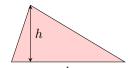
A UNDERSTANDING FORMULAS

A.1 FINDING THE SUBJECTS



MCQ 1: For a triangle

we have the

formula

$$A = \frac{b \times h}{2}.$$

Find the subject of the formula.

- \boxtimes A: the area
- \Box b: the base
- \square h: the height

Answer: The subject of the formula is the area A.

MCQ 2: In an electrical circuit, we have the formula

$$U = RI$$
.

Find the subject of the formula.

- $\boxtimes U$: the voltage
- \square R: the resistance
- \square I: the current

Answer: The subject of the formula is the voltage U.

MCQ 3: In physics, we have the formula

$$v = \frac{d}{t}$$
.

Find the subject of the formula.

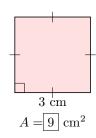
- $\boxtimes v$: the velocity
- \Box d: the distance
- \Box t: the time

Answer: The subject of the formula is the velocity v.

B SUBSTITUTING INTO FORMULAS

B.1 CALCULATING MEASURES IN GEOMETRY

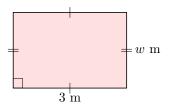
Ex 4: Find the area of the figure:



Answer:

$$A = s^2$$
 (identifying the formula)
= 3^2 (substituting $s = 3$)
= 9 cm^2 (solving)

Ex 5: The area of the rectangle is $6 \,\mathrm{m}^2$ and the length of one side is $3 \,\mathrm{m}$.



Find the width of the rectangle.

$$w = \boxed{2} \text{ m}$$

Answer:

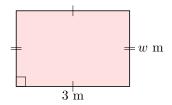
$$A = L \times w \quad \text{(identifying the formula)}$$

$$6 = 3 \times w \quad \text{(substituting } A = 6 \text{ and } L = 3\text{)}$$

$$w = \frac{6}{3} \quad \text{(dividing by 3)}$$

$$w = 2 \, \text{m}$$

Ex 6: The perimeter of the rectangle is 10 m.



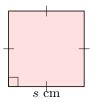
Find the width of the rectangle.

$$w = \boxed{2} \text{ m}$$

Answer:

$$P=2L+2w$$
 (identifying the formula)
 $10=2\times 3+2w$ (substituting $P=10$ and $L=3$)
 $10=6+2w$
 $2w=10-6$ (subtracting 6)
 $2w=4$
 $w=\frac{4}{2}=2\,\mathrm{m}$

Ex 7: The area of a square is $10 \,\mathrm{cm}^2$.

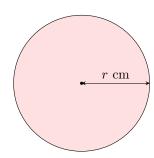


Find the length of the side of the square (round to 2 decimal places).

$$s = 3.16$$
 cm

$$A=s^2$$
 (identifying the formula)
$$10=s^2$$
 (substituting $A=10$)
$$s=\sqrt{10}$$
 (taking the square root)
$$s\approx 3.16\,\mathrm{cm}$$
 (rounding to 2 decimal places)

Ex 8: The area of a circle is $10 \,\mathrm{cm}^2$. Recall the formula: $A = \pi r^2$.



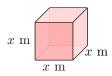
Find the radius of the circle (round your answer to 2 decimal places).

$$r = \boxed{1.78} \text{ cm}$$

Answer:

$$A = \pi r^2$$
 (recall the formula)
 $10 = \pi r^2$ (substitute $A = 10$)
 $r^2 = \frac{10}{\pi}$ (divide both sides by π)
 $r = \sqrt{\frac{10}{\pi}}$ (take the square root)
 $r \approx 1.78 \, \mathrm{cm}$ (rounded to 2 decimal places)

Ex 9: The volume of the cube is $10 \,\mathrm{m}^3$.



Find the length of the side of the cube (round to 2 decimal places and $\sqrt[3]{10} = 2.1544...$).

$$x = 2.15 \text{ m}$$

Answer:

$$V=x^3$$
 (identifying the formula)
 $10=x^3$ (substituting $V=10$)
 $x=\sqrt[3]{10}$ (taking the cube root of both sides)
 $x\approx 2.15\,\mathrm{m}$ (rounding to 2 decimal places)

B.2 CALCULATING MEASURES IN PHYSICS

Ex 10: A car travels at a constant speed of 120 km/h for 2 hours.

Find the distance traveled by the car. Recall the formula: $v = \frac{d}{t}$.

$$d = 240 \text{ km}$$

Answer:

$$v = \frac{d}{t}$$
 (identifying the formula)
$$20 = \frac{d}{2}$$
 (substituting $v = 120$ and $t = 2$)
$$d = 120 \times 2$$
 (multiplying both sides by 2)
$$d = 240 \text{ km}$$

Ex 11: A circuit has a resistance of 5 Ω and a voltage of 20 V

Find the current flowing through the circuit. Recall the formula: U=RI.

$$I = \boxed{4}$$
 A

Answer:

$$U=RI$$
 (identifying the formula)
 $20=5I$ (substituting $U=20$ and $R=5$)
 $I=\frac{20}{5}$ (dividing both sides by 5)
 $I=4$ A

Ex 12: The formula $F = \frac{9}{5}C + 32$ converts a temperature from Celsius (C) to Fahrenheit (F).

Given a temperature of 68°F, find the temperature in Celsius.

$$C = 20$$
°C

Answer:

$$F = \frac{9}{5}C + 32 \quad \text{(identifying the formula)}$$

$$68 = \frac{9}{5}C + 32 \quad \text{(substituting } F = 68\text{)}$$

$$68 - 32 = \frac{9}{5}C \quad \text{(subtracting 32 from both sides)}$$

$$36 = \frac{9}{5}C$$

$$C = 36 \times \frac{5}{9} \quad \text{(multiplying both sides by } \frac{5}{9}\text{)}$$

Ex 13: The formula $E_p = mgh$ calculates the gravitational potential energy (E_p) of an object, where m is the mass in kilograms, g is the acceleration due to gravity (9.8 m/s²), and h is the height in meters.

Given an object with a mass of 10 kg and a gravitational potential energy of 490 J, find the height at which the object is located.

$$h = \boxed{5}$$
 m

$$\begin{split} E_p &= mgh & \text{(identifying the formula)} \\ 490 &= 10 \times 9.8 \times h & \text{(substituting } E_p = 490, \ m = 10, \ g = 9.8) \\ 490 &= 98h & \text{(calculating } 10 \times 9.8) \\ h &= \frac{490}{98} & \text{(dividing both sides by } 98) \\ h &= 5 \text{ m} \end{split}$$

C REARRANGING FORMULAS

C.1 REARRANGING LINEAR EQUATIONS

Ex 14: Rearrange the equation 3x + 4y = 13 to make y the subject.

$$y = \boxed{\frac{13 - 3x}{4}}$$

Answer:

$$3x + 4y = 13$$

 $4y = 13 - 3x$ (subtracting $3x$ from both sides)
 $y = \frac{13 - 3x}{4}$ (dividing both sides by 4)

Ex 15: Rearrange the equation 5x - 2y = 10 to make y the subject.

$$y = \boxed{\frac{5x - 10}{2}}$$

Answer:

$$5x - 2y = 10$$

$$-2y = 10 - 5x (subtracting 5x from both sides)$$

$$-2y = -5x + 10 (reordering)$$

$$y = \frac{-5x + 10}{-2} (dividing both sides by -2)$$

$$y = \frac{5x - 10}{2} (simplifying)$$

Ex 16: Rearrange the equation 3y + 2x = -x + 3 to make y the subject.

$$y = \boxed{1 - x}$$

Answer:

$$3y + 2x = -x + 3$$

$$3y = -x + 3 - 2x$$
 (subtracting 2x from both sides)
$$3y = 3 - 3x$$

$$y = \frac{3 - 3x}{3}$$
 (dividing both sides by 3)
$$y = 1 - x$$
 (simplifying)

Ex 17: Rearrange the equation 7y - 5x = 2x + 14 to make y the subject.

$$y = \boxed{2+x}$$

Answer:

$$7y - 5x = 2x + 14$$

$$7y = 2x + 14 + 5x \quad \text{(adding } 5x \text{ to both sides)}$$

$$7y = 7x + 14$$

$$y = \frac{7x + 14}{7} \quad \text{(dividing both sides by 7)}$$

$$y = x + 2 \quad \text{(simplifying)}$$

C.2 REARRANGING GEOMETRIC FORMULAE

MCQ 18: The formula for the circumference (perimeter) of a circle is $C=2\pi r$.

Rearrange the formula to make r the subject.

Choose the correct answer:

$$\Box r = 2\pi C$$

$$\Box 2\pi r = C$$

$$\boxtimes r = \frac{C}{2\pi}$$

Answer:

$$C=2\pi r$$
 (given formula)
$$r=\frac{C}{2\pi}$$
 (dividing both sides by 2π)

MCQ 19: The formula for the volume of a cube is $V = s^3$. Rearrange the formula to make s the subject.

Choose the correct answer:

$$\square \ s = V^3$$

$$\square s = \frac{V}{3}$$

$$\boxtimes s = \sqrt[3]{V}$$

Answer:

$$V=s^3$$
 (given formula)
$$s=\sqrt[3]{V}$$
 (taking the cube root of both sides)

MCQ 20: The formula for the area of a triangle is $A = \frac{1}{2}bh$. Rearrange the formula to make h the subject.

Choose the correct answer:

$$\Box h = \frac{b}{2A}$$

$$\Box \ h = \frac{A}{2b}$$

$$\boxtimes h = \frac{2A}{b}$$

Answer:

$$A = \frac{1}{2}bh$$
 (given formula)
 $2A = bh$ (multiplying both sides by 2)
 $h = \frac{2A}{b}$ (dividing both sides by b)

C.3 REARRANGING PHYSICS FORMULAE

MCQ 21: The formula $F = \frac{9}{5}C + 32$ converts a temperature from Celsius (C) to Fahrenheit (F).

Rearrange the formula to make C the subject.

Choose the correct answer:

$$\boxtimes C = \frac{5}{9}(F - 32)$$

$$\Box C = \frac{9}{5}(F - 32)$$

$$\Box \ C = \frac{5}{9}F + 32$$

Answer:

$$F = \frac{9}{5}C + 32 \qquad \text{(given formula)}$$

$$F - 32 = \frac{9}{5}C \qquad \text{(subtracting 32 from both sides)}$$

$$C = \frac{5}{9}(F - 32) \quad \text{(multiplying both sides by } \frac{5}{9}\text{)}$$

MCQ 22: The formula $v = \frac{d}{t}$ relates speed (v), distance (d), and time (t).

Rearrange the formula to make t the subject.

Choose the correct answer:

$$\boxtimes t = \frac{d}{v}$$

$$\Box t = \frac{v}{d}$$

$$\Box t = dv$$

Answer:

$$v = \frac{d}{t}$$
 (given formula)
$$vt = d$$
 (multiplying both sides by t)
$$t = \frac{d}{v}$$
 (dividing both sides by v)

MCQ 23: The formula U = RI relates voltage (U), resistance (R), and current (I).

Rearrange the formula to make I the subject.

Choose the correct answer:

$$\boxtimes I = \frac{U}{R}$$

$$\square I = UR$$

$$\square I = \frac{R}{U}$$

Answer:

$$U = RI \quad \text{(given formula)}$$

$$I = \frac{U}{R} \quad \text{(dividing both sides by } R\text{)}$$

MCQ 24: The formula $E = \frac{1}{2}mv^2$ relates kinetic energy (E), mass (m), and speed (v).

Rearrange the formula to make v the subject.

Choose the correct answer:

$$\boxtimes v = \sqrt{\frac{2E}{m}}$$

$$\square \ v = \frac{E}{m}$$

$$\square \ v = \frac{2E}{m}$$

Answer:

$$E = \frac{1}{2}mv^2$$
 (given formula)

$$2E = mv^2$$
 (multiplying both sides by 2)

$$v^2 = \frac{2E}{m}$$
 (dividing both sides by m)

$$v = \sqrt{\frac{2E}{m}}$$
 (taking the square root of both sides)

C.4 REARRANGING RATIO EQUATIONS

Ex 25: Rearrange the equation $\frac{2}{y} = \frac{x}{6}$ to make y the subject:

$$y = \boxed{\frac{12}{x}}$$

Answer:

$$\frac{2}{y}$$
 $\frac{x}{6}$

 $y \times x = 2 \times 6$ (cross multiplication)

$$y = \frac{12}{x}$$
 (dividing both sides by x)

Ex 26: Rearrange the equation $\frac{1}{x} = \frac{2}{y}$ to make y the subject:

$$y = 2x$$

Answer:



 $1 \times y = x \times 2$ (cross multiplication)

$$y = 2x$$

Ex 27: Rearrange the equation $\frac{x}{2} = \frac{4}{y}$ to make y the subject:

$$y = \boxed{\frac{8}{x}}$$

Answer:

$$\frac{x}{2}$$
 $\frac{4}{y}$

 $x \times y = 2 \times 4$ (cross multiplication)

$$y = \frac{8}{x}$$
 (dividing by x)

Ex 28: Rearrange the equation $\frac{y}{x} = \frac{4}{3}$ to make y the subject:

$$y = \boxed{\frac{4x}{3}}$$

Answer:



 $y \times 3 = x \times 4$ (cross multiplication)

$$y = \frac{4x}{3}$$

D CONSTRUCTING FORMULAS

D.1 MODELING LINEAR RELATIONSHIPS WITH ALGEBRA

Ex 29: A mechanic charges a \$40 call-out fee and \$30 per hour thereafter

Find the mechanic's fee M for a job which takes x hours.

$$M = 40 + 30x$$

Answer: The total fee is \$40 plus \$30 for each of x hours.

- For x = 1, $M = 40 + 30 \times 1$
- For x = 2, $M = 40 + 30 \times 2$
- •
- For x, $M = 40 + 30 \times x$

Ex 30: A car rental company includes 50 km in the rental and adds/allows 15 km for each extra hour.

Find the total distance D the car can travel in terms of the rental time x (in hours).

$$D = 50 + 15x$$

Answer: The total distance is 50 km plus 15 km for each extra hour.

- For x = 1, $D = 50 + 15 \times 1$
- For x = 2, $D = 50 + 15 \times 2$
- •
- For x, $D = 50 + 15 \times x$

Ex 31: A gym membership includes a one-time joining fee of \$25 and a monthly fee of \$40.

Find the total cost C after x months.

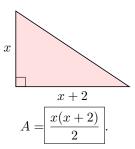
$$C = 25 + 40x$$

 ${}_{Answer:}$ The total cost is the joining fee of \$25 plus \$40 for each of x months.

- For x = 1, $C = 25 + 40 \times 1$
- For x = 2, $C = 25 + 40 \times 2$
- :
- For x, $C = 25 + 40 \times x$

D.2 MODELING AREAS AND VOLUMES WITH ALGEBRA: LEVEL 1

Ex 32: A right-angled triangle has a base length of x + 2 units and a height of x units. Find the area A of the triangle.



Answer: The area of a triangle is half the product of its base and height.

$$A = \frac{\text{base} \times \text{height}}{2}$$
$$= \frac{x(x+2)}{2}$$

Ex 33: A garden is in the shape of a semi-circle with diameter x meters.



Find the area A of the garden in terms of the diameter x of the semi-circle.

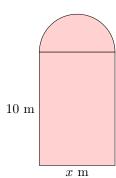
$$A = \boxed{\frac{\pi x^2}{8}} \quad \text{m}^2.$$

Answer:

- The radius r is half of the diameter: $r = \frac{x}{2}$.
- The area of a circle is πr^2 , where r is the radius.

Area of semi-circle
$$=\frac{1}{2}$$
Area of circle $=\frac{1}{2}\pi r^2$ $=\frac{1}{2}\pi\left(\frac{x}{2}\right)^2$ $=\frac{1}{2}\pi\frac{x^2}{4}$ $=\frac{\pi x^2}{9}$

Ex 34: The door is composed of a rectangle with height 10 meters and width x meters, topped with a semi-circle of diameter x meters.



Find the area A of the door in terms of x.

$$A = \boxed{10x + \frac{\pi x^2}{8}} \quad \text{m}^2.$$

Answer:

- The area of the rectangle is width times height: $x \times 10 = 10x$.
- The radius r of the semi-circle is half of the diameter: $r = \frac{x}{2}$.
- The area of a full circle is πr^2 , so the area of the semi-circle is half of that.

Area of semi-circle
$$= \frac{1}{2}\pi r^2$$

$$= \frac{1}{2}\pi \left(\frac{x}{2}\right)^2$$

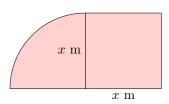
$$= \frac{1}{2}\pi \frac{x^2}{4}$$

$$= \frac{\pi x^2}{8}$$

• The total area of the door is the sum of the rectangle and semi-circle areas:

$$A = 10x + \frac{\pi x^2}{8}$$

Ex 35: The door consists of a square with side length x meters and a quarter-circle with radius x meters.



Find the area A of the door in terms of x.

$$A = \boxed{x^2 + \frac{\pi x^2}{4}} \quad \mathbf{m}^2.$$

Answer:

- The area of the square is side times side: $x \times x = x^2$.
- The radius r of the quarter-circle is r = x.
- The area of a full circle is πr^2 , so the area of the quarter-circle is one-fourth of that.

Area of quarter-circle =
$$\frac{1}{4}\pi r^2$$

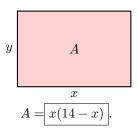
= $\frac{1}{4}\pi x^2$

• The total area of the door is the sum of the square and quarter-circle areas:

$$A = x^2 + \frac{\pi x^2}{4}$$

D.3 MODELING AREAS AND VOLUMES WITH ALGEBRA: LEVEL 2

Ex 36: You have 28 meters of fencing to enclose a rectangular vegetable garden. Let x be the length of the rectangle and y be the width. Find the area A of the garden in terms of x.



Answer:

• The perimeter of the rectangle is 28 m, so:

$$2x + 2y = 28$$

• Solve for y in terms of x:

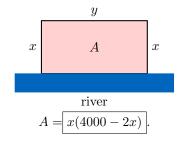
$$2x + 2y = 28$$
$$2y = 28 - 2x$$
$$y = 14 - x$$

• The area of the rectangle is $A = x \times y$. Substituting y = 14 - x.

$$A = x(14 - x)$$

Ex 37: A farmer has 4000 meters of fencing to enclose a rectangular field along a river. Because one side is along the river, fencing is required on only three sides.

Let x be the length perpendicular to the river and y the length parallel to the river. Find the area A of the field in terms of x.



Answer:

• Since the fence is on three sides only, the total fencing is:

$$x + x + y = 4000$$
 or $2x + y = 4000$

• Solve for y in terms of x:

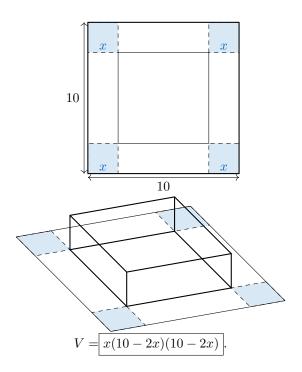
$$2x + y = 4000$$
$$y = 4000 - 2x$$

• The area of the rectangle is $A = x \times y$. Substituting y = 4000 - 2x:

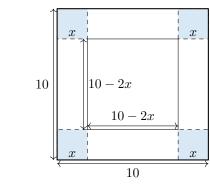
$$A = x(4000 - 2x)$$

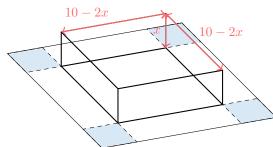
Ex 38: A sheet of paper $10 \text{ cm} \times 10 \text{ cm}$ is made into an open box by cutting x-cm squares out of each corner and folding up the sides.

Find the volume V of the box in terms of x.



• After cutting out x-cm squares, the height of the box is x. The new length is 10 - 2x and the new width is 10 - 2x.





• The volume of a box is

 $V = \text{height} \times \text{length} \times \text{width}.$

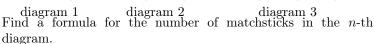
• Substituting the dimensions:

$$V = x(10 - 2x)(10 - 2x).$$

D.4 FINDING PATTERNS AND WRITING FORMULAE

Ex 39: Look at the following matchstick pattern:





Number of matchsticks = 3n + 1

Answer: Counting the matchsticks in each diagram:

- Diagram 1: $4 = 3 \times 1 + 1$
- Diagram 2: $7 = 3 \times 2 + 1$
- Diagram 3: $10 = 3 \times 3 + 1$
- :
- Diagram $n: 3 \times n + 1$

Ex 40: Look at the following triangular matchstick pattern:







diagram 1 diagram 2 diagram 3 Find a formula for the number of matchsticks in the n-th diagram.

Number of matchsticks = 2n+1

Answer: Counting the matchsticks in each diagram:

- Diagram 1: $3 = 2 \times 1 + 1$
- Diagram 2: $5 = 2 \times 2 + 1$
- Diagram 3: $7 = 2 \times 3 + 1$
- •
- Diagram $n: 2 \times n + 1$

Ex 41: Find the *n*-th term of the sequence $5, 10, 15, 20, 25, \ldots$

$$n$$
-th term = $5n$.

Answer:

- 1^{st} term: $5 = 5 \times 1$
- 2^{nd} term: $10 = 5 \times 2$
- 3^{rd} term: $15 = 5 \times 3$
- •
- n^{th} term: $5 \times n$

Ex 42: Find the n-th term of the sequence 6, 12, 18, 24, 30, 36, ...

$$n$$
-th term = $6n$

Answer:

- 1^{st} term: $6 = 6 \times 1$
- 2^{nd} term: $12 = 6 \times 2$
- $3^{\rm rd}$ term: $18 = 6 \times 3$
- . :
- n^{th} term: $6 \times n$

Ex 43: Find the *n*-th term of the sequence $1, 3, 5, 7, 9, 11, \ldots$

$$n$$
-th term = $2n-1$.

• 1^{st} term: $1 = 2 \times 1 - 1$

• 2^{nd} term: $3 = 2 \times 2 - 1$

• 3^{rd} term: $5 = 2 \times 3 - 1$

• :

• n^{th} term: $2 \times n - 1$

Ex 44: Find the *n*-th term of the sequence $2, 4, 8, 16, 32, 64, \ldots$

$$n$$
-th term = 2^n .

Answer:

• 1^{st} term: $2 = 2^1$

• 2^{nd} term: $4 = 2^2$

• 3^{rd} term: $8 = 2^3$

• :

• n^{th} term: 2^n

Ex 45: Find the *n*-th term of the sequence $1, 4, 9, 16, 25, 36, \ldots$

$$n$$
-th term = n^2 .

Answer:

• 1^{st} term: $1 = 1^2$

• 2^{nd} term: $4 = 2^2$

• 3^{rd} term: $9 = 3^2$

• :

• n^{th} term: n^2

Ex 46: Find the *n*-th term of the sequence $\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \dots$

$$n$$
-th term = $\left\lceil \frac{1}{2n} \right\rceil$.

Answer:

• 1st term: $\frac{1}{2} = \frac{1}{2 \times 1}$

• 2^{nd} term: $\frac{1}{4} = \frac{1}{2 \times 2}$

• 3^{rd} term: $\frac{1}{6} = \frac{1}{2 \times 3}$

• n^{th} term: $\frac{1}{2n}$