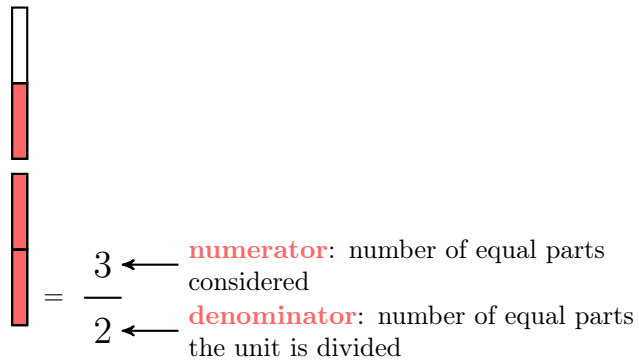


FRACTIONS

A DEFINITIONS

Definition Fraction

A **fraction** includes two numbers: the **numerator** and the **denominator**, separated by a bar.

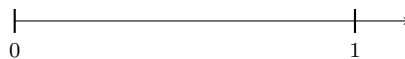


B ON THE NUMBER LINE

Method Representing a Fraction on the Number Line

To represent the fraction $\frac{2}{3}$ on a number line.

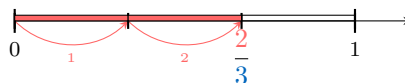
1. Draw a straight line and mark the points 0 and 1.



2. Divide the line between 0 and 1 into 3 equal parts.



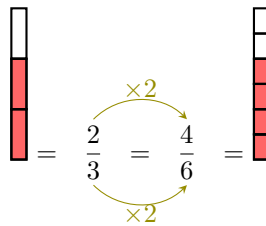
3. Count 2 parts from 0 and mark the point.



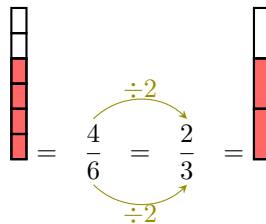
C EQUIVALENT FRACTIONS

Definition Equivalent Fractions

- When you multiply the numerator and the denominator by the same number, the fractions are equals.



- When you divide the numerator and the denominator by the same number, the fractions are equals.

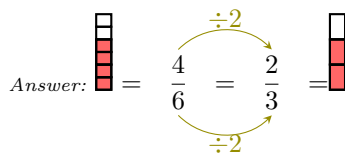


D SIMPLIFICATION

Method Simplifying a fraction

To simplify a fraction, we find an equivalent fraction with the smallest possible numerator and denominator.

Ex: Simplify $\frac{4}{6}$



E CROSS MULTIPLICATION

Proposition Cross Multiplication Property

$$\frac{a}{b} = \frac{c}{d} \text{ if and only if } a \times d = b \times c$$

Ex: Solve x for $\frac{10}{5} = \frac{x}{8}$.

Answer:

$$\begin{array}{l} \frac{10}{5} = \frac{x}{8} \\ 5 \times x = 10 \times 8 \quad (\text{cross multiplication}) \\ x = 10 \times 8 \div 5 \quad (\text{dividing both sides by 5}) \\ x = 16 \end{array}$$

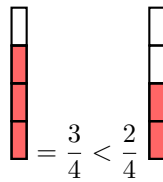
F ORDERING FRACTIONS

Definition Ordering Fractions with the Same Denominator

For two fractions with the same denominator, the fraction with the larger numerator is larger.

Ex: Compare $\frac{3}{4}$ and $\frac{2}{4}$.

Answer:



Method Comparing Fractions with Different Denominators

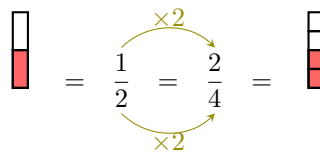
To compare two fractions with different denominators:

- Find a **common denominator**.
- Convert each fraction to an equivalent fraction with that denominator.
- Compare the numerators.

Ex: Compare $\frac{1}{2}$ and $\frac{3}{4}$.

Answer:

- Since $\frac{1}{2}$ and $\frac{3}{4}$ have different denominators, we change $\frac{1}{2}$ into an equivalent fraction with denominator 4:



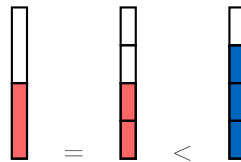
- Compare the numerators:

$$\frac{2}{4} < \frac{3}{4}$$

- Therefore,

$$\frac{1}{2} < \frac{3}{4}$$

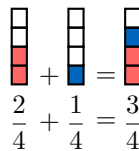
- In pictures:



G ADDITION AND SUBTRACTION WITH COMMON DENOMINATORS

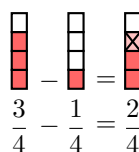
Definition Addition of Fractions with Common Denominators

When we **add** fractions with common denominators, we keep the denominator the same and add the numerators:



Definition Subtraction of Fractions with Common Denominators

When we **subtract** fractions with common denominators, we keep the denominator the same and subtract the numerators:



H ADDITION AND SUBTRACTION WITH DIFFERENT DENOMINATORS

Method Addition or Subtraction of Fractions with Different Denominators

To add or subtract fractions with different denominators:

- **Find a common denominator:** Choose a common multiple of the denominators.
- **Convert each fraction:** Rewrite each fraction so it has the common denominator.
- **Add or subtract the numerators:** Add or subtract the numerators and keep the denominator the same.

Ex: Calculate $\frac{3}{4} + \frac{5}{6}$.

Answer:

- **Find a common denominator:** To add fractions, they must have the same denominator.

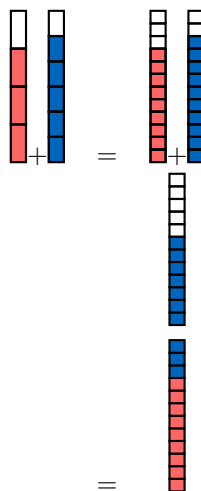
– Multiples of 4: 4, 8, **12**, 16, 20, ...

– Multiples of 6: 6, **12**, 18, 24, ...

– The smallest common denominator is **12**.

$$\begin{aligned}
 \bullet \quad \frac{3}{4} + \frac{5}{6} &= \frac{3 \times 3}{4 \times 3} + \frac{5 \times 2}{6 \times 2} \\
 &= \frac{9}{12} + \frac{10}{12} && \text{(common denominator = 12)} \\
 &= \frac{9+10}{12} && \text{(adding numerators)} \\
 &= \frac{19}{12}
 \end{aligned}$$

- **Visual representation:**



I FRACTION AS QUOTIENT

Proposition Fraction as Quotient

A fraction is a quotient that represents the result of **division**. It tells us how much of something we have when we divide it into equal parts.

- **The top number (numerator)** is the whole.
- **The bottom number (denominator)** is the number of equal parts the whole is divided into.

The fraction $\frac{2}{3}$ is the same as saying "**2 divided by 3**".

$$2 \div 3 = \frac{2}{3}$$

The fraction $\frac{2}{3}$ is the number which, when multiplied by 3, gives 2:

$$\frac{2}{3} \times 3 = 2$$

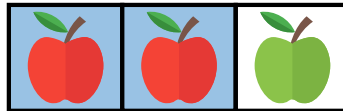
J FRACTION AS RATIO

Definition Fractions as Ratios

A fraction can represent the ratio of part to the whole:

$$\frac{\text{Part}}{\text{Whole}}$$

Ex: There are 3 apples in Hugo's basket. 2 of the apples are red.



The fraction (ratio) of red apples is:

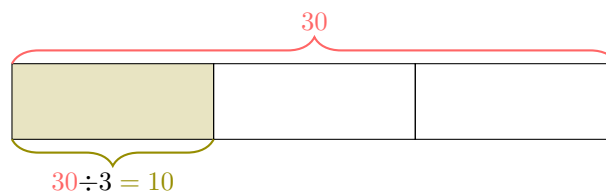
$$\frac{2}{3}$$

Method Finding a Quantity from a Fraction (Ratio)

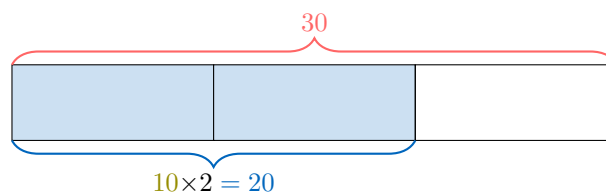
To calculate $\frac{2}{3}$ of 30:

- **Method 1 (unitary method):**

– Divide the total by the denominator to find the amount for one part:



– Multiply the result by the numerator to find the desired quantity:



- **Method 2** (calculation using a formula):

$$\begin{aligned} \frac{2}{3} \text{ of } 30 &= \frac{2}{3} \times 30 \\ &= (2 \div 3) \times 30 \\ &= 20 \end{aligned}$$

K FRACTION AS DECIMAL NUMBER

Method Converting a Fraction to a Decimal

- **Division Method:** Perform the division of the numerator by the denominator.
- **Power of 10 Denominator Method:** Find an equivalent fraction where the denominator is a power of 10.

Ex: Convert $\frac{3}{4}$ to a decimal number.

Answer:

- **Division Method:**

$$\frac{3}{4} = 3 \div 4 \\ = 0.75$$

$$\begin{array}{r} 0.75 \\ 4 \overline{) 3.00} \\ \underline{2.8} \\ 20 \\ \underline{20} \\ 0 \end{array}$$

- **Power of 10 Denominator Method:**

$$\frac{3}{4} = \frac{3 \times 25}{4 \times 25} \\ = \frac{75}{100} \\ = 75 \div 100 \\ = 0.75$$

Method Converting Decimal to Fraction

- Multiply the decimal by a power of 10 (10, 100, 1000, ...) to eliminate the decimal point.
- Write the result over the same power of 10 to form a fraction.

Ex: Convert 1.3 to a fraction.

Answer:

$$1.3 = \frac{1.3 \times 10}{10} \\ = \frac{13}{10}$$

L PROPER AND IMPROPER FRACTIONS

Definition Proper and improper fractions

A fraction which has numerator less than its denominator is called a **proper fraction**.

A fraction which has numerator greater than its denominator is called an **improper fraction**.

Ex:

- $\frac{2}{3} =$  is a proper fraction.

- $\frac{5}{3} = 1 + \frac{2}{3} =$  is an improper fraction.

Definition Mixed Number

A **mixed number** is a representation of a number that combines a whole number and a proper fraction. By standard convention, the addition symbol is implied and thus not explicitly written:

$$1\frac{2}{3} \text{ is understood as } 1 + \frac{2}{3} =$$

