FACTORIZATION OF ALGEBRAIC EXPRESSIONS

Factorization is the reverse process of expansion. While expanding converts a product into a sum, factorization converts a sum into a product.
Mastering factorization is a crucial skill for simplifying complex expressions and solving higher-degree equations. Unless stated otherwise, we work with real numbers in this chapter.

A COMMON FACTOR LAWS

The **common factor** laws are the reverse of the distributive laws: instead of expanding brackets, we look for a common factor to "pull out" and introduce brackets.

Proposition Common Factor Laws

$$ab + ac = a(b + c) \quad \text{and} \quad ab - ac = a(b - c)$$

Ex: Factorize $2x + 2$.

Answer:

$$\begin{aligned} 2x + 2 &= 2 \times x + 2 \times 1 \\ &= 2(x + 1). \end{aligned}$$

B DIFFERENCE OF SQUARES

Proposition Difference of Squares

$$a^2 - b^2 = (a - b)(a + b)$$

Ex: Factorize $x^2 - 9$.

Answer:

$$\begin{aligned} x^2 - 9 &= x^2 - 3^2 \\ &= (x - 3)(x + 3). \end{aligned}$$

Note Any non-negative real number c can be written as a perfect square: $c = (\sqrt{c})^2$. This allows us to rewrite expressions such as $x^2 - c$ as a difference of squares and apply the formula above.

Ex: Factorize $x^2 - 3$.

Answer: We can write 3 as $(\sqrt{3})^2$ to create a difference of squares.

$$\begin{aligned} x^2 - 3 &= x^2 - \left(\sqrt{3}\right)^2 \\ &= \left(x - \sqrt{3}\right)\left(x + \sqrt{3}\right). \end{aligned}$$

Note A sum of squares $a^2 + b^2$ cannot be factored over the real numbers.

Ex: Factorize if possible: $x^2 + 1$.

Answer: Since $x^2 + 1 = x^2 + 1^2$ is a sum of squares, it cannot be factored into linear factors with real coefficients.

C PERFECT SQUARE TRINOMIALS

Proposition Perfect Square Trinomials

$$\begin{aligned} a^2 + 2ab + b^2 &= (a + b)^2 \\ a^2 - 2ab + b^2 &= (a - b)^2 \end{aligned}$$

Ex: Factorize $x^2 + 2x + 1$.

Answer: We check if this fits the form $a^2 + 2ab + b^2$.

- The first term is x^2, so let $a = x$.
- The last term is $1 = 1^2$, so let $b = 1$.
- We check if the middle term is $2ab$: $2(x)(1) = 2x$. It matches.

Therefore, the expression is a perfect square:

$$\begin{aligned} x^2 + 2x + 1 &= x^2 + 2(x)(1) + 1^2 \\ &= (x + 1)^2. \end{aligned}$$