FORMULAS

A UNDERSTANDING FORMULAS

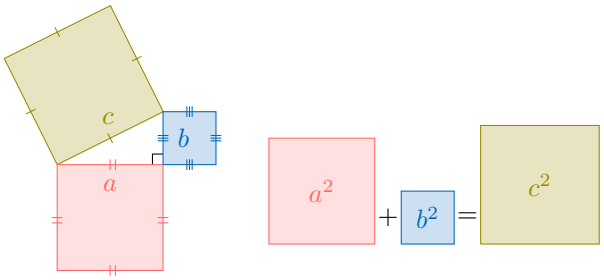
A **formula** is a special type of equation that describes a relationship between two or more variables. Formulas are powerful tools in mathematics and science because they provide a concise rule for modelling real-world phenomena, from calculating the area of a shape to predicting the distance an object travels.

Definition Formula

A **formula** is an equation that expresses one variable in terms of others. The single variable that is isolated on one side of the equation is called the **subject** of the formula.

Ex: 

In the formula for the area of a circle, $A = \pi r^2$, the subject is the area, A.

Ex: 

In the Pythagorean theorem, $c^2 = a^2 + b^2$, the subject is c^2.

B SUBSTITUTING INTO FORMULAS

Method Substituting Values

To find the value of a formula's subject, we can **substitute** known values for the other variables and evaluate the expression.

1. **Identify the formula** required for the problem.
2. **Substitute** the given values in place of their corresponding variables.
3. **Evaluate** the resulting numerical expression to find the value of the subject.

Ex: The formula for the area of a square is $A = s^2$. Find the area of a square with a side length of 4 cm.

Answer:

1. The formula is given: $A = s^2$.
2. Substitute the known value $s = 4$: $A = (4)^2$.
3. Evaluate the expression: $A = 16$.

The area of the square is 16 cm^2.

C REARRANGING FORMULAS

Method Changing the Subject

We can **rearrange** a formula to make a different variable the subject. This is also known as "solving for a variable". The process uses the same rules as solving equations: apply inverse operations to both sides of the formula to isolate the desired variable.

Ex: The formula for the area of a square is $A = s^2$. Rearrange the formula to make the side length, s, the subject.

Answer: To isolate s, we must undo the "squaring" operation by taking the square root of both sides.

$$A = s^2$$
$$\sqrt{A} = \sqrt{s^2}$$
$$\sqrt{A} = s$$

Since length must be positive, we only take the positive square root. The rearranged formula is $s = \sqrt{A}$.

D CONSTRUCTING FORMULAS

Method Constructing a Formula

To **construct a formula** from a description or pattern:

1. **Define variables:** Assign symbols (e.g., x, y, C) to represent the quantities involved.
2. **Identify the relationship:** Describe in words how the variables are connected. Look for a fixed starting amount (a constant) and a rate of change (a coefficient).
3. **Translate to algebra:** Write the relationship as an equation.

Ex: A taxi charges a \$3 initial fee, plus \$2 for every kilometre travelled. Construct a formula for the total cost, C, of a journey of d kilometres.

Answer:

1. Variables are defined: C for total cost and d for distance in kilometres.
2. The relationship is: The total cost is the fixed fee of \$3 plus \$2 multiplied by the number of kilometres.
3. The formula is:
$$C = 2d + 3$$

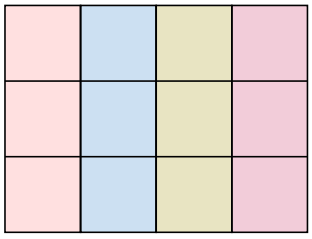
Method Constructing a Formula from a Pattern

One of the most powerful skills in mathematics is the ability to observe a pattern and generalize it into a formula.

1. **Analyze specific cases:** Start by collecting data from the first few examples in the pattern. Organize this data in a table.
2. **Identify the relationship:** Look for a rule that connects the case number (e.g., diagram number, n) to the result (e.g., number of matchsticks, M). Look for a starting value and a common difference.
3. **Generalize the rule:** Write down your observation as a formula using variables.
4. **Test your formula:** Check if your formula works for the cases you analyzed and see if it can predict the next case.

Ex: Derive the formula for the area of a rectangle.

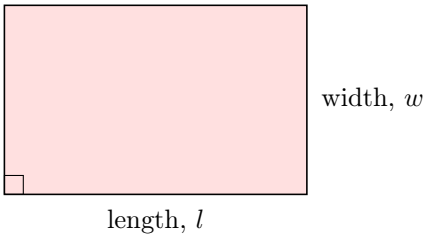
- **Specific Case**: Consider a rectangle with a width of 4 units and a height of 3 units. We can calculate its area by counting the unit squares inside, which can be seen as 4 columns of 3 squares each.



$$\text{Area} = \underbrace{3 + 3 + 3 + 3}_{4 \text{ times}} = 4 \times 3$$

  The area is the product of its width and height: $A = 4 \times 3 = 12$.

- **General Case**: We generalize this observation. For any rectangle with width w and length l, the area A is given by the product of its dimensions.



The formula is:

$$A = l \times w$$

Ex: Examine the pattern of squares made from matchsticks:

Diagram 1 Diagram 2 Diagram 3

Construct a formula for the number of matchsticks, M, in Diagram n.

Answer:

1. **Analyze specific cases**: We count the matchsticks in each diagram and create a table.

Diagram Number (n)	1	2	3
Number of Matchsticks (M)	4	7	10

2. **Identify the relationship**: We start with 4 matchsticks. To get to the next diagram, we add 3 more matchsticks each time. The common difference is 3. This indicates a linear relationship.
   - Diagram 1: 4
   - Diagram 2: $4 + 3$
   - Diagram 3: $4 + 3 + 3 = 4 + 2 \times 3$
3. **Generalize the rule**: For Diagram n, we start with 4 and add 3 a total of $(n - 1)$ times.

   $$M = 4 + 3(n - 1)$$

   By expanding and simplifying, we get:

   $$M = 4 + 3n - 3 \implies M = 3n + 1$$

4. **Test the formula**:
   - For $n = 1$: $M = 3(1) + 1 = 4$. Correct.
   - For $n = 2$: $M = 3(2) + 1 = 7$. Correct.
   - For $n = 3$: $M = 3(3) + 1 = 10$. Correct.

The formula for the number of matchsticks in Diagram n is $M = 3n + 1$.