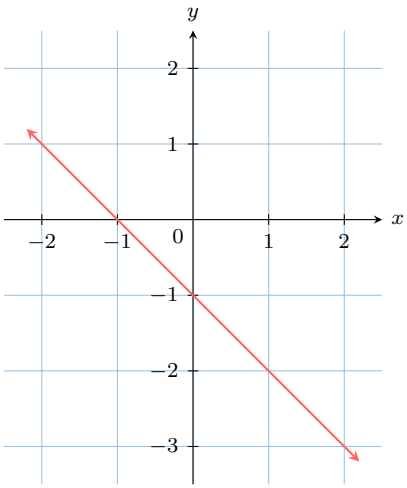
$f(2) =$ ☐

Ex 77: The graph of $y = f(x)$ is:



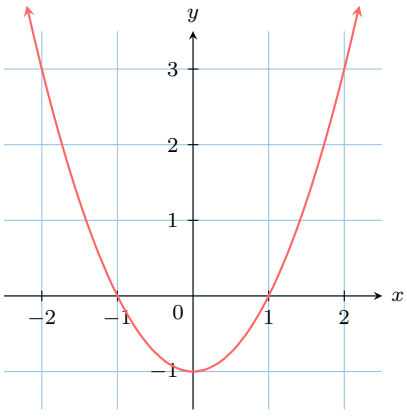
$f(1) =$ ☐

Ex 78: The graph of $y = f(x)$ is:



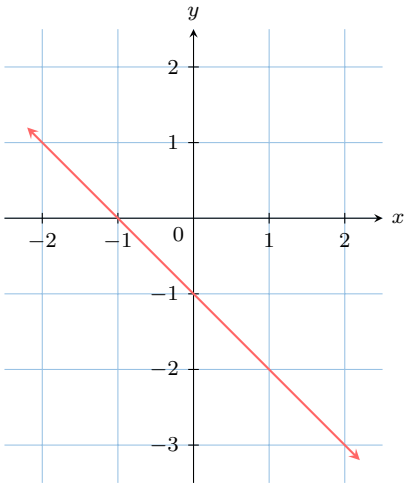
$f(-2) =$ ☐

Ex 79: The graph of $y = f(x)$ is:



$f(1) =$ ☐

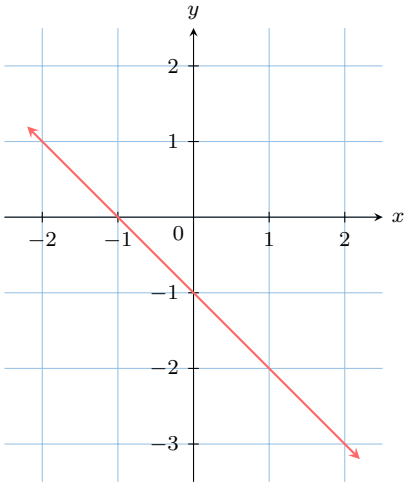
Ex 80: The graph of $y = f(x)$ is:



$f(1) =$ ☐

A.4.3 FINDING INPUTS FROM OUTPUTS ON A GRAPH

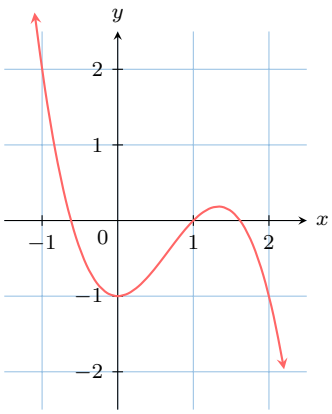
Ex 81: The graph of $y = f(x)$ is:



Find all x such that $f(x) = -2$.

$x =$ ☐

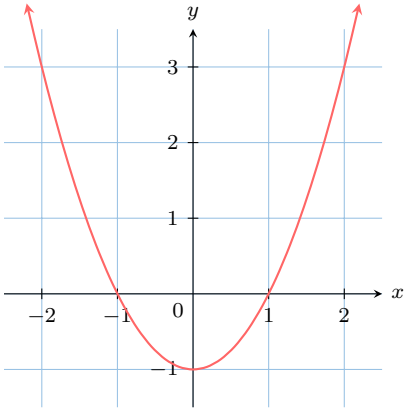
Ex 82: The graph of $y = f(x)$ is:



Find all x such that $f(x) = 2$.

$x =$ ☐

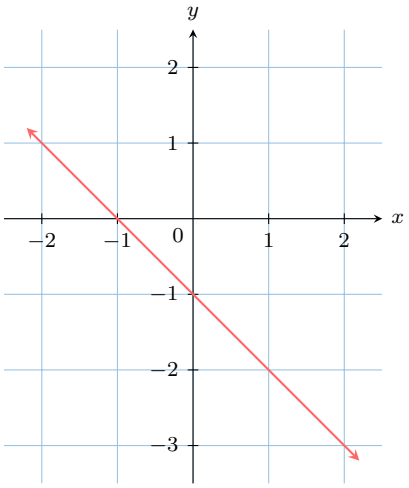
Ex 83: The graph of $y = f(x)$ is:



Find all x such that $f(x) = 3$.
Give your answers in increasing order:

$x =$ ☐ or $x =$ ☐

Ex 84: The graph of $y = f(x)$ is:



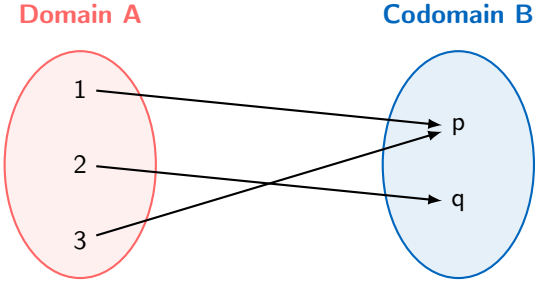
Find all x such that $f(x) = 1$.

$x =$ ☐

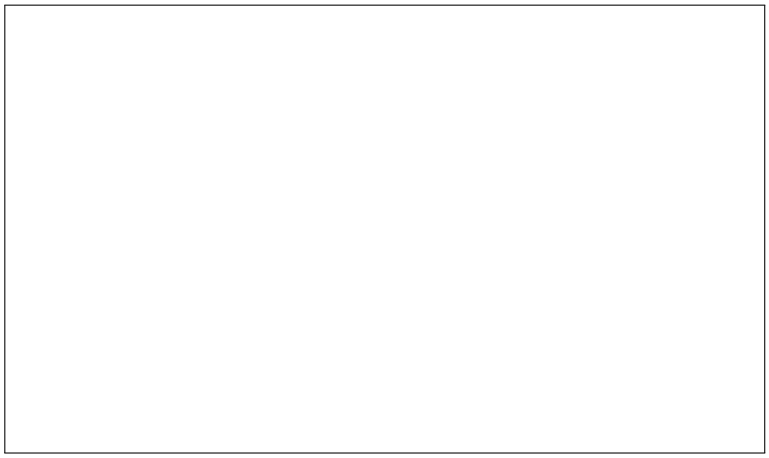
A.5 BIJECTIVE FUNCTIONS

A.5.1 ANALYZING FUNCTION PROPERTIES FROM MAPPING DIAGRAMS

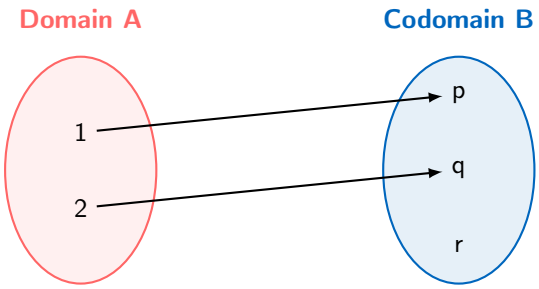
Ex 85: Let $A = \{1, 2, 3\}$ and $B = \{p, q\}$. A function $f : A \to B$ is defined by the mapping diagram below.



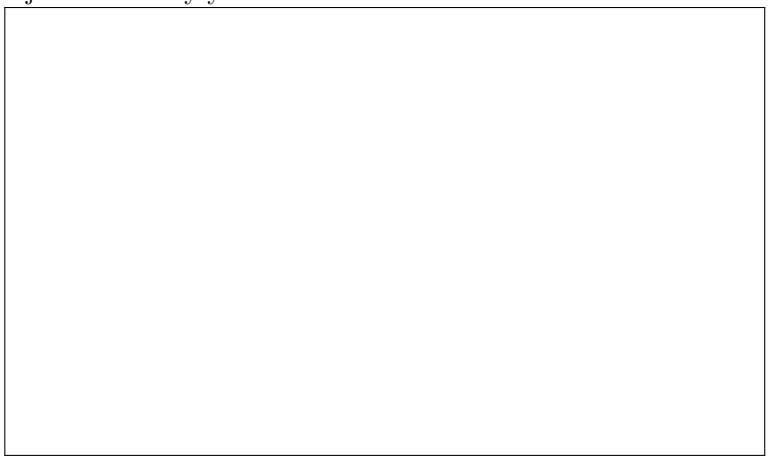
Determine if the function f is injective, surjective, and/or bijective. Justify your answers.



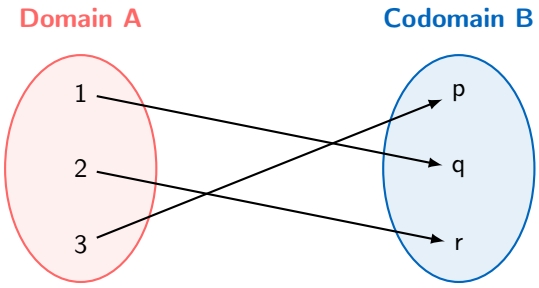
Ex 86: Let $A = \{1, 2\}$ and $B = \{p, q, r\}$. A function $g : A \to B$ is defined by the mapping diagram below.



Determine if the function g is injective, surjective, and/or bijective. Justify your answers.

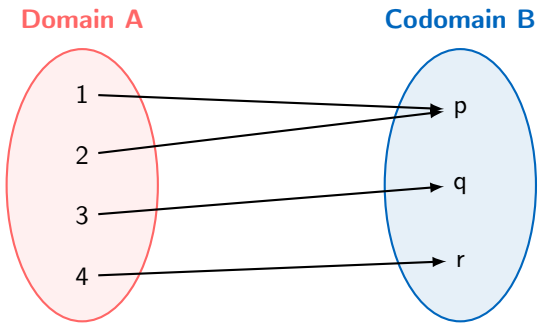


Ex 87: Let $A = \{1, 2, 3\}$ and $B = \{p, q, r\}$. A function $h : A \to B$ is defined by the mapping diagram below.



Determine if the function h is injective, surjective, and/or bijective. Justify your answers.

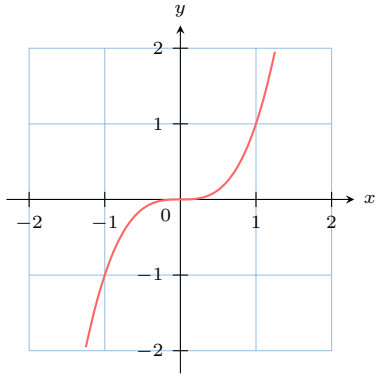
Ex 88: Let $A = \{1, 2, 3, 4\}$ and $B = \{p, q, r\}$. A function $k : A \to B$ is defined by the mapping diagram below.



Determine if the function k is injective, surjective, and/or bijective. Justify your answers.

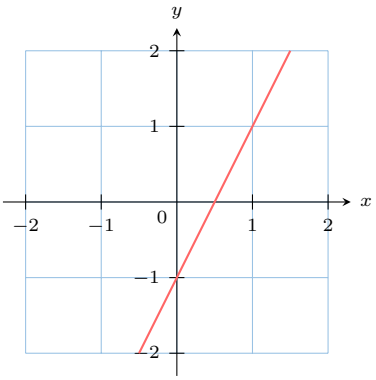
A.5.2 APPLYING THE HORIZONTAL LINE TEST

Ex 89: Consider the function $f : \mathbb{R} \longrightarrow \mathbb{R}$, graphed below.
$x \longmapsto x^3$



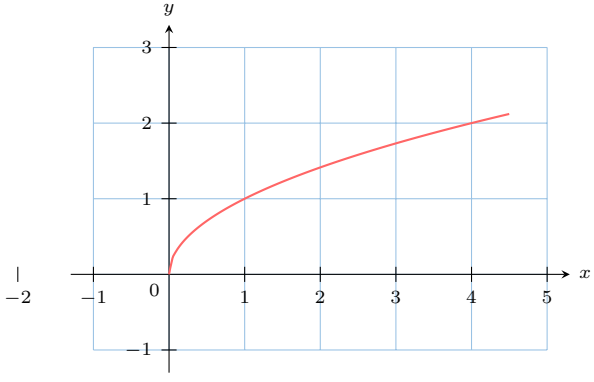
Determine if the function f is injective, surjective, and/or bijective. Justify your answers.

Ex 90: Consider the function $h : \mathbb{R} \longrightarrow \mathbb{R}$, graphed below.
$x \longmapsto 2x - 1$



Determine if the function h is injective, surjective, and/or bijective. Justify your answers.

Ex 91: Consider the function $f : [0, \infty) \longrightarrow [0, \infty)$, graphed below.
$x \longmapsto \sqrt{x}$



Determine if the function f is injective, surjective, and/or bijective. Justify your answers.

B OPERATIONS ON FUNCTIONS

B.1 ALGEBRA OF FUNCTIONS

B.1.1 ADDING, SUBTRACTING, AND MULTIPLYING FUNCTIONS

Ex 92: For $f(x) = 2x + 2$ and $g(x) = 3 - x$, find in simplest form:

1. $f(3) + g(3) =$
2. $f(-1) + g(-1) =$
3. $f(x) + g(x) =$
4. $g(x) + f(x) =$

Ex 93: For $f(x) = x^2 - 2$ and $g(x) = x - 2$, find in simplest form:

1. $f(0) + g(0) =$
2. $f(-2) + g(-2) =$
3. $f(x) + g(x) =$
4. $f(x) - g(x) =$

Ex 94: Let $f(x) = 3x - 2$ and $g(x) = x^2$. Find in factorized form:

$$f(x) \times g(x) =$$

Ex 95: Let $f(x) = 2x + 5$ and $g(x) = x - 4$. Find in factorized form:

$$f(x) \times g(x) =$$

B.1.2 DECOMPOSING EXPRESSIONS INTO FUNCTIONS

Ex 96: Find two functions f and g such that $f(x) \times g(x) = (x + 3)^2(x - 2)$.

- $f(x) =$
- $g(x) =$

Ex 97: Find two functions f and g such that $f(x) \times g(x) = (x^2 + 4)(3x - 7)$.

- $f(x) =$
- $g(x) =$

Ex 98: Find two functions f and g such that $f(x) + g(x) = (x - 2)^2 + \sqrt{x}$.

- $f(x) =$
- $g(x) =$

Ex 99: Find two functions f and g such that $f(x) + g(x) = \dfrac{1}{x} + (x + 1)^2$.

- $f(x) =$
- $g(x) =$

B.1.3 OPERATIONS ON FUNCTIONS AND THEIR DOMAINS

Ex 100: Let the functions f and g be defined by the rules $f(x) = \sqrt{x + 2}$ and $g(x) = \sqrt{3 - x}$. Let $h = f + g$.

1. Find the domain of f and the domain of g.
2. Find the domain of the combined function h.
3. Calculate $h(-1)$.

Ex 101: Let the functions f and g be defined by the rules $f(x) = \dfrac{1}{x - 4}$ and $g(x) = \sqrt{x - 1}$. Let $h = f \times g$.

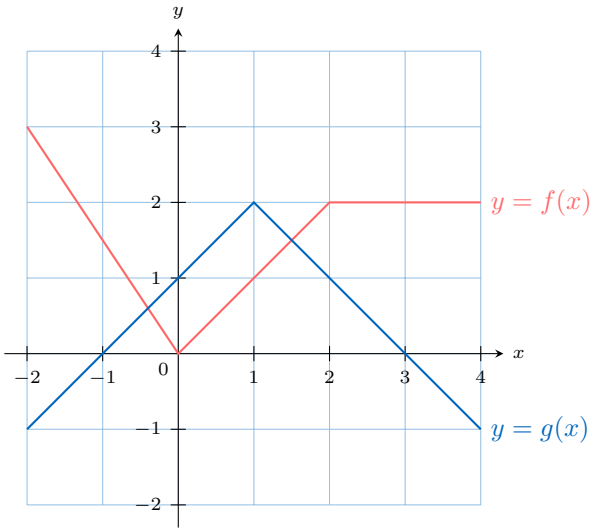
1. Find the domain of f and the domain of g.

2. Find the domain of the combined function h.

3. Calculate $h(5)$.

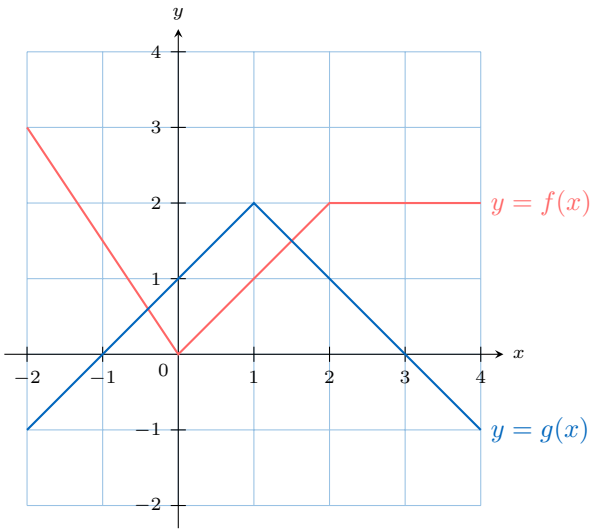
B.1.4 GRAPHICAL COMBINATION OF FUNCTIONS

Ex 102: The graphs of two functions, f and g, are shown below.



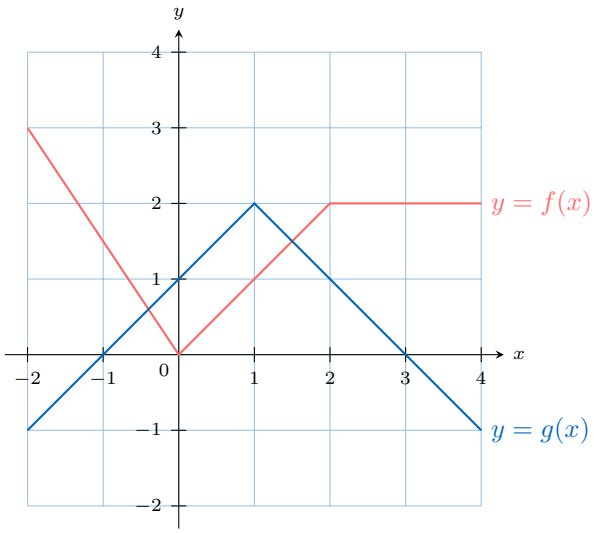
Plot the graph of the function $f + g$.

Ex 103: The graphs of two functions, f and g, are shown below.



Plot the graph of the function $f - g$.

Ex 104: The graphs of two functions, f and g, are shown below.



Plot the graph of the function $f \times g$.

B.2 COMPOSITION OF FUNCTIONS

B.2.1 EVALUATING COMPOSITE FUNCTIONS

Ex 105: For $f(x) = 2x + 2$ and $g(x) = 3 - x$, find in simplest form:

1. $f(g(3)) =$ ☐
2. $f(g(-1)) =$ ☐
3. $f(g(x)) =$ ☐
4. $g(f(x)) =$ ☐

Ex 106: For $f(x) = x^2 + 2x$ and $g(x) = 2 - x$, find in simplest form:

1. $f(g(3)) =$ ☐
2. $f(g(-1)) =$ ☐
3. $f(g(x)) =$ ☐
4. $g(f(x)) =$ ☐

Ex 107: For $f(x) = 3x - 5$, find in simplest form:

1. $f(f(-1)) =$ ☐
2. $f(f(x)) =$ ☐

B.2.2 DECOMPOSING FUNCTIONS INTO COMPOSITIONS

Ex 108: Find two functions f and g such that $f(g(x)) = \sqrt{2x-1}$ and $g(x) \neq x$.

- $f(x) =$ ☐
- $g(x) =$ ☐

Ex 109: Find two functions f and g such that $f(g(x)) = (x+2)^5$ and $g(x) \neq x$.

- $f(x) =$ $\square$
- $g(x) =$ $\square$

Ex 110: Find two functions f and g such that $f(g(x)) = \dfrac{1}{x^2+1}$ and $g(x) \neq x$.

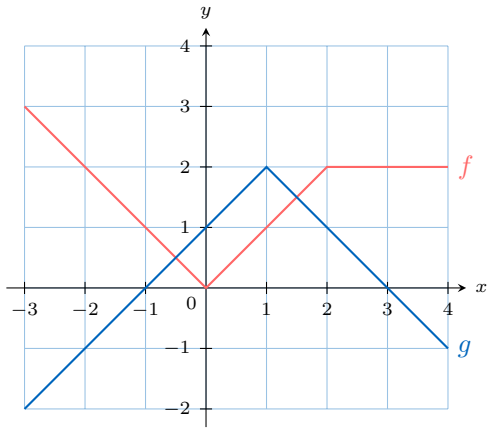
- $f(x) =$ $\square$
- $g(x) =$ $\square$

Ex 111: Find two functions f and g such that $f(g(x)) = (x^3 - 2)^{-4}$ and $g(x) \neq x$.

- $f(x) =$ $\square$
- $g(x) =$ $\square$

B.2.3 EVALUATING COMPOSITE FUNCTIONS FROM GRAPHS

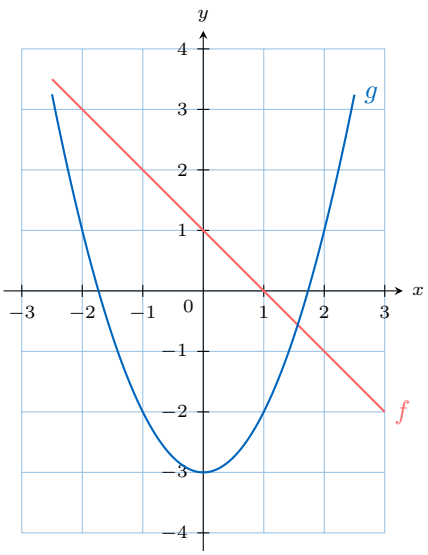
Ex 112: The graphs of two functions, f and g, are shown below.



Use the graphs to find the values of:

1. $(f \circ g)(1) =$ $\square$
2. $(g \circ f)(-2) =$ $\square$
3. $(f \circ f)(0) =$ $\square$

Ex 113: The graphs of two functions, f and g, are shown.



Find the values of:

1. $(f \circ g)(2) =$ $\square$
2. $(g \circ f)(-1) =$ $\square$

B.2.4 SOLVING EQUATIONS WITH COMPOSITE FUNCTIONS

Ex 114: Let $f(x) = x^2 - 3$ and $g(x) = 2x - 1$. Find all values of x such that $(f \circ g)(x) = 6$.

Ex 115: Let $f(x) = 2x - 5$ and $g(x) = \dfrac{x+1}{3}$. Find all values of x such that $(g \circ f)(x) = x$.

Ex 116: Let $f(x) = x^2 - 4x + 5$ and $g(x) = x - 1$. Find all values of x such that $(f \circ g)(x) = 2$.

B.3 INVERSE FUNCTIONS

B.3.1 FINDING AND CHECKING INVERSES

Ex 117:

1. Find the inverse of $f(x) = x + 3$.

$$f^{-1}(x) = \boxed{}$$

2. Evaluate

$$f^{-1}(f(x)) = \boxed{}$$
$$f(f^{-1}(x)) = \boxed{}$$

Ex 118:

1. Find the inverse of $f(x) = 4x - 8$.

$$f^{-1}(x) = \boxed{}$$

2. Evaluate

$$f^{-1}(f(x)) = \boxed{}$$
$$f(f^{-1}(x)) = \boxed{}$$

Ex 119:

1. Find the inverse of $f(x) = \dfrac{x}{2} - 3$.

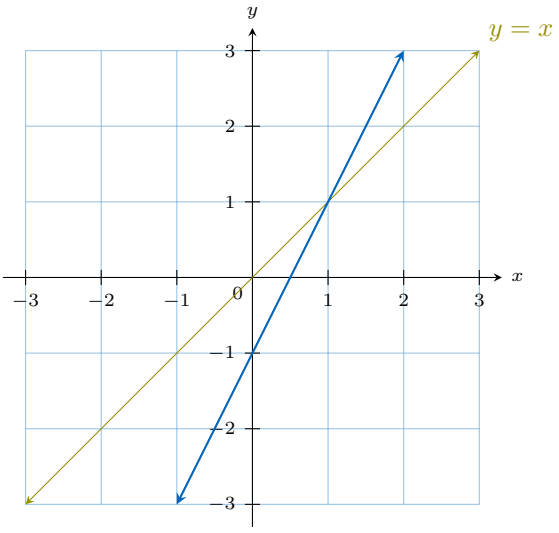
$$f^{-1}(x) = \boxed{}$$

2. Evaluate

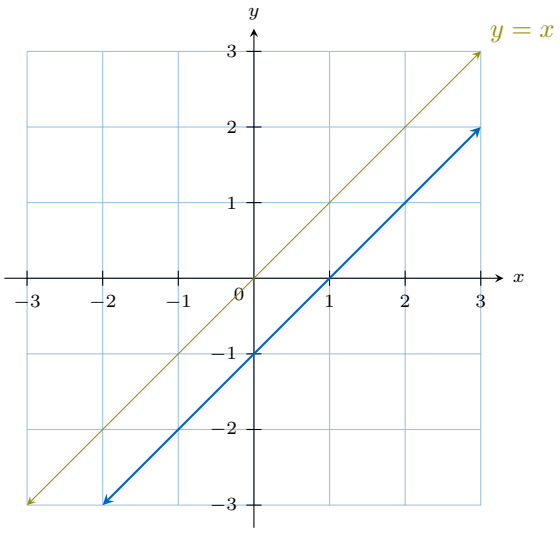
$$f^{-1}(f(x)) = \boxed{}$$
$$f(f^{-1}(x)) = \boxed{}$$

B.3.2 GRAPHING THE INVERSE FUNCTION BY REFLECTION

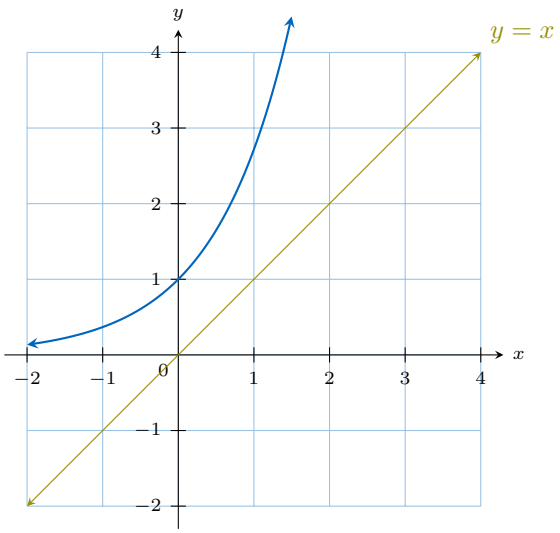
Ex 120: Draw the graph of the inverse function of the blue graph:



Ex 121: Draw the graph of the inverse function of the blue graph:



Ex 122: Draw the graph of the inverse function of the blue graph:



B.3.3 FINDING INVERSES OF VARIOUS FUNCTION TYPES

Ex 123: Let the function f be defined by

$$\begin{aligned} f : [1, \infty) &\longrightarrow [2, \infty) \\ x &\longmapsto (x-1)^2 + 2 \end{aligned}$$

1. State the domain and range of f.

2. Find an expression for $f^{-1}(x)$.

3. State the domain and range of f^{-1}.

Ex 124: Let the function f be defined by $f(x) = \dfrac{2x+1}{x-3}$ for $x \neq 3$.

1. Find an expression for $f^{-1}(x)$.
2. Solve the equation $f^{-1}(x) = 4$.

Ex 125: Consider the function $f(x) = \dfrac{5}{x}$ for $x \neq 0$.

1. Find the inverse function, $f^{-1}(x)$.
2. Compare $f(x)$ and $f^{-1}(x)$ and explain what it means for a function to be its own inverse.