FRACTIONS

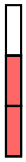
A DEFINITIONS

Definition Fraction

A **fraction** consists of two numbers: the **numerator**, a, and the **denominator**, $b \neq 0$, separated by a horizontal bar:

$$\frac{a \leftarrow \text{numerator: number of equal parts considered}}{b \leftarrow \text{denominator: number of equal parts the unit is divided}}$$

A fraction can be represented as:

- Symbol : $\frac{2}{3}$
- Words : two thirds or two over three
- Linear model : 

B FRACTION AS QUOTIENT

Proposition Fraction as Quotient

A fraction is a quotient that represents the result of **division**. It tells us how much of something we have when we divide it into equal parts.

- **The top number (numerator)** is the whole.
- **The bottom number (denominator)** is the number of equal parts the whole is divided into.

The fraction $\frac{a}{b}$ is the same as saying **"a divided by b"**.

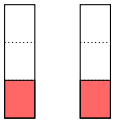
$$\frac{a}{b} = a \div b$$

The fraction $\frac{a}{b}$ is the number which, when multiplied by b, gives a:

$$\frac{a}{b} \times b = a$$

Ex:

$$2 \div 3 = \quad = \frac{2}{3}$$

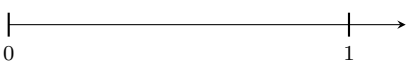


C ON THE NUMBER LINE

Method Representing a Fraction on the Number Line

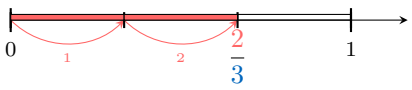
To represent the fraction $\frac{2}{3}$ on a number line.

1. Draw a straight line and mark the points 0 and 1.



2. Divide the line between 0 and 1 into 3 equal parts.

3. Count 2 parts from 0 and mark the point.



D EQUIVALENT FRACTIONS

Definition Equivalent Fractions

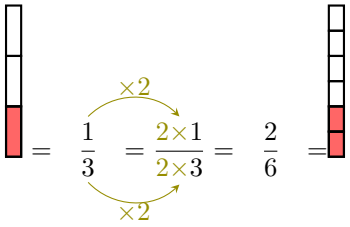
- When you multiply the numerator and the denominator by the same number, the fractions are equals.

$$\frac{a}{b} \overset{\times k}{=} \frac{k\times a}{k\times b}$$

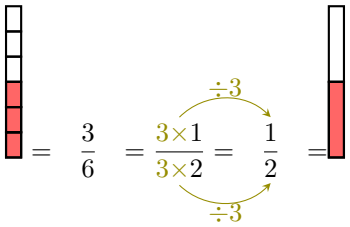
- When you divide the numerator and the denominator by the same number, the fractions are equals.

$$\frac{k\times a}{k\times b} \overset{\div k}{=} \frac{a}{b}$$

Ex:



Ex:



E SIMPLIFICATION

Definition Simplest form

A fraction is in **simplest form** if it is written with the smallest possible whole number numerator and denominator, that is, if its numerator and denominator have no common factors other than 1.

Ex:

- $\frac{2}{3}$ is in simplest form.
- $\frac{4}{6}$ is **not** in simplest form because we can write $\frac{4}{6} = \frac{2}{3}$.

Method **Simplifying a fraction**

To simplify a fraction (or to write a fraction in its simplest form), we cancel the greatest common factor of the numerator and the denominator .

Ex: Simplify $\frac{4}{6}$.

Answer:

$$\frac{4}{6} = \frac{2 \times \cancel{2}}{3 \times \cancel{2}} = \frac{2}{3}$$

F CROSS MULTIPLICATION

Proposition **Cross Multiplication Property**

$$\frac{a}{b} = \frac{c}{d} \quad \text{if and only if} \quad a \times d = b \times c$$

Ex: Solve x for $\frac{10}{5} = \frac{x}{8}$.

Answer:

$$\begin{aligned} \frac{10}{5} = \frac{x}{8} & \\ 5 \times x &= 10 \times 8 \quad \text{(cross mutiplication)} \\ x &= 10 \times 8 \div 5 \quad \text{(dividing both sides by 5)} \\ x &= 16 \end{aligned}$$

G ADDITION AND SUBTRACTION

Definition **Addition and Subtraction of Fractions with Common Denominators**

- When we **add** fractions with common denominators, we keep the denominator the same and add the numerators:

$$\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b}$$

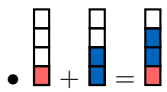
- When we **subtract** fractions with common denominators, we keep the denominator the same and subtract the numerators:

$$\frac{a}{b} - \frac{c}{b} = \frac{a-c}{b}$$

Ex: Calculate $\frac{1}{4} + \frac{2}{4}$.

Answer:

- $$\frac{1}{4} + \frac{2}{4} = \frac{1+2}{4} = \frac{3}{4}$$

- 

Method **Addition or Subtraction of Fractions with Different Denominators**

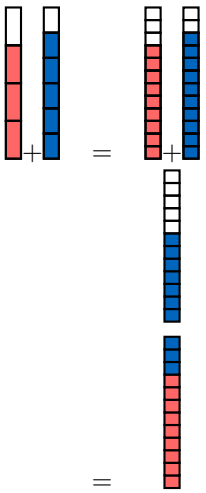
To add or subtract fractions with different denominators:

- **Find a common denominator**: Choose a common multiple of the denominators.
- **Convert each fraction**: Rewrite each fraction so it has the common denominator.
- **Add or subtract the numerators**: Add or subtract the numerators and keep the denominator the same.

Ex: Calculate $\frac{3}{4} + \frac{5}{6}$.

Answer:

- **Find a common denominator**:To add fractions, they must have the same denominator.
  - Multiples of 4: 4, 8, **12**, 16, 20, ...
  - Multiples of 6: 6, **12**, 18, 24, ...
  - The smallest common denominator is **12**.
- $\frac{3}{4}+\frac{5}{6}=\frac{3\times 3}{4\times 3}+\frac{5\times 2}{6\times 2}$
  $=\frac{9}{12}+\frac{10}{12}$ (common denominator = 12)
  $=\frac{9+10}{12}$ (adding numerators)
  $=\frac{19}{12}$
- **Visual representation**:



H MULTIPLICATION OF A FRACTION BY A NUMBER

Definition **Multiplication of a Fraction by a Number**

To multiply a fraction by a whole number:

1. Multiply the numerator by the number.
2. Keep the denominator the same.

$$a\times\frac{b}{c}=\frac{a\times b}{c}$$

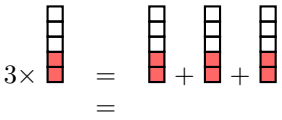
Ex: Calculate $3\times\frac{2}{5}$.

Answer:

- Mathematical calculation:

$$3\times\frac{2}{5}=\frac{3\times 2}{5}$$
$$=\frac{6}{5}$$

- Visual representation:



I MULTIPLICATION OF FRACTIONS

Definition Multiplication of Fractions

To multiply fractions, **tu multiplies** the numerators and **tu multiplies** the denominators:

$$\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$$

Ex: Calculate $\frac{5}{2} \times \frac{3}{4}$.

Answer:

$$\begin{aligned} \frac{5}{2} \times \frac{3}{4} &= \frac{5 \times 3}{2 \times 4} \\ &= \frac{15}{8} \end{aligned}$$

Method Canceling Common Factors

To make multiplication easier, **tu peux annuler** any common factors in the numerators and denominators before multiplying.

Ex: Calculate $\frac{31}{7} \times \frac{12}{31}$.

Answer:

$$\begin{aligned} \frac{31}{7} \times \frac{12}{31} &= \frac{\cancel{31} \times 12}{7 \times \cancel{31}} \quad \text{(cancel the common factor 31)} \\ &= \frac{12}{7} \end{aligned}$$

J DIVISION OF FRACTIONS

Definition Reciprocal

The **reciprocal** of a number is a number that, when multiplied by the original number, gives 1.

Proposition Reciprocal of a fraction

The reciprocal of the fraction $\frac{a}{b}$ is $\frac{b}{a}$.

Ex: State the reciprocal of $\frac{5}{7}$.

Answer: The reciprocal of $\frac{5}{7}$ is $\frac{7}{5}$.

Definition Division of fractions

To divide by a fraction, you multiply by its reciprocal:

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c},$$

or equivalently,

$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \times \frac{d}{c}.$$

Ex: Calculate $\frac{2}{3} \div \frac{5}{7}$.

Answer:

$$\begin{aligned} \frac{2}{3} \div \frac{5}{7} &= \frac{2}{3} \times \frac{7}{5} \quad \text{(multiply by the reciprocal)} \\ &= \frac{2 \times 7}{3 \times 5} \quad \text{(multiply numerators and denominators)} \\ &= \frac{14}{15}. \end{aligned}$$

K SIGN RULES

Proposition Sign rules

$$\frac{-a}{b} = \frac{a}{-b} = -\frac{a}{b},$$

and

$$\frac{-a}{-b} = \frac{a}{b}.$$

Ex: Simplify $\frac{-4}{-6}$.

Answer:

$$\begin{aligned} \frac{-4}{-6} &= \frac{4}{6} && \text{(a negative divided by a negative is positive)} \\ &= \frac{2 \times 2}{3 \times 2} && \text{(cancel the common factor 2)} \\ &= \frac{2}{3}. \end{aligned}$$

L ORDER OF OPERATIONS

Definition Order of Operations

The division line in a fraction acts as a grouping symbol (like parentheses). This means that, according to the order of operations (PEMDAS), you must first evaluate the numerator and the denominator before performing the division.

Ex: Simplify $\frac{1+7}{3\times4}$.

Answer:

$$\begin{aligned} \frac{1+7}{3 \times 4} &= \frac{8}{12} && \text{(evaluate numerator and denominator)} \\ &= \frac{2 \times \cancel{4}}{3 \times \cancel{4}} && \text{(cancel common factor)} \\ &= \frac{2}{3} \end{aligned}$$