FUNCTION TRANSFORMATIONS

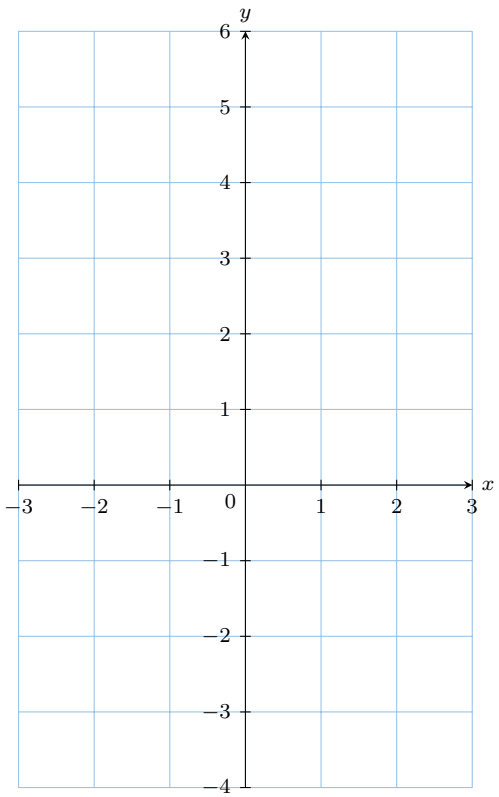
A TRANSLATION

A.1 TRANSLATING GRAPHS VERTICALLY

Ex 1: For the functions $f(x) = x$ and $g(x) = x + 3$:

1. On the same set of axes, sketch the graphs of f and g. (You may fill in a table of values for $x = -3, -2, -1, 0, 1, 2, 3$.)

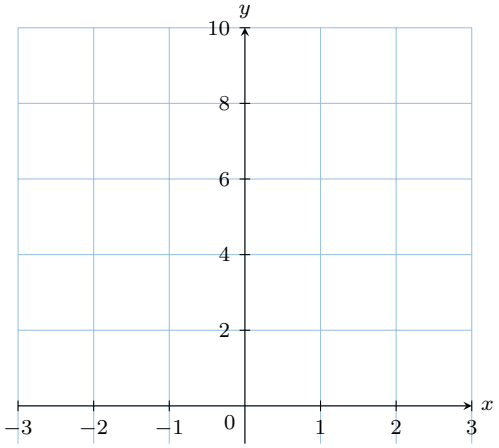
2. Find the geometrical transformation between these two graphs.



Ex 2: For the functions $f(x) = x^2$ and $g(x) = x^2 + 2$:

1. On the same set of axes, sketch the graphs of f and g. (You may fill in a table of values for $x = -3, -2, -1, 0, 1, 2, 3$.)

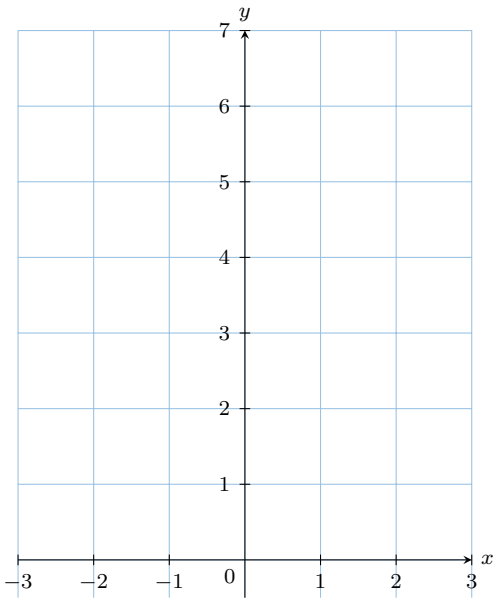
2. Find the geometrical transformation between these two graphs.



Ex 3: For the functions $f(x) = \dfrac{4}{1+x^2}$ and $g(x) = \dfrac{4}{1+x^2} + 3$:

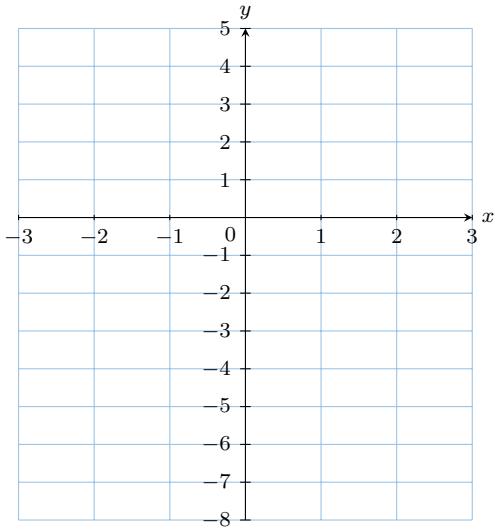
1. On the same set of axes, sketch the graphs of f and g. (You may fill in a table of values for $x = -3, -2, -1, 0, 1, 2, 3$.)

2. Find the geometrical transformation between these two graphs.



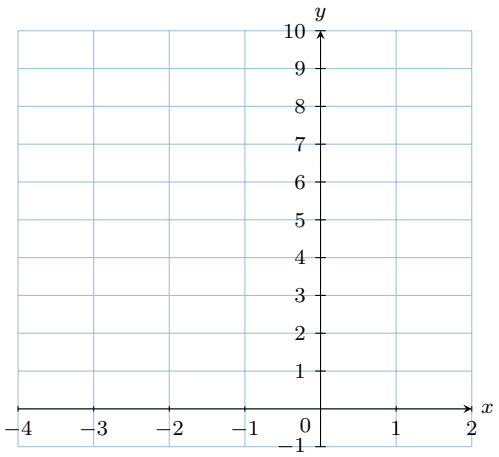
Ex 4: For the functions $f(x) = -(x-2)(x+2)$ and $g(x) = -(x-2)(x+2) - 2$:

1. On the same set of axes, sketch the graphs of f and g. (You may fill in a table of values for $x = -3, -2, -1, 0, 1, 2, 3$.)
2. Find the geometrical transformation between these two graphs.



Ex 6: For the functions $f(x) = x^2$ and $g(x) = (x+2)^2$:

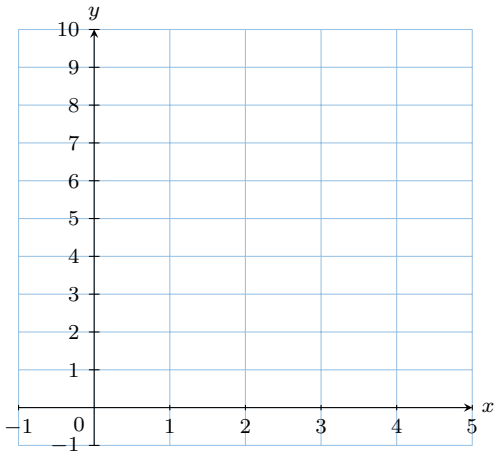
1. On the same set of axes, sketch the graphs of f and g. (You may fill in a table of values for $x = -4, -3, -2, -1, 0, 1, 2$.)
2. Find the geometrical transformation between these two graphs.



A.2 TRANSLATING GRAPHS HORIZONTALLY

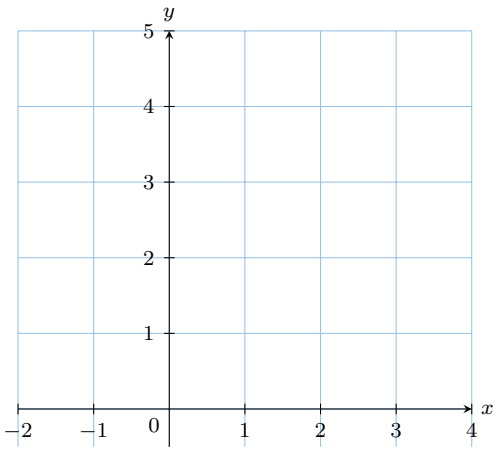
Ex 5: For the functions $f(x) = x^2$ and $g(x) = (x-3)^2$:

1. On the same set of axes, sketch the graphs of f and g. (You may fill in a table of values for $x = -1, 0, 1, 2, 3, 4, 5$.)
2. Find the geometrical transformation between these two graphs.



Ex 7: For the functions $f(x) = \dfrac{4}{1+x^2}$ and $g(x) = \dfrac{4}{1+(x-2)^2}$:

1. On the same set of axes, sketch the graphs of f and g. (You may fill in a table of values for $x = -2, -1, 0, 1, 2, 3, 4$.)
2. Find the geometrical transformation between these two graphs.



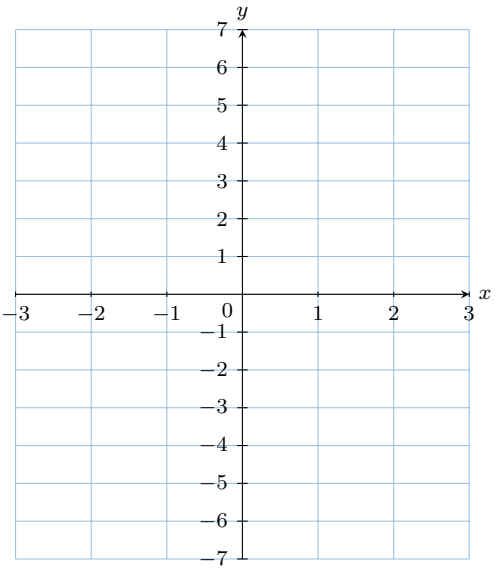
B DILATION

B.1 DILATING GRAPHS VERTICALLY

Ex 8: For the functions $f(x) = x$ and $g(x) = 2x$:

1. On the same set of axes, sketch the graphs of f and g. (You may fill in a table of values for $x = -3, -2, -1, 0, 1, 2, 3$.)

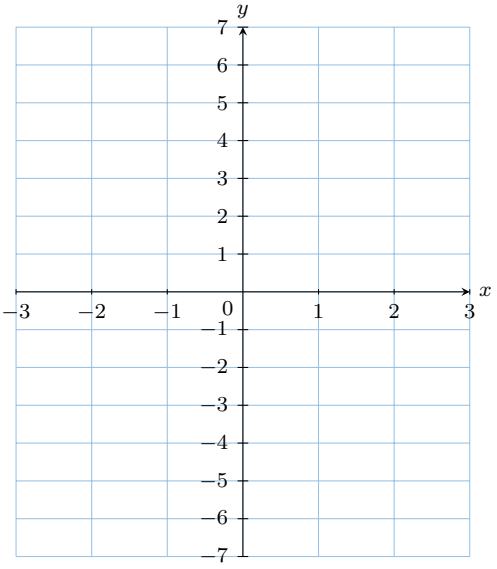
2. Find the geometrical transformation between these two graphs.



Ex 9: For the functions $f(x) = \dfrac{x}{2}$ and $g(x) = 2x$:

1. On the same set of axes, sketch the graphs of f and g. (You may fill in a table of values for $x = -3, -2, -1, 0, 1, 2, 3$.)

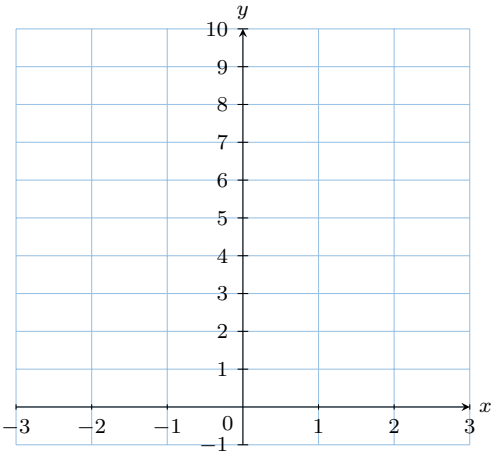
2. Find the geometrical transformation between these two graphs.



Ex 10: For the functions $f(x) = x^2$ and $g(x) = \dfrac{x^2}{2}$:

1. On the same set of axes, sketch the graphs of f and g. (You may fill in a table of values for $x = -3, -2, -1, 0, 1, 2, 3$.)

2. Find the geometrical transformation between these two graphs.

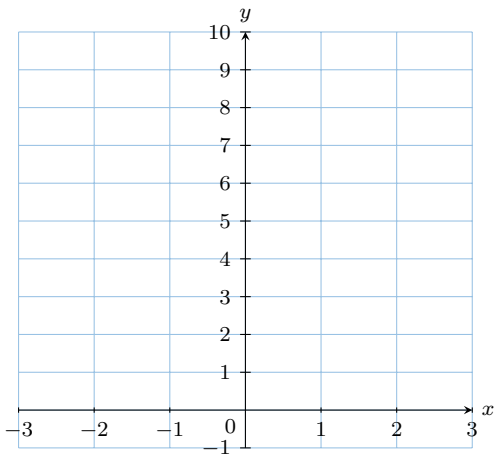


B.2 DILATING GRAPHS HORIZONTALLY

Ex 11: For the functions $f(x) = x^2$ and $g(x) = (2x)^2$:

1. On the same set of axes, sketch the graphs of f and g.

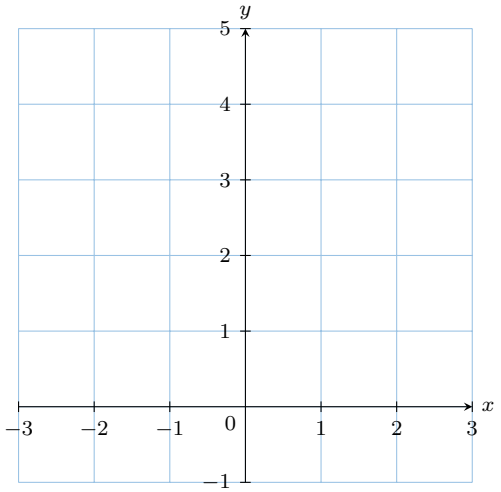
2. Find the geometrical transformation that maps the graph of f to the graph of g.



Ex 12: For the functions $f(x) = x^2$ and $g(x) = (\frac{1}{2}x)^2$:

1. On the same set of axes, sketch the graphs of f and g.

2. Find the geometrical transformation that maps the graph of f to the graph of g.



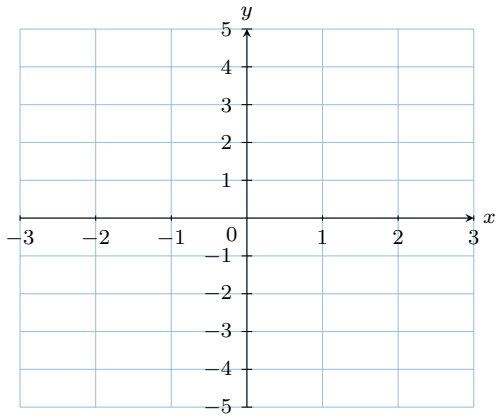
C REFLECTION

C.1 REFLECTING GRAPHS

Ex 13: For the functions $f(x) = x - 1$ and $g(x) = -(x - 1)$:

1. On the same set of axes, sketch the graphs of f and g. (You may fill in a table of values for $x = -3, -2, -1, 0, 1, 2, 3$.)

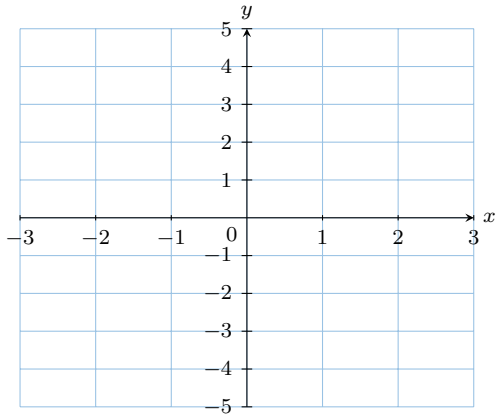
2. Find the geometrical transformation between these two graphs.



Ex 14: For the functions $f(x) = x - 1$ and $g(x) = -x - 1$:

1. On the same set of axes, sketch the graphs of f and g. (You may fill in a table of values for $x = -3, -2, -1, 0, 1, 2, 3$.)

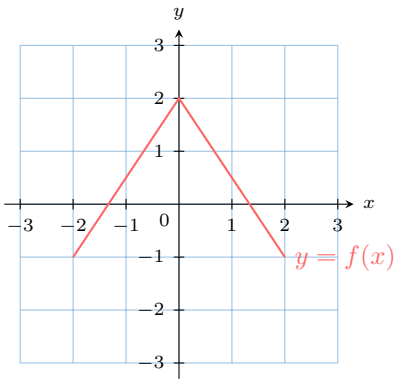
2. Find the geometrical transformation between these two graphs.



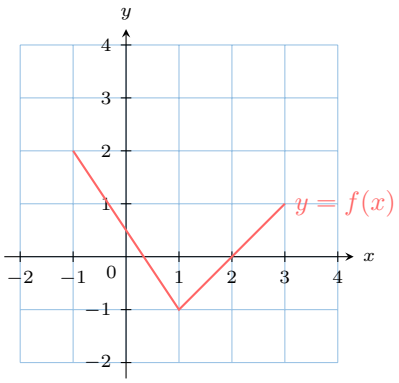
D COMBINING TRANSFORMATIONS

D.1 APPLYING COMBINED TRANSFORMATIONS

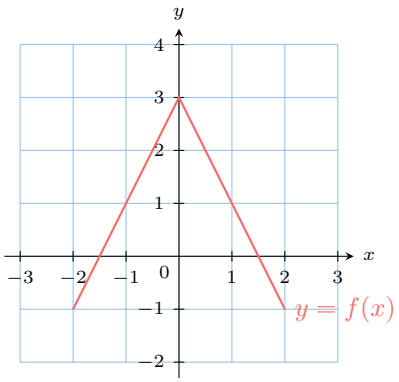
Ex 15: The graph of $y = f(x)$ is shown. On the same axes, sketch the graph of $y = f(x + 1) - 2$.



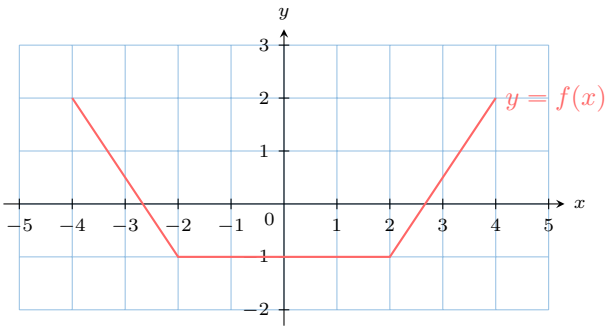
Ex 16: The graph of $y = f(x)$ is shown. On the same axes, sketch the graph of $y = 2f(x-1)$.



Ex 17: The graph of $y = f(x)$ is shown. On the same axes, sketch the graph of $y = -f(x) + 1$.



Ex 18: The graph of $y = f(x)$ is shown. On the same axes, sketch the graph of $y = f(2x) - 1$.



D.2 FINDING EQUATIONS FROM A SEQUENCE OF TRANSFORMATIONS

Ex 19: Consider a function $y = f(x)$. Find the equation of the resulting function, $g(x)$, if the graph of f is transformed by the following sequence:

1. A reflection in the y-axis.
2. A horizontal stretch by a factor of 2.

Ex 20: Consider a function $y = f(x)$. Find the equation of the resulting function, $g(x)$, if the graph of f is transformed by the following sequence:

1. A vertical stretch by a factor of 3.
2. A reflection in the x-axis.

Ex 21: Consider a function $y = f(x)$. Find the equation of the resulting function, $g(x)$, if the graph of f is transformed by the following sequence:

1. A horizontal translation of 5 units to the left.
2. A horizontal compression by a factor of $\frac{1}{2}$.