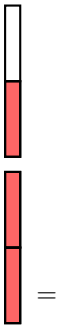
FRACTIONS

A DEFINITIONS

Definition Fraction

A **fraction** includes two numbers: the **numerator** and the **denominator**, separated by a bar.



$= \dfrac{3}{2}$

numerator: number of equal parts considered

denominator: number of equal parts the unit is divided

B ON THE NUMBER LINE

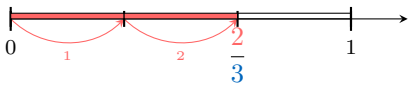
Method Representing a Fraction on the Number Line

To represent the fraction $\frac{2}{3}$ on a number line.

1. Draw a straight line and mark the points 0 and 1.

2. Divide the line between 0 and 1 into 3 equal parts.

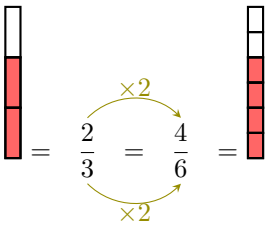
3. Count 2 parts from 0 and mark the point.



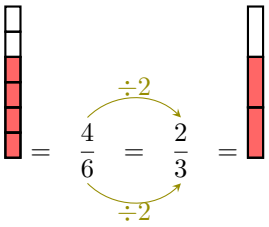
C EQUIVALENT FRACTIONS

Definition Equivalent Fractions

- When you multiply the numerator and the denominator by the same number, the fractions are equals.



- When you divide the numerator and the denominator by the same number, the fractions are equals.

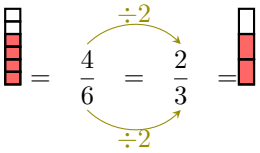


D SIMPLIFICATION

Method Simplifying a fraction

To simplify a fraction, we find an equivalent fraction with the smallest possible numerator and denominator.

Ex: Simplify $\frac{4}{6}$

Answer: 

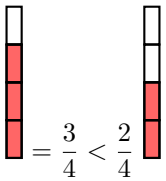
E ORDERING FRACTIONS

Definition Ordering Fractions with the Same Denominator

For two fractions with the same denominator, the fraction with the larger numerator is larger.

Ex: Compare $\frac{3}{4}$ and $\frac{2}{4}$.

Answer:



Method Comparing Fractions with Different Denominators

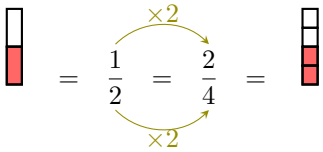
To compare two fractions with different denominators:

- Find a **common denominator**.
- Convert each fraction to an equivalent fraction with that denominator.
- Compare the numerators.

Ex: Compare $\frac{1}{2}$ and $\frac{3}{4}$.

Answer:

- Since $\frac{1}{2}$ and $\frac{3}{4}$ have different denominators, we change $\frac{1}{2}$ into an equivalent fraction with denominator 4:



$$\frac{1}{2} = \frac{2}{4}$$

- Compare the numerators:

$$\frac{2}{4} < \frac{3}{4}$$

- Therefore,

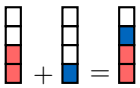
$$\frac{1}{2} < \frac{3}{4}$$

- In pictures:

F ADDITION AND SUBTRACTION WITH COMMON DENOMINATORS

Definition Addition of Fractions with Common Denominators

When we **add** fractions with common denominators, we keep the denominator the same and add the numerators:



$$\frac{2}{4} + \frac{1}{4} = \frac{3}{4}$$

Definition Subtraction of Fractions with Common Denominators

When we **subtract** fractions with common denominators, we keep the denominator the same and subtract the numerators:

$$\frac{3}{4} - \frac{1}{4} = \frac{2}{4}$$

G ADDITION AND SUBTRACTION WITH DIFFERENT DENOMINATORS

Method Addition or Subtraction of Fractions with Different Denominators

To add or subtract fractions with different denominators:

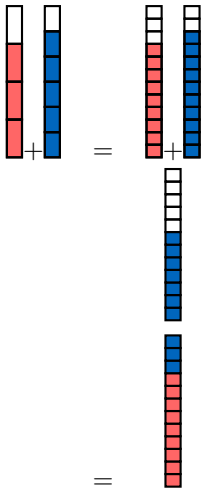
- **Find a common denominator**: Choose a common multiple of the denominators.
- **Convert each fraction**: Rewrite each fraction so it has the common denominator.
- **Add or subtract the numerators**: Add or subtract the numerators and keep the denominator the same.

Ex: Calculate $\frac{3}{4} + \frac{5}{6}$.

Answer:

- **Find a common denominator**:To add fractions, they must have the same denominator.
  - Multiples of 4: 4, 8, **12**, 16, 20, ...
  - Multiples of 6: 6, **12**, 18, 24, ...
  - The smallest common denominator is **12**.

- $\frac{3}{4}+\frac{5}{6}=\frac{3\times 3}{4\times 3}+\frac{5\times 2}{6\times 2}$
  $=\frac{9}{12}+\frac{10}{12}$ (common denominator = 12)
  $=\frac{9+10}{12}$ (adding numerators)
  $=\frac{19}{12}$
- **Visual representation**:



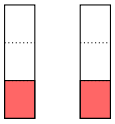
H FRACTION AS QUOTIENT

Proposition **Fraction as Quotient**

A fraction is a quotient that represents the result of **division**. It tells us how much of something we have when we divide it into equal parts.

- **The top number (numerator)** is the whole.
- **The bottom number (denominator)** is the number of equal parts the whole is divided into.

The fraction $\frac{2}{3}$ is the same as saying **"2 divided by 3"**.

$$2\div 3=$$  $$=\frac{2}{3}$$

The fraction $\frac{2}{3}$ is the number which, when multiplied by 3, gives 2:

$$\frac{2}{3}\times 3=2$$

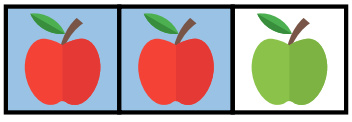
I FRACTION AS RATIO

Definition **Fractions as Ratios**

A fraction can represent the ratio of part to the whole:

$$\frac{\text{Part}}{\text{Whole}}$$

Ex: There are 3 apples in Hugo's basket. 2 of the apples are red.



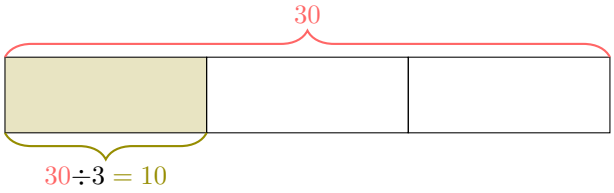
The fraction (ratio) of red apples is:

$$\frac{2}{3}$$

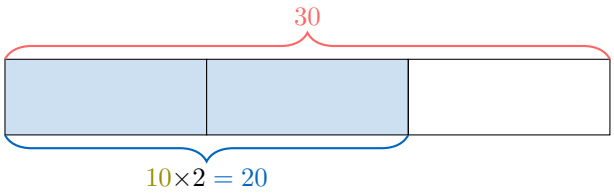
Method Finding a Quantity from a Fraction (Ratio)

To calculate $\frac{2}{3}$ of 30:

- **Method 1 (unitary method)**:
  - Divide the total by the denominator to find the amount for one part:



  - Multiply the result by the numerator to find the desired quantity:



- **Method 2** (calculation using a formula):

$$\begin{aligned}\frac{2}{3} \text{ of } 30 &= \frac{2}{3} \times 30 \\ &= (2 \div 3) \times 30 \\ &= 20\end{aligned}$$

J FRACTION AS DECIMAL NUMBER

Method Converting a Fraction to a Decimal

- **Division Method**: Perform the division of the numerator by the denominator.
- **Power of 10 Denominator Method**: Find an equivalent fraction where the denominator is a power of 10.

Ex: Convert $\frac{3}{4}$ to a decimal number.

Answer:

- **Division Method**:

$$\begin{aligned}\frac{3}{4} &= 3 \div 4 \\ &= 0.75\end{aligned}$$

$$\begin{array}{r} 0.75 \\ 4\overline{)3.00} \\ \underline{2.8} \\ 20 \\ \underline{20} \\ 0 \end{array}$$

- **Power of 10 Denominator Method**:

$$\begin{aligned}\frac{3}{4} &= \frac{3\times25}{4\times25} \\ &= \frac{75}{100} \\ &= 75 \div 100 \\ &= 0.75\end{aligned}$$

Method **Converting Decimal to Fraction**

- Multiply the decimal by a power of 10 (10, 100, 1000, ...) to eliminate the decimal point.
- Write the result over the same power of 10 to form a fraction.

Ex: Convert 1.3 to a fraction.

Answer:

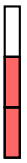
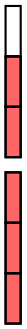
$$1.3 = \frac{1.3 \times 10}{10} = \frac{13}{10}$$

K PROPER AND IMPROPER FRACTIONS

Definition **Proper and improper fractions**

A fraction which has numerator less than its denominator is called a **proper fraction**.
A fraction which has numerator greater than its denominator is called an **improper fraction**.

Ex:

- $\frac{2}{3} =$  is a proper fraction.
- $\frac{5}{3} = 1 + \frac{2}{3} =$  is an improper fraction.

Definition **Mixed Number**

A **mixed number** is a representation of a number that combines a whole number and a proper fraction.
By standard convention, the addition symbol is implied and thus not explicitly written:

$1\frac{2}{3}$ is understood as $1 + \frac{2}{3} =$