EXPONENTS

A POSITIVE EXPONENTS

A.1 WRITING REPEATED MULTIPLICATION IN EXPONENT FORM

Ex 1: Write in exponent form:

$$2 \times 2 \times 2 = \boxed{2^3}$$

Answer: $\overbrace{2 \times 2 \times 2}^{3 \text{ factors}} = 2^3$

Ex 2: Write in exponent form:

$$3 \times 3 \times 3 \times 3 = \boxed{3^4}$$

Answer: $\overbrace{3 \times 3 \times 3 \times 3}^{4 \text{ factors}} = 3^4$

Ex 3: Write in exponent form:

$$5 \times 5 = \boxed{5^2}$$

Answer: $\overbrace{5 \times 5}^{2 \text{ factors}} = 5^2$

Ex 4: Write in exponent form:

$$7 \times 7 \times 7 = \boxed{7^3}$$

Answer: $\overbrace{7 \times 7 \times 7}^{3 \text{ factors}} = 7^3$

Ex 5: Write in exponent form:

$$10 \times 10 \times 10 \times 10 \times 10 = \boxed{10^5}$$

Answer: $\overbrace{10 \times 10 \times 10 \times 10 \times 10}^{5 \text{ factors}} = 10^5$

A.2 WRITING IN EXPONENT FORM FROM VERBAL EXPRESSIONS

Ex 6: Write in exponent form:

$$2 \text{ raised to the power of } 3 = \boxed{2^3}$$

Answer: 2 raised to the power of 3 $= 2^3$

Ex 7: Write in exponent form:

$$5 \text{ raised to the power of } 2 = \boxed{5^2}$$

Answer: 5 raised to the power of 2 $= 5^2$

Ex 8: Write in exponent form:

$$7 \text{ raised to the power of } 4 = \boxed{7^4}$$

Answer: 7 raised to the power of 4 $= 7^4$

Ex 9: Write in exponent form:

$$10 \text{ raised to the power of } 5 = \boxed{10^5}$$

Answer: 10 raised to the power of 5 $= 10^5$

A.3 CALCULATING POWERS

Ex 10: Evaluate the power:

$$2^3 = \boxed{8}$$

Answer:

$$\begin{aligned} 2^3 &= 2 \times 2 \times 2 \\ &= 8 \end{aligned}$$

Ex 11: Evaluate the power:

$$5^2 = \boxed{25}$$

Answer:

$$\begin{aligned} 5^2 &= 5 \times 5 \\ &= 25 \end{aligned}$$

Ex 12: Evaluate the power:

$$3^4 = \boxed{81}$$

Answer:

$$\begin{aligned} 3^4 &= 3 \times 3 \times 3 \times 3 \\ &= 81 \end{aligned}$$

Ex 13: Evaluate the power:

$$10^3 = \boxed{1000}$$

Answer:

$$\begin{aligned} 10^3 &= 10 \times 10 \times 10 \\ &= 1000 \end{aligned}$$

A.4 EXPRESSING NUMBERS IN EXPONENT FORM

Ex 14: Write in exponent form:

$$8 = \boxed{2^3}$$

Answer:

$$\begin{aligned} 8 &= 2 \times 2 \times 2 \\ &= 2^3 \end{aligned}$$

Ex 15: Write in exponent form:

$$27 = \boxed{3^3}$$

Answer:

$$\begin{aligned} 27 &= 3 \times 3 \times 3 \\ &= 3^3 \end{aligned}$$

Ex 16: Write in exponent form:

$$16 = \boxed{2^4}$$

Answer:

$$\begin{aligned} 16 &= 2 \times 2 \times 2 \times 2 \\ &= 2^4 \end{aligned}$$

Ex 17: Write in exponent form:

$$100 = \boxed{10^2}$$

Answer:

$$\begin{aligned} 100 &= 10 \times 10 \\ &= 10^2 \end{aligned}$$

A.5 INTERPRETING POWERS

MCQ 18: Determine if the following statement is True or False:

$$2^3 = 2 + 2 + 2$$

☐ True

☒ False

Answer:

- The expression 2^3 represents $2 \times 2 \times 2$, not $2 + 2 + 2$.
- Therefore, the statement $2^3 = 2 + 2 + 2$ is **False**.

MCQ 19: Determine if the following statement is True or False:

$$3^2 = 2 \times 2 \times 2$$

☐ True

☒ False

Answer:

- The expression 3^2 represents 3×3, not $2 \times 2 \times 2$.
- Therefore, the statement $3^2 = 2 \times 2 \times 2$ is **False**.

MCQ 20: Determine if the following statement is True or False:

$$4^3 = 4 \times 4 \times 4$$

☒ True

☐ False

Answer:

- The expression 4^3 represents $4 \times 4 \times 4$.
- Therefore, the statement $4^3 = 4 \times 4 \times 4$ is **True**.

MCQ 21: Determine if the following statement is True or False:

$$3 \times 4 = 4 + 4 + 4$$

☒ True

☐ False

Answer:

- The expression 3×4 represents 3 groups of 4, which is $4 + 4 + 4$.
- Therefore, the statement $3 \times 4 = 4 + 4 + 4$ is **True**.

A.6 EVALUATING EXPRESSIONS WITH POWERS

Ex 22: Evaluate the expression:

$$2^3 \times 3^2 = \boxed{72}$$

Answer:

$$\begin{aligned} 2^3 \times 3^2 &= (2 \times 2 \times 2) \times (3 \times 3) \\ &= 8 \times 9 \\ &= 72 \end{aligned}$$

Ex 23: Evaluate the expression:

$$3^2 \times 10^2 = \boxed{900}$$

Answer:

$$\begin{aligned} 3^2 \times 10^2 &= (3 \times 3) \times (10 \times 10) \\ &= 9 \times 100 \\ &= 900 \end{aligned}$$

Ex 24: Evaluate the expression:

$$6 \times 10^3 = \boxed{6000}$$

Answer:

$$\begin{aligned} 6 \times 10^3 &= 6 \times (10 \times 10 \times 10) \\ &= 6 \times 1000 \\ &= 6\,000 \end{aligned}$$

Ex 25: Evaluate the expression:

$$2.5 \times 10^2 = \boxed{250}$$

Answer:

$$\begin{aligned} 2.5 \times 10^2 &= 2.5 \times (10 \times 10) \\ &= 2.5 \times 100 \\ &= 250 \end{aligned}$$

A.7 CHECKING EQUALITY BETWEEN PRODUCTS AND POWERS

MCQ 26: Determine if the following statement is True or False:

$$2 \times 2 \times 3 \times 3 = 2^4$$

☐ True

☒ False

Answer:

- The expression $2 \times 2 \times 3 \times 3$ is equal to $2^2 \times 3^2 = 4 \times 9 = 36$.
- The expression $2^4 = 2 \times 2 \times 2 \times 2 = 16$.
- Therefore, the statement $2 \times 2 \times 3 \times 3 = 2^4$ is **False**.

MCQ 27: Determine if the following statement is True or False:

$$2 \times 2 \times 2 = 3^2$$

☐ True

☒ False

Answer:

- $2 \times 2 \times 2 = 8$
- $3^2 = 3 \times 3 = 9$
- Therefore, the statement $2 \times 2 \times 2 = 3^2$ is **False**.

MCQ 28: Determine if the following statement is True or False:

$$2 \times 3 \times 2 \times 3 = 2^2 \times 3^2$$

☒ True

☐ False

Answer:

$$\begin{aligned} 2 \times 3 \times 2 \times 3 &= (2 \times 2) \times (3 \times 3) \\ &= 2^2 \times 3^2 \end{aligned}$$

Therefore, the statement $2 \times 3 \times 2 \times 3 = 2^2 \times 3^2$ is **True**.

MCQ 29: Determine if the following statement is True or False:

$$5 \times 5 \times 5 \times 4 = 5^3 \times 2^2$$

☒ True

☐ False

Answer:

$$\begin{aligned} 5 \times 5 \times 5 \times 4 &= (5 \times 5 \times 5) \times 4 \\ &= 5^3 \times 4 \\ &= 5^3 \times (2 \times 2) \\ &= 5^3 \times 2^2 \end{aligned}$$

Therefore, the statement $5 \times 5 \times 5 \times 4 = 5^3 \times 2^2$ is **True**.

A.8 WRITING REPEATED MULTIPLICATION OF AN ALGEBRAIC EXPRESSION IN EXPONENT FORM

Ex 30: Write in exponent form:

$$x \times x \times x = \boxed{x^3}$$

Answer: $\overbrace{x \times x \times x}^{3 \text{ factors}} = x^3$

Ex 31: Write in exponent form:

$$x \times x = \boxed{x^2}$$

Answer: $\overbrace{x \times x}^{2 \text{ factors}} = x^2$

MCQ 32: Which expressions are equal to x?
Choose all answers that apply:

☐ x^2

☒ x^1

☐ 1

Answer:

- x^2 means $x \times x$, which is not equal to x (unless $x = 1$ or $x = 0$).
- $x^1 = x$: **(bonne réponse)**
- 1 is only equal to x if $x = 1$, but in general, $x \neq 1$.

Ex 33: Write in exponent form:

$$x \times x \times x \times x = \boxed{x^4}$$

Answer: $\overbrace{x \times x \times x \times x}^{4 \text{ factors}} = x^4$

A.9 WRITING ALGEBRAIC EXPRESSIONS IN EXPONENT FORM FROM VERBAL DESCRIPTIONS

Ex 34: Write in exponent form:

$$x \text{ squared} = \boxed{x^2}$$

Answer: x squared $= x^2$

Ex 35: Write in exponent form:

$$x \text{ to the power of } 4 = \boxed{x^4}$$

Answer: x to the power of $4 = x^4$

Ex 36: Write in exponent form:

$$x \text{ cubed} = \boxed{x^3}$$

Answer: x cubed $= x^3$

Ex 37: Write in exponent form:

$$x \text{ to the power of } 5 = \boxed{x^5}$$

Answer: x to the power of $5 = x^5$

B NEGATIVE EXPONENTS

B.1 WRITING NEGATIVE EXPONENTS AS FRACTIONS

Ex 38: Write as a fraction:

$$3^{-2} = \boxed{\frac{1}{9}}$$

Answer:

$$\begin{aligned} 3^{-2} &= \frac{1}{3 \times 3} \\ &= \frac{1}{9} \end{aligned}$$

Ex 39: Write as a fraction:

$$10^{-3} = \boxed{\frac{1}{1000}}$$

Answer:

$$\begin{aligned} 10^{-3} &= \frac{1}{10 \times 10 \times 10} \\ &= \frac{1}{1000} \end{aligned}$$

Ex 40: Write as a fraction:

$$2^{-1} = \boxed{\frac{1}{2}}$$

Answer:

$$2^{-1} = \frac{1}{2}$$

Ex 41: Write as a fraction:

$$5^{-2} = \boxed{\frac{1}{25}}$$

Answer:

$$\begin{aligned} 5^{-2} &= \frac{1}{5 \times 5} \\ &= \frac{1}{25} \end{aligned}$$

B.2 WRITING FRACTIONS AS NEGATIVE EXPONENTS

Ex 42: Write using a negative exponent:

$$\frac{1}{4} = \boxed{2^{-2}}$$

Answer:

$$\begin{aligned} \frac{1}{4} &= \frac{1}{2 \times 2} \\ &= 2^{-2} \end{aligned}$$

Ex 43: Write using a negative exponent:

$$\frac{1}{27} = \boxed{3^{-3}}$$

Answer:

$$\begin{aligned} \frac{1}{27} &= \frac{1}{3 \times 3 \times 3} \\ &= 3^{-3} \end{aligned}$$

Ex 44: Write using a negative exponent:

$$\frac{1}{1000} = \boxed{10^{-3}}$$

Answer:

$$\begin{aligned} \frac{1}{1000} &= \frac{1}{10 \times 10 \times 10} \\ &= 10^{-3} \end{aligned}$$

Ex 45: Write using a negative exponent:

$$\frac{1}{25} = \boxed{5^{-2}}$$

Answer:

$$\begin{aligned} \frac{1}{25} &= \frac{1}{5 \times 5} \\ &= 5^{-2} \end{aligned}$$

C RATIONAL EXPONENTS

C.1 EXPRESSING ROOTS USING EXPONENTS

Ex 46: Write in exponent form:

$$\sqrt{3} = \boxed{3^{\frac{1}{2}}}$$

Answer:

$$\sqrt{3} = 3^{\frac{1}{2}}$$

Ex 47: Write in exponent form:

$$\frac{1}{\sqrt{7}} = \boxed{7^{-\frac{1}{2}}}$$

Answer:

$$\frac{1}{\sqrt{7}} = 7^{-\frac{1}{2}}$$

Ex 48: Write in exponent form:

$$\sqrt{7} = \boxed{7^{\frac{1}{2}}}$$

Answer:

$$\sqrt{7} = 7^{\frac{1}{2}}$$

Ex 49: Write in exponent form:

$$\frac{1}{\sqrt{3}} = \boxed{3^{-\frac{1}{2}}}$$

Answer:

$$\frac{1}{\sqrt{3}} = 3^{-\frac{1}{2}}$$

Ex 50: Write in exponent form:

$$\sqrt{x} = \boxed{x^{\frac{1}{2}}}$$

Answer:

$$\sqrt{x} = x^{\frac{1}{2}}$$

C.2 CALCULATING POWERS AND ROUNDING

Ex 51: Calculate:

$$3^{\frac{1}{2}} = \boxed{1.73} \text{ (rounded to 2 decimal places)}$$

Answer: Using a calculator, we find:

$$3^{\frac{1}{2}} \approx 1.73$$

Ex 52: Calculate:

$$2^{\frac{1}{2}} = \boxed{1.41} \text{ (rounded to 2 decimal places)}$$

Answer: Using a calculator, we find:

$$2^{\frac{1}{2}} \approx 1.41$$

Ex 53: Calculate:

$$2^{-\frac{1}{2}} = \boxed{0.71} \text{ (rounded to 2 decimal places)}$$

Answer: Using a calculator, we find:

$$2^{-\frac{1}{2}} \approx 0.71$$

Ex 54: Calculate:

$$100^{-\frac{1}{2}} = \boxed{0.10} \text{ (rounded to 2 decimal places)}$$

Answer: Using a calculator, we find:

$$100^{-\frac{1}{2}} = 0.10$$

D EXPONENT LAW 1

D.1 SIMPLIFYING PRODUCTS OF POWERS

Ex 55: Simplify:

$$7^3 \times 7^2 = \boxed{7^5}$$

Answer:

$$\begin{aligned} 7^3 \times 7^2 &= \overbrace{7 \times 7 \times 7}^{3 \text{ factors}} \times \overbrace{7 \times 7}^{2 \text{ factors}} \\ &= \overbrace{7 \times 7 \times 7 \times 7 \times 7}^{3+2 \text{ factors}} \\ &= 7^{3+2} \\ &= 7^5 \end{aligned}$$

Ex 56: Simplify:

$$2^4 \times 2^3 = \boxed{2^7}$$

Answer:

$$\begin{aligned} 2^4 \times 2^3 &= \overbrace{2 \times 2 \times 2 \times 2}^{4 \text{ factors}} \times \overbrace{2 \times 2 \times 2}^{3 \text{ factors}} \\ &= \overbrace{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2}^{4+3 \text{ factors}} \\ &= 2^{4+3} \\ &= 2^7 \end{aligned}$$

Ex 57: Simplify:

$$3^5 \times 3^2 = \boxed{3^7}$$

Answer:

$$\begin{aligned} 3^5 \times 3^2 &= \overbrace{3 \times 3 \times 3 \times 3 \times 3}^{5 \text{ factors}} \times \overbrace{3 \times 3}^{2 \text{ factors}} \\ &= \overbrace{3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3}^{5+2 \text{ factors}} \\ &= 3^{5+2} \\ &= 3^7 \end{aligned}$$

Ex 58: Simplify:

$$10^6 \times 10^2 = \boxed{10^8}$$

Answer:

$$\begin{aligned} 10^6 \times 10^2 &= \overbrace{10 \times \cdots \times 10}^{6 \text{ factors}} \times \overbrace{10 \times 10}^{2 \text{ factors}} \\ &= \overbrace{10 \times \cdots \times 10}^{6+2 \text{ factors}} \\ &= 10^{6+2} \\ &= 10^8 \end{aligned}$$

Ex 59: Simplify:

$$2^3 \times 2 = \boxed{2^4}$$

Answer:

$$\begin{aligned} 2^3 \times 2 &= 2^3 \times 2^1 \\ &= \overbrace{2 \times 2 \times 2}^{3 \text{ factors}} \times \overbrace{2}^{1 \text{ factor}} \\ &= \overbrace{2 \times 2 \times 2 \times 2}^{3+1 \text{ factors}} \\ &= 2^{3+1} \\ &= 2^4 \end{aligned}$$

Ex 60: Simplify:

$$3 \times 3^4 = \boxed{3^5}$$

Answer:

$$\begin{aligned} 3 \times 3^4 &= 3^1 \times 3^4 \\ &= \overbrace{3}^{1 \text{ factor}} \times \overbrace{3 \times 3 \times 3 \times 3}^{4 \text{ factors}} \\ &= \overbrace{3 \times 3 \times 3 \times 3 \times 3}^{1+4 \text{ factors}} \\ &= 3^{1+4} \\ &= 3^5 \end{aligned}$$

D.2 SIMPLIFYING PRODUCTS OF ALGEBRAIC POWERS

Ex 61: Simplify:

$$x^2 \times x^3 = \boxed{x^5}$$

Answer:

$$\begin{aligned} x^2 \times x^3 &= \overbrace{x \times x}^{2 \text{ factors}} \times \overbrace{x \times x \times x}^{3 \text{ factors}} \\ &= \overbrace{x \times x \times x \times x \times x}^{2+3 \text{ factors}} \\ &= x^{2+3} \\ &= x^5 \end{aligned}$$

Ex 62: Simplify:

$$x \times x^2 = \boxed{x^3}$$

Answer:

$$\begin{aligned} x \times x^2 &= x^1 \times x^2 \\ &= \overbrace{x}^{1 \text{ factor}} \times \overbrace{x \times x}^{2 \text{ factors}} \\ &= \overbrace{x \times x \times x}^{1+2 \text{ factors}} \\ &= x^{1+2} \\ &= x^3 \end{aligned}$$

Ex 63: Simplify:

$$x^2 \times x^2 = \boxed{x^4}$$

Answer:

$$\begin{aligned} x^2 \times x^2 &= \overbrace{x \times x}^{2 \text{ factors}} \times \overbrace{x \times x}^{2 \text{ factors}} \\ &= \overbrace{x \times x \times x \times x}^{2+2 \text{ factors}} \\ &= x^{2+2} \\ &= x^4 \end{aligned}$$

Ex 64: Simplify:

$$x^3 \times x = \boxed{x^4}$$

Answer:

$$\begin{aligned} x^3 \times x &= x^3 \times x^1 \\ &= \overbrace{x \times x \times x}^{3 \text{ factors}} \times \overbrace{x}^{1 \text{ factor}} \\ &= \overbrace{x \times x \times x \times x}^{3+1 \text{ factors}} \\ &= x^{3+1} \\ &= x^4 \end{aligned}$$

D.3 IDENTIFYING CORRECT EXPONENTIAL EXPRESSIONS

MCQ 65: Which expressions are equal to $2^2 + 2^1$?
Choose all answers that apply:

☒ 6

☐ 2^3

☐ 4^3

Answer:

- $2^2 + 2^1 = 4 + 2 = 6$:**(correct)**
- $2^3 = 8$ which is not equal to $2^2 + 2^1 = 6$.
- $4^3 = 64$ which is not equal to $2^2 + 2^1 = 6$.

MCQ 66: Which expressions are equal to $5^2 \times 5^1$?
Choose all answers that apply:

☐ 25

☒ 125

☒ 5^3

Answer:

- $5^2 \times 5^1 = 5^{2+1} = 5^3 = 125$: **(correct)**
- 25 is just 5^2, not $5^2 \times 5^1$.
- 5^3 is the same as $5^2 \times 5^1$ by the law of exponents : **(correct)**

MCQ 67: Which expressions are equal to $3^2 + 3^1$?
Choose all answers that apply:

☒ 12

☐ 3^3

☐ 9^3

Answer:

- $3^2 + 3^1 = 9 + 3 = 12$: **(correct)**
- $3^3 = 27$ which is not equal to $3^2 + 3^1 = 12$.
- $9^3 = 729$ which is not equal to $3^2 + 3^1 = 12$.

MCQ 68: Which expressions are equal to $4^3 \times 4^2$?
Choose all answers that apply:

☒ 4^5

☐ 64

☒ 1024

Answer:

- $4^3 \times 4^2 = 4^{3+2} = 4^5$: **(correct)**
- 64 is 4^3, not $4^3 \times 4^2$.
- 1024 is 4^5, and thus equal to $4^3 \times 4^2$: **(correct)**

D.4 SIMPLIFYING EXPRESSIONS OF POWERS

Ex 69: Simplify:

$$x^{-2}\, x^3 = \boxed{x}$$

Answer:

$$\begin{aligned} x^{-2}\, x^3 &= x^{(-2)+3} \\ &= x^1 \\ &= x \end{aligned}$$

Ex 70: Simplify:

$$2^2\, 2^{-3}\, 2^{-3} = \boxed{2^{-4}}$$

Answer:

$$\begin{aligned} 2^2\, 2^{-3}\, 2^{-3} &= 2^{2+(-3)+(-3)} \\ &= 2^{(-1)+(-3)} \\ &= 2^{-4} \end{aligned}$$

Ex 71: Simplify:

$$x\, x^3\, x^{-2} = \boxed{x^2}$$

Answer:

$$\begin{aligned} x\, x^3\, x^{-2} &= x^{1+3+(-2)} \\ &= x^{4+(-2)} \\ &= x^2 \end{aligned}$$

Ex 72: Simplify:

$$x^3 \times x^{-3} = \boxed{1}$$

Answer:

$$\begin{aligned} x^3 \times x^{-3} &= x^{3+(-3)} \\ &= x^0 \\ &= 1 \end{aligned}$$

E EXPONENT LAW 2

E.1 SIMPLIFYING FRACTIONS OF POWERS

Ex 73: Simplify:

$$\frac{7^5}{7^2} = \boxed{7^3}$$

Answer:

$$\begin{aligned}\frac{7^5}{7^2} &= \frac{\overbrace{\cancel{7}\times\cancel{7}\times 7\times 7\times 7}^{5\text{ factors}}}{\underbrace{\cancel{7}\times\cancel{7}}_{2\text{ factors}}}\\ &= \overbrace{7\times 7\times 7}^{5-2\text{ factors}}\\ &= 7^{5-2}\\ &= 7^3\end{aligned}$$

Ex 74: Simplify:

$$\frac{5^6}{5^4} = \boxed{5^2}$$

Answer:

$$\begin{aligned}\frac{5^6}{5^4} &= \frac{\overbrace{\cancel{5}\times\cancel{5}\times\cancel{5}\times\cancel{5}\times 5\times 5}^{6\text{ factors}}}{\underbrace{\cancel{5}\times\cancel{5}\times\cancel{5}\times\cancel{5}}_{4\text{ factors}}}\\ &= \overbrace{5\times 5}^{6-4\text{ factors}}\\ &= 5^{6-4}\\ &= 5^2\end{aligned}$$

Ex 75: Simplify:

$$\frac{2^3}{2^5} = \boxed{2^{-2}}$$

Answer:

$$\begin{aligned}\frac{2^3}{2^5} &= \frac{\overbrace{\cancel{2}\times\cancel{2}\times\cancel{2}}^{3\text{ facteurs}}}{\underbrace{\cancel{2}\times\cancel{2}\times\cancel{2}\times 2\times 2}_{5\text{ facteurs}}}\\ &= \frac{1}{2\times 2}\\ &= 2^{-2} \quad (= 2^{3-5})\end{aligned}$$

Ex 76: Simplify:

$$\frac{3}{3^5} = \boxed{3^{-4}}$$

Answer:

$$\begin{aligned}\frac{3}{3^5} &= \frac{\overbrace{\cancel{3}}^{1\text{ facteur}}}{\underbrace{\cancel{3}\times 3\times 3\times 3\times 3}_{5\text{ facteurs}}}\\ &= \frac{1}{3\times 3\times 3\times 3}\\ &= 3^{-4} \quad (= 3^{1-5})\end{aligned}$$

Ex 77: Simplify:

$$\frac{7^2}{7^6} = \boxed{7^{-4}}$$

Answer:

$$\begin{aligned}\frac{7^2}{7^6} &= \frac{\overbrace{\cancel{7}\times\cancel{7}}^{2\text{ facteurs}}}{\underbrace{\cancel{7}\times\cancel{7}\times 7\times 7\times 7\times 7}_{6\text{ facteurs}}}\\ &= \frac{1}{7\times 7\times 7\times 7}\\ &= 7^{-4} \quad (= 7^{2-6})\end{aligned}$$

E.2 SIMPLIFYING FRACTIONS OF ALGEBRAIC POWERS

Ex 78: Simplify:

$$\frac{x^5}{x^2} = \boxed{x^3}$$

Answer:

$$\begin{aligned}\frac{x^5}{x^2} &= \frac{\overbrace{\cancel{x}\times\cancel{x}\times x\times x\times x}^{5\text{ factors}}}{\underbrace{\cancel{x}\times\cancel{x}}_{2\text{ factors}}}\\ &= \overbrace{x\times x\times x}^{5-2\text{ factors}}\\ &= x^{5-2}\\ &= x^3\end{aligned}$$

Ex 79: Simplify:

$$\frac{x^6}{x^4} = \boxed{x^2}$$

Answer:

$$\begin{aligned}\frac{x^6}{x^4} &= \frac{\overbrace{\cancel{x}\times\cancel{x}\times\cancel{x}\times\cancel{x}\times x\times x}^{6\text{ factors}}}{\underbrace{\cancel{x}\times\cancel{x}\times\cancel{x}\times\cancel{x}}_{4\text{ factors}}}\\ &= \overbrace{x\times x}^{6-4\text{ factors}}\\ &= x^{6-4}\\ &= x^2\end{aligned}$$

Ex 80: Simplify:

$$\frac{x^3}{x^5} = \boxed{x^{-2}}$$

Answer:

$$\begin{aligned}\frac{x^3}{x^5} &= \frac{\overbrace{\cancel{x}\times\cancel{x}\times\cancel{x}}^{3\text{ facteurs}}}{\underbrace{\cancel{x}\times\cancel{x}\times\cancel{x}\times x\times x}_{5\text{ facteurs}}}\\ &= \frac{1}{x\times x}\\ &= x^{-2} \quad (= x^{3-5})\end{aligned}$$

Ex 81: Simplify:

$$\frac{x}{x^5} = \boxed{x^{-4}}$$

Answer:

$$\frac{x}{x^5} = \frac{\overbrace{\cancel{x}}^{1\text{ facteur}}}{\underbrace{\cancel{x}\times x\times x\times x\times x}_{5\text{ facteurs}}}$$
$$= \frac{1}{x\times x\times x\times x}$$
$$= x^{-4} \quad (= x^{1-5})$$

Ex 82: Simplify:

$$\frac{x^2}{x^6} = \boxed{x^{-4}}$$

Answer:

$$\frac{x^2}{x^6} = \frac{\overbrace{\cancel{x}\times\cancel{x}}^{2\text{ facteurs}}}{\underbrace{\cancel{x}\times\cancel{x}\times x\times x\times x\times x}_{6\text{ facteurs}}}$$
$$= \frac{1}{x\times x\times x\times x}$$
$$= x^{-4} \quad (= x^{2-6})$$

F EXPONENT LAW 3

F.1 SIMPLIFYING POWERS OF POWERS

Ex 83: Simplify:

$$\left(5^2\right)^3 = \boxed{5^6}$$

Answer:

$$\begin{aligned}\left(5^2\right)^3 &= (5\times 5)^3\\ &= (5\times 5)\times(5\times 5)\times(5\times 5)\\ &= 5^6 \quad (= 5^{2\times 3})\end{aligned}$$

Ex 84: Simplify:

$$\left(7^3\right)^2 = \boxed{7^6}$$

Answer:

$$\begin{aligned}\left(7^3\right)^2 &= (7\times 7\times 7)^2\\ &= (7\times 7\times 7)\times(7\times 7\times 7)\\ &= 7^6 \quad (= 7^{3\times 2})\end{aligned}$$

Ex 85: Simplify:

$$\left(3^2\right)^4 = \boxed{3^8}$$

Answer:

$$\begin{aligned}\left(3^2\right)^4 &= (3\times 3)^4\\ &= (3\times 3)\times(3\times 3)\times(3\times 3)\times(3\times 3)\\ &= 3^8 \quad (= 3^{2\times 4})\end{aligned}$$

Ex 86: Simplify:

$$\left(2^5\right)^2 = \boxed{2^{10}}$$

Answer:

$$\begin{aligned}\left(2^5\right)^2 &= (2\times 2\times 2\times 2\times 2)^2\\ &= (2\times 2\times 2\times 2\times 2)\times(2\times 2\times 2\times 2\times 2)\\ &= 2^{10} \quad (= 2^{5\times 2})\end{aligned}$$

F.2 SIMPLIFYING POWERS OF POWERS

Ex 87: Simplify:

$$\left(x^2\right)^3 = \boxed{x^6}$$

Answer:

$$\begin{aligned}\left(x^2\right)^3 &= (x\times x)^3\\ &= (x\times x)\times(x\times x)\times(x\times x)\\ &= x^6 \quad (= x^{2\times 3})\end{aligned}$$

Ex 88: Simplify:

$$\left(x^3\right)^2 = \boxed{x^6}$$

Answer:

$$\begin{aligned}\left(x^3\right)^2 &= (x\times x\times x)^2\\ &= (x\times x\times x)\times(x\times x\times x)\\ &= x^6 \quad (= x^{3\times 2})\end{aligned}$$

Ex 89: Simplify:

$$\left(x^2\right)^4 = \boxed{x^8}$$

Answer:

$$\begin{aligned}\left(x^2\right)^4 &= (x\times x)^4\\ &= (x\times x)\times(x\times x)\times(x\times x)\times(x\times x)\\ &= x^8 \quad (= x^{2\times 4})\end{aligned}$$

Ex 90: Simplify:

$$\left(x^5\right)^2 = \boxed{x^{10}}$$

Answer:

$$\begin{aligned}\left(x^5\right)^2 &= (x\times x\times x\times x\times x)^2\\ &= (x\times x\times x\times x\times x)\times(x\times x\times x\times x\times x)\\ &= x^{10} \quad (= x^{5\times 2})\end{aligned}$$

G EXPONENT LAW 4

G.1 SIMPLIFYING POWERS OF PRODUCTS

Ex 91: Simplify:

$$(3\times 5)^2 = \boxed{3^2\times 5^2}$$

Answer:

$$\begin{aligned}(3\times 5)^2 &= (3\times 5)\times(3\times 5)\\ &= (3\times 3)\times(5\times 5)\\ &= 3^2\times 5^2\end{aligned}$$

Ex 92: Simplify:

$$(2\times 3)^4 = \boxed{2^4\times 3^4}$$

Answer:

$$\begin{aligned}(2\times 3)^4 &= (2\times 3)\times(2\times 3)\times(2\times 3)\times(2\times 3)\\ &= (2\times 2\times 2\times 2)\times(3\times 3\times 3\times 3)\\ &= 2^4\times 3^4\end{aligned}$$

Ex 93: Simplify:

$$(3\times 7)^3 = \boxed{3^3\times 7^3}$$

Answer:

$$\begin{aligned}(3\times 7)^3 &= (3\times 7)\times(3\times 7)\times(3\times 7)\\ &= (3\times 3\times 3)\times(7\times 7\times 7)\\ &= 3^3\times 7^3\end{aligned}$$

Ex 94: Simplify:

$$(3\times 5\times 7)^2 = \boxed{3^2\times 5^2\times 7^2}$$

Answer:

$$\begin{aligned}(3\times 5\times 7)^2 &= (3\times 5\times 7)\times(3\times 5\times 7)\\ &= (3\times 3)\times(5\times 5)\times(7\times 7)\\ &= 3^2\times 5^2\times 7^2\end{aligned}$$

G.2 SIMPLIFYING POWERS OF PRODUCTS

Ex 95: Simplify:

$$(2\times x)^3 = \boxed{2^3\times x^3}$$

Answer:

$$\begin{aligned}(2\times x)^3 &= (2\times x)\times(2\times x)\times(2\times x)\\ &= (2\times 2\times 2)\times(x\times x\times x)\\ &= 2^3\times x^3\end{aligned}$$

Ex 96: Simplify:

$$(x\times 3)^2 = \boxed{x^2\times 3^2}$$

Answer:

$$\begin{aligned}(x\times 3)^2 &= (x\times 3)\times(x\times 3)\\ &= (x\times x)\times(3\times 3)\\ &= x^2\times 3^2\end{aligned}$$

Ex 97: Simplify:

$$(5\times x)^4 = \boxed{5^4\times x^4}$$

Answer:

$$\begin{aligned}(5\times x)^4 &= (5\times x)\times(5\times x)\times(5\times x)\times(5\times x)\\ &= (5\times 5\times 5\times 5)\times(x\times x\times x\times x)\\ &= 5^4\times x^4\end{aligned}$$

Ex 98: Simplify:

$$(x\times 2)^5 = \boxed{x^5\times 2^5}$$

Answer:

$$\begin{aligned}(x\times 2)^5 &= (x\times 2)\times(x\times 2)\times(x\times 2)\times(x\times 2)\times(x\times 2)\\ &= (x\times x\times x\times x\times x)\times(2\times 2\times 2\times 2\times 2)\\ &= x^5\times 2^5\end{aligned}$$

H EXPONENT LAW 5

H.1 SIMPLIFYING POWERS OF FRACTIONS

Ex 99: Simplify:

$$\left(\frac{5}{3}\right)^2 = \boxed{\frac{5^2}{3^2}}$$

Answer:

$$\begin{aligned}\left(\frac{5}{3}\right)^2 &= \frac{5}{3}\times\frac{5}{3}\\ &= \frac{5\times 5}{3\times 3}\\ &= \frac{5^2}{3^2}\end{aligned}$$

Ex 100: Simplify:

$$\left(\frac{2}{7}\right)^3 = \boxed{\frac{2^3}{7^3}}$$

Answer:

$$\begin{aligned}\left(\frac{2}{7}\right)^3 &= \frac{2}{7}\times\frac{2}{7}\times\frac{2}{7}\\ &= \frac{2\times 2\times 2}{7\times 7\times 7}\\ &= \frac{2^3}{7^3}\end{aligned}$$

Ex 101: Simplify:

$$\left(\frac{1}{2}\right)^2 = \boxed{\frac{1^2}{2^2}}$$

Answer:

$$\begin{aligned}\left(\frac{1}{2}\right)^2 &= \frac{1}{2}\times\frac{1}{2}\\ &= \frac{1\times 1}{2\times 2}\\ &= \frac{1^2}{2^2}\\ &= \frac{1}{4}\end{aligned}$$

Ex 102: Simplify:

$$\left(\frac{1}{3}\right)^3 = \boxed{\frac{1^3}{3^3}}$$

Answer:

$$\begin{aligned}\left(\frac{1}{3}\right)^3 &= \frac{1}{3}\times\frac{1}{3}\times\frac{1}{3}\\ &= \frac{1\times 1\times 1}{3\times 3\times 3}\\ &= \frac{1^3}{3^3}\\ &= \frac{1}{27}\end{aligned}$$

H.2 SIMPLIFYING POWERS OF ALGEBRAIC FRACTIONS

Ex 103: Simplify:

$$\left(\frac{x}{2}\right)^4 = \boxed{\frac{x^4}{2^4}}$$

Answer:

$$\begin{aligned}\left(\frac{x}{2}\right)^4 &= \frac{x}{2} \times \frac{x}{2} \times \frac{x}{2} \times \frac{x}{2} \\ &= \frac{x \times x \times x \times x}{2 \times 2 \times 2 \times 2} \\ &= \frac{x^4}{2^4}\end{aligned}$$

Ex 104: Simplify:

$$\left(\frac{1}{x}\right)^3 = \boxed{\frac{1^3}{x^3}}$$

Answer:

$$\begin{aligned}\left(\frac{1}{x}\right)^3 &= \frac{1}{x} \times \frac{1}{x} \times \frac{1}{x} \\ &= \frac{1 \times 1 \times 1}{x \times x \times x} \\ &= \frac{1^3}{x^3} \\ &= \frac{1}{x^3}\end{aligned}$$

Ex 105: Simplify:

$$\left(\frac{2}{x}\right)^4 = \boxed{\frac{2^4}{x^4}}$$

Answer:

$$\begin{aligned}\left(\frac{2}{x}\right)^4 &= \frac{2}{x} \times \frac{2}{x} \times \frac{2}{x} \times \frac{2}{x} \\ &= \frac{2 \times 2 \times 2 \times 2}{x \times x \times x \times x} \\ &= \frac{2^4}{x^4} \\ &= \frac{16}{x^4}\end{aligned}$$

Ex 106: Simplify:

$$\left(\frac{x}{10}\right)^2 = \boxed{\frac{x^2}{10^2}}$$

Answer:

$$\begin{aligned}\left(\frac{x}{10}\right)^2 &= \frac{x}{10} \times \frac{x}{10} \\ &= \frac{x \times x}{10 \times 10} \\ &= \frac{x^2}{10^2} \\ &= \frac{x^2}{100}\end{aligned}$$

I EXPONENT LAW 6

I.1 EXPRESSING NEGATIVE EXPONENTS AS FRACTIONS

Ex 107: Write as a fraction:

$$\left(\frac{4}{7}\right)^{-1} = \boxed{\frac{7}{4}}$$

Answer:

$$\begin{aligned}\left(\frac{4}{7}\right)^{-1} &= \left(\frac{7}{4}\right)^1 \\ &= \frac{7}{4}\end{aligned}$$

Ex 108: Write as a fraction:

$$\left(\frac{5}{3}\right)^{-2} = \boxed{\frac{9}{25}}$$

Answer:

$$\begin{aligned}\left(\frac{5}{3}\right)^{-2} &= \left(\frac{3}{5}\right)^2 \\ &= \frac{3^2}{5^2} \\ &= \frac{9}{25}\end{aligned}$$

Ex 109: Write as a fraction:

$$\left(\frac{1}{2}\right)^{-3} = \boxed{8}$$

Answer:

$$\begin{aligned}\left(\frac{1}{2}\right)^{-3} &= \left(\frac{2}{1}\right)^3 \\ &= 2^3 \\ &= 8\end{aligned}$$

Ex 110: Write as a fraction:

$$\left(\frac{2}{3}\right)^{-3} = \boxed{\frac{27}{8}}$$

Answer:

$$\begin{aligned}\left(\frac{2}{3}\right)^{-3} &= \left(\frac{3}{2}\right)^3 \\ &= \frac{3^3}{2^3} \\ &= \frac{27}{8}\end{aligned}$$

I.2 MULTIPLYING BY THE INVERSE

Ex 111: Simplify:

$$\frac{3}{2} \times \left(\frac{3}{2}\right)^{-1} = \boxed{1}$$

Answer:

$$\begin{aligned}\frac{3}{2} \times \left(\frac{3}{2}\right)^{-1} &= \frac{3}{2} \times \frac{2}{3} \\ &= \frac{3 \times 2}{2 \times 3} \\ &= 1\end{aligned}$$

Ex 112: Simplify:

$$\frac{x}{2} \times \left(\frac{x}{2}\right)^{-1} = \boxed{1}$$

Answer:

$$\begin{aligned}\frac{x}{2} \times \left(\frac{x}{2}\right)^{-1} &= \frac{x}{2} \times \frac{2}{x} \\ &= \frac{x \times 2}{2 \times x} \\ &= \frac{2x}{2x} \\ &= 1\end{aligned}$$

Ex 113: Simplify:

$$\frac{a}{b} \times \left(\frac{a}{b}\right)^{-1} = \boxed{1}$$

Answer:

$$\begin{aligned}\frac{a}{b} \times \left(\frac{a}{b}\right)^{-1} &= \frac{a}{b} \times \frac{b}{a} \\ &= \frac{a \times b}{b \times a} \\ &= \frac{ab}{ab} \\ &= 1\end{aligned}$$

J ORDER OF OPERATIONS

J.1 EVALUATING EXPRESSIONS WITH EXPONENTS IN 2 STEPS

Ex 114: Evaluate this expression:

$$2 \times 5^2 = \boxed{50}$$

Answer:

$$\begin{aligned}2 \times 5^2 &= 2 \times 5^2 && \text{(exponent: } 5^2 = 25\text{)} \\ &= 2 \times 25 && \text{(multiplication: } 2 \times 25 = 50\text{)} \\ &= 50\end{aligned}$$

Ex 115: Evaluate this expression:

$$2^3 - 1 = \boxed{7}$$

Answer:

$$\begin{aligned}2^3 - 1 &= 2^3 - 1 && \text{(exponent: } 2^3 = 8\text{)} \\ &= 8 - 1 && \text{(subtraction: } 8 - 1 = 7\text{)} \\ &= 7\end{aligned}$$

Ex 116: Evaluate this expression:

$$(2+1)^2 = \boxed{9}$$

Answer:

$$\begin{aligned}(2+1)^2 &= (2+1)^2 && \text{(parentheses: } 2+1 = 3\text{)} \\ &= 3^2 && \text{(exponent: } 3^2 = 9\text{)} \\ &= 9\end{aligned}$$

Ex 117: Evaluate this expression:

$$2^3 \div 4 = \boxed{2}$$

Answer:

$$\begin{aligned}2^3 \div 4 &= 2^3 \div 4 && \text{(exponent: } 2^3 = 8\text{)} \\ &= 8 \div 4 && \text{(division: } 8 \div 4 = 2\text{)} \\ &= 2\end{aligned}$$

Ex 118: Evaluate this expression:

$$(5-2)^2 = \boxed{9}$$

Answer:

$$\begin{aligned}(5-2)^2 &= (5-2)^2 && \text{(parentheses: } 5-2 = 3\text{)} \\ &= 3^2 && \text{(exponent: } 3^2 = 9\text{)} \\ &= 9\end{aligned}$$

J.2 EVALUATING EXPRESSIONS WITH EXPONENTS IN 3 STEPS

Ex 119: Evaluate this expression:

$$2^3 \times (8-6) = \boxed{16}$$

Answer:

$$\begin{aligned}2^3 \times (8-6) &= 2^3 \times (8-6) && \text{(parentheses: } 8-6 = 2\text{)} \\ &= 2^3 \times 2 && \text{(exponent: } 2^3 = 8\text{)} \\ &= 8 \times 2 && \text{(multiplication: } 8 \times 2 = 16\text{)} \\ &= 16\end{aligned}$$

Ex 120: Evaluate this expression:

$$(2+1)^2 - 1 = \boxed{8}$$

Answer:

$$\begin{aligned}(2+1)^2 - 1 &= (2+1)^2 - 1 && \text{(parentheses: } 2+1 = 3\text{)} \\ &= 3^2 - 1 && \text{(exponent: } 3^2 = 9\text{)} \\ &= 9 - 1 && \text{(subtraction: } 9 - 1 = 8\text{)} \\ &= 8\end{aligned}$$

Ex 121: Evaluate this expression:

$$(3^2 - 1) \times 4 = \boxed{32}$$

Answer:

$$\begin{aligned}(3^2 - 1) \times 4 &= (3^2 - 1) \times 4 && \text{(evaluate the parentheses: } 3^2 = 9\text{)} \\ &= (9 - 1) \times 4 && \text{(evaluate the parentheses: } 9 - 1 = 8\text{)} \\ &= 8 \times 4 && \text{(multiplication: } 8 \times 4 = 32\text{)} \\ &= 32\end{aligned}$$

Ex 122: Evaluate this expression:

$$\frac{3^2 - 1}{2} = \boxed{4}$$

Answer:

$$\begin{aligned}\frac{3^2 - 1}{2} &= \frac{3^2 - 1}{2} && \text{(evaluate the numerator: } 3^2 = 9\text{)} \\ &= \frac{9 - 1}{2} && \text{(evaluate the numerator: } 9 - 1 = 8\text{)} \\ &= \frac{8}{2} && \text{(division: } 8 \div 2 = 4\text{)} \\ &= 4\end{aligned}$$

J.3 FINDING THE OPERATORS

Ex 123:

$$3^3 \boxed{-} 2^2 = 23$$

Answer:

- $3^3 + 2^2 = 27 + 4 = 31$, so it's not true.
- $3^3 - 2^2 = 27 - 4 = 23$, so it's true.
- $3^3 \times 2^2 = 27 \times 4 = 108$, so it's not true.
- $3^3 \div 2^2 = 27 \div 4 = 6.75$, so it's not true.

Ex 124:

$$2^4 \boxed{\times} 3^2 = 144$$

Answer:

- $2^4 + 3^2 = 16 + 9 = 25$, so it's not true.
- $2^4 - 3^2 = 16 - 9 = 7$, so it's not true.
- $2^4 \times 3^2 = 16 \times 9 = 144$, so it's true.
- $2^4 \div 3^2 = 16 \div 9 \approx 1.78$, so it's not true.

Ex 125:

$$2^3 \boxed{\div} 4 = 2$$

Answer:

- $2^3 + 4 = 8 + 4 = 12$, so it's not true.
- $2^3 - 4 = 8 - 4 = 4$, so it's not true.
- $2^3 \times 4 = 8 \times 4 = 32$, so it's not true.
- $2^3 \div 4 = 8 \div 4 = 2$, so it's true.

Ex 126:

$$(2+1)^2 \boxed{+} 1 = 10$$

Answer:

- $(2+1)^2 + 1 = 9 + 1 = 10$, so it's true.
- $(2+1)^2 - 1 = 9 - 1 = 8$, so it's not true.
- $(2+1)^2 \times 1 = 9 \times 1 = 9$, so it's not true.
- $(2+1)^2 \div 1 = 9 \div 1 = 9$, so it's not true.

J.4 COMBINING NEGATIVE POWERS WITH ARITHMETIC

Ex 127: Write as a fraction:

$$1 + 2^{-1} = \boxed{\frac{3}{2}}$$

Answer:

$$\begin{aligned} 1 + 2^{-1} &= 1 + \frac{1}{2} \\ &= \frac{2}{2} + \frac{1}{2} \\ &= \frac{3}{2} \end{aligned}$$

Ex 128: Write as a fraction:

$$3^{-1} - 1 = \boxed{-\frac{2}{3}}$$

Answer:

$$\begin{aligned} 3^{-1} - 1 &= \frac{1}{3} - 1 \\ &= \frac{1}{3} - \frac{3}{3} \\ &= \frac{1-3}{3} \\ &= -\frac{2}{3} \end{aligned}$$

Ex 129: Write as a fraction:

$$5 \times 3^{-2} = \boxed{\frac{5}{9}}$$

Answer:

$$\begin{aligned} 5 \times 3^{-2} &= 5 \times \frac{1}{3 \times 3} \\ &= 5 \times \frac{1}{9} \\ &= \frac{5}{9} \end{aligned}$$

Ex 130: Write as a fraction:

$$\frac{4}{5} \times 2^{-2} = \boxed{\frac{1}{5}}$$

Answer:

$$\begin{aligned} \frac{4}{5} \times 2^{-2} &= \frac{4}{5} \times \frac{1}{2 \times 2} \\ &= \frac{4}{5} \times \frac{1}{4} \\ &= \frac{4 \times 1}{5 \times 4} \\ &= \frac{4}{20} \\ &= \frac{1}{5} \end{aligned}$$

J.5 SIMPLIFYING ALGEBRAIC EXPRESSIONS

Ex 131: Simplify the expression:

$$2x^2+3x^2=\boxed{5x^2}$$

Answer:

$$\begin{aligned}2x^2+3x^2&=(2+3)x^2 \quad \text{(combine like terms)}\\&=5x^2\end{aligned}$$

Ex 132: Simplify the expression:

$$3x^2-x^2=\boxed{2x^2}$$

Answer:

$$\begin{aligned}3x^2-x^2&=(3-1)x^2 \quad \text{(combine like terms)}\\&=2x^2\end{aligned}$$

Ex 133: Simplify the expression:

$$2x^2+3x+x=\boxed{2x^2+4x}$$

Answer:

$$\begin{aligned}2x^2+3x+x&=2x^2+(3+1)x \quad \text{(combine like terms)}\\&=2x^2+4x\end{aligned}$$

Ex 134: Simplify the expression:

$$x^2+2x+x^2+5x+1=\boxed{2x^2+7x+1}$$

Answer:

$$\begin{aligned}x^2+2x+x^2+5x+1&=(x^2+x^2)+(2x+5x)+1\\&=2x^2+7x+1\end{aligned}$$

Ex 135: Simplify the expression:

$$3x^2+4+2x+x^2+6x+1=\boxed{4x^2+8x+5}$$

Answer:

$$\begin{aligned}3x^2+4+2x+x^2+6x+1&=(3x^2+x^2)+(2x+6x)+(4+1)\\&=4x^2+8x+5\end{aligned}$$

Ex 136: Simplify the expression:

$$(2x-x)^2=\boxed{x^2}$$

Answer:

$$\begin{aligned}(2x-x)^2&=(x)^2 \quad \text{(combine like terms in the parentheses)}\\&=x^2\end{aligned}$$

J.6 SIMPLIFYING EXPRESSIONS OF POWERS

Ex 137: Simplify:

$$\frac{2^3}{2}\times 2^3=\boxed{2^5}$$

Answer:

$$\begin{aligned}\frac{2^3}{2}\times 2^3&=2^{3-1}\times 2^3\\&=2^2\times 2^3\\&=2^{2+3}\\&=2^5\end{aligned}$$

Ex 138: Simplify:

$$x^3\times\frac{x^4}{x^2}=\boxed{x^5}$$

Answer:

$$\begin{aligned}x^3\times\frac{x^4}{x^2}&=x^3\times x^{4-2}\\&=x^3\times x^2\\&=x^{3+2}\\&=x^5\end{aligned}$$

Ex 139: Simplify:

$$\frac{x}{x^2}\,x^{-1}=\boxed{x^{-2}}$$

Answer:

$$\begin{aligned}\frac{x}{x^2}\,x^{-1}&=x^{1-2}\times x^{-1}\\&=x^{-1}\times x^{-1}\\&=x^{(-1)+(-1)}\\&=x^{-2}\end{aligned}$$

Ex 140: Simplify:

$$\frac{2^2}{2\times 2^3}=\boxed{2^{-2}}$$

Answer:

$$\begin{aligned}\frac{2^2}{2\times 2^3}&=\frac{2^2}{2^1\times 2^3}\\&=\frac{2^2}{2^{1+3}}\\&=\frac{2^2}{2^4}\\&=2^{2-4}\\&=2^{-2}\end{aligned}$$

Ex 141: Simplify:

$$\left(\frac{x}{2}\right)^2\times 4=\boxed{x^2}$$

Answer:

$$\begin{aligned}\left(\frac{x}{2}\right)^2\times 4&=\frac{x^2}{2^2}\times 4\\&=\frac{x^2}{4}\times 4\\&=x^2\end{aligned}$$

Ex 142: Simplify:

$$\frac{x^3 \times \left(x^2\right)^2}{x^4} = \boxed{x^3}$$

Answer:

$$\begin{aligned}\frac{x^3 \times \left(x^2\right)^2}{x^4} &= \frac{x^3 \times x^{2\times 2}}{x^4}\\ &= \frac{x^3 \times x^4}{x^4}\\ &= \frac{x^{3+4}}{x^4}\\ &= \frac{x^7}{x^4}\\ &= x^{7-4}\\ &= x^3\end{aligned}$$

J.7 EVALUATING TO AN INTEGER

Ex 143: Express as an integer:

$$\sqrt{2} \times 2^{\frac{1}{2}} = \boxed{2}$$

Answer:

$$\begin{aligned}\sqrt{2} \times 2^{\frac{1}{2}} &= 2^{\frac{1}{2}} \times 2^{\frac{1}{2}}\\ &= 2^{\frac{1}{2}+\frac{1}{2}}\\ &= 2^1\\ &= 2\end{aligned}$$

Ex 144: Express as an integer:

$$\frac{2^{\frac{3}{2}}}{\sqrt{2}} = \boxed{2}$$

Answer:

$$\begin{aligned}\frac{2^{\frac{3}{2}}}{\sqrt{2}} &= \frac{2^{\frac{3}{2}}}{2^{\frac{1}{2}}}\\ &= 2^{\frac{3}{2}-\frac{1}{2}}\\ &= 2^1\\ &= 2\end{aligned}$$

Ex 145: Express as an integer:

$$\left(\sqrt{2}\right)^4 = \boxed{4}$$

Answer:

$$\begin{aligned}\left(\sqrt{2}\right)^4 &= \left(2^{\frac{1}{2}}\right)^4\\ &= 2^{\frac{1}{2}\times 4}\\ &= 2^2\\ &= 4\end{aligned}$$

Ex 146: Express as an integer:

$$(3\sqrt{2})^2 = \boxed{18}$$

Answer:

$$\begin{aligned}(3\sqrt{2})^2 &= 3^2 \times (\sqrt{2})^2\\ &= 9 \times 2\\ &= 18\end{aligned}$$

K SCIENTIFIC NOTATION

K.1 WRITING NUMBERS AS POWERS OF TEN

Ex 147: Write in exponent form:

$$100 = \boxed{10^2}$$

Answer:

$$\begin{aligned}100 &= 10 \times 10\\ &= 10^2\end{aligned}$$

Ex 148: Write in exponent form:

$$1\,000 = \boxed{10^3}$$

Answer:

$$\begin{aligned}1\,000 &= 10 \times 10 \times 10\\ &= 10^3\end{aligned}$$

Ex 149: Write in exponent form:

$$0.01 = \boxed{10^{-2}}$$

Answer:

$$\begin{aligned}0.01 &= \frac{1}{100}\\ &= \frac{1}{10^2}\\ &= 10^{-2}\end{aligned}$$

Ex 150: Write in exponent form:

$$0.000\,1 = \boxed{10^{-4}}$$

Answer:

$$\begin{aligned}0.000\,1 &= \frac{1}{10\,000}\\ &= \frac{1}{10^4}\\ &= 10^{-4}\end{aligned}$$

K.2 EXPRESSING NUMBERS IN SCIENTIFIC NOTATION

Ex 151: Write in scientific notation:

$$123 = \boxed{1.23} \times \boxed{10^2}$$

Answer:

$$\begin{aligned}123 &= 1.23 \times 100\\ &= 1.23 \times 10^2\end{aligned}$$

Ex 152: Write in scientific notation:

$$1\,200 = \boxed{1.2} \times \boxed{10^3}$$

Answer:

$$\begin{aligned}1\,200 &= 1.2 \times 1\,000\\ &= 1.2 \times 10^3\end{aligned}$$

Ex 153: Write in scientific notation:

$$5\,000\,000 = \boxed{5} \times \boxed{10^6}$$

Answer:

$$\begin{aligned} 5\,000\,000 &= 5 \times 1\,000\,000 \\ &= 5 \times 10^6 \end{aligned}$$

Ex 154: Write in scientific notation:

$$8\,100\,000\,000 = \boxed{8.1} \times \boxed{10^9}$$

Answer:

$$\begin{aligned} 8\,100\,000\,000 &= 8.1 \times 1\,000\,000\,000 \\ &= 8.1 \times 10^9 \end{aligned}$$

Ex 155: Write in scientific notation:

$$0.05 = \boxed{5} \times \boxed{10^{-2}}$$

Answer:

$$\begin{aligned} 0.05 &= 5 \times \frac{1}{100} \\ &= 5 \times 10^{-2} \end{aligned}$$

Ex 156: Write in scientific notation:

$$0.12 = \boxed{1.2} \times \boxed{10^{-1}}$$

Answer:

$$\begin{aligned} 0.12 &= 1.2 \times \frac{1}{10} \\ &= 1.2 \times 10^{-1} \end{aligned}$$

Ex 157: Write in scientific notation:

$$0.000\,59 = \boxed{5.9} \times \boxed{10^{-4}}$$

Answer:

$$\begin{aligned} 0.000\,59 &= 5.9 \times \frac{1}{10^4} \\ &= 5.9 \times 10^{-4} \end{aligned}$$

K.3 EXPRESSING IN DECIMAL FORM

Ex 158: Write in decimal form:

$$8.2 \times 10^2 = \boxed{820}$$

Answer:

$$\begin{aligned} 8.2 \times 10^2 &= 8.2 \times 100 \\ &= 820 \end{aligned}$$

Ex 159: Write in decimal form:

$$1.25 \times 10^3 = \boxed{1250}$$

Answer:

$$\begin{aligned} 1.25 \times 10^3 &= 1.25 \times 1\,000 \\ &= 1\,250 \end{aligned}$$

Ex 160: Write in decimal form:

$$5 \times 10^6 = \boxed{5000000}$$

Answer:

$$\begin{aligned} 5 \times 10^6 &= 5 \times 1\,000\,000 \\ &= 5\,000\,000 \end{aligned}$$

Ex 161: Write in decimal form:

$$2 \times 10^{-2} = \boxed{0.02}$$

Answer:

$$\begin{aligned} 2 \times 10^{-2} &= 2 \times \frac{1}{100} \\ &= 0.02 \end{aligned}$$

Ex 162: Write in decimal form:

$$8.5 \times 10^{-1} = \boxed{0.85}$$

Answer:

$$\begin{aligned} 8.5 \times 10^{-1} &= 8.5 \times \frac{1}{10} \\ &= 0.85 \end{aligned}$$

Ex 163: Write in decimal form:

$$9.1 \times 10^{-5} = \boxed{0.000091}$$

Answer:

$$\begin{aligned} 9.1 \times 10^{-5} &= 9.1 \times \frac{1}{100\,000} \\ &= 0.000\,091 \end{aligned}$$

K.4 EXPRESSING REAL-WORLD QUANTITIES IN SCIENTIFIC NOTATION

Ex 164: There are approximately 4 million red blood cells in a drop of blood. Write the quantity in scientific notation:

$$\boxed{4} \times \boxed{10^6} \text{ red blood cells}$$

Answer:

$$\begin{aligned} 4\,000\,000 &= 4 \times 1\,000\,000 \\ &= 4 \times 10^6 \end{aligned}$$

Ex 165: There are approximately 3 billion stars in the galaxy. Write the quantity in scientific notation:

$$\boxed{3} \times \boxed{10^9} \text{ stars}$$

Answer:

$$\begin{aligned} 3\,000\,000\,000 &= 3 \times 1\,000\,000\,000 \\ &= 3 \times 10^9 \end{aligned}$$

Ex 166: There are approximately 7.5 billion people on Earth. Write the quantity in scientific notation:

$$\boxed{7.5} \times \boxed{10^9} \text{ people}$$

Answer:

$$\begin{aligned} 7\,500\,000\,000 &= 7.5 \times 1\,000\,000\,000 \\ &= 7.5 \times 10^9 \end{aligned}$$

Ex 167: The distance from the Earth to the Sun is approximately 150 million kilometers. Write the quantity in scientific notation:

$$\boxed{1.5} \times \boxed{10^8} \text{ kilometers}$$

Answer:

$$\begin{aligned} 150\,000\,000 &= 1.5 \times 100\,000\,000 \\ &= 1.5 \times 10^8 \end{aligned}$$

L EXPONENTIAL EXPRESSION

L.1 SIMPLIFYING USING EXPONENT LAWS

Ex 168: Simplify:

$$3^{x-1} \times 3^{x+1} = \boxed{9^x}$$

Answer: **Method: Using the Product Rule for Exponents**

$$\begin{aligned} 3^{x-1} \times 3^{x+1} &= 3^{(x-1)+(x+1)} && \text{(Apply the product rule } a^m \cdot a^n = a^{m+n}) \\ &= 3^{2x} && \text{(Simplify the exponent)} \\ &= (3^2)^x && \text{(Apply the power rule } a^{mn} = (a^m)^n) \\ &= 9^x \end{aligned}$$

Ex 169: Simplify:

$$\frac{2^{x+2}}{2} = \boxed{2^{x+1}}$$

Answer: **Method: Using Exponent Laws**

$$\begin{aligned} \frac{2^{x+2}}{2} &= \frac{2^{x+2}}{2^1} && \text{(Rewrite the denominator)} \\ &= 2^{(x+2)-1} && \text{(Apply the quotient rule } \frac{a^m}{a^n} = a^{m-n}) \\ &= 2^{x+1} && \text{(Simplify the exponent)} \end{aligned}$$

Ex 170: Simplify:

$$\frac{4^{x+1}}{2^x} = \boxed{2^{x+2}}$$

Answer: **Method: Using Exponent Laws**

$$\begin{aligned} \frac{4^{x+1}}{2^x} &= \frac{(2^2)^{x+1}}{2^x} && \text{(Rewrite the base 4 as } 2^2) \\ &= \frac{2^{2(x+1)}}{2^x} && \text{(Apply the power rule } (a^m)^n = a^{mn}) \\ &= \frac{2^{2x+2}}{2^x} && \text{(Distribute in the exponent)} \\ &= 2^{(2x+2)-x} && \text{(Apply the quotient rule } \frac{a^m}{a^n} = a^{m-n}) \\ &= 2^{x+2} \end{aligned}$$

Ex 171: Simplify:

$$(2^x \cdot 3^x)^2 = \boxed{36^x}$$

Answer: **Method: Using Exponent Laws**

$$\begin{aligned} (2^x \cdot 3^x)^2 &= ((2 \cdot 3)^x)^2 && \text{(Apply the product rule } a^n b^n = (ab)^n) \\ &= (6^x)^2 && \text{(Simplify the base)} \\ &= 6^{2x} && \text{(Apply the power rule } (a^m)^n = a^{mn}) \\ &= (6^2)^x \\ &= 36^x \end{aligned}$$

L.2 SIMPLIFYING EXPONENTIAL EXPRESSIONS

Ex 172: Simplify:

$$\frac{3^x + 6^x}{3^x} = \boxed{1 + 2^x}$$

Answer:

- **Method 1: Splitting the Fraction**

$$\begin{aligned} \frac{3^x + 6^x}{3^x} &= \frac{3^x}{3^x} + \frac{6^x}{3^x} && \text{(Splitting the fraction)} \\ &= 1 + \left(\frac{6}{3}\right)^x && \text{(Using the exponent law)} \\ &= 1 + 2^x && \text{(Simplifying the base)} \end{aligned}$$

- **Method 2: Factorization**

$$\begin{aligned} \frac{3^x + 6^x}{3^x} &= \frac{3^x + (2 \times 3)^x}{3^x} && \text{(Rewrite 6 as } 2\times3) \\ &= \frac{3^x + 2^x \cdot 3^x}{3^x} && \text{(Apply exponent law)} \\ &= \frac{3^x(1 + 2^x)}{3^x} && \text{(Factor out the common term } 3^x) \\ &= 1 + 2^x && \text{(Cancel the common factor)} \end{aligned}$$

Ex 173: Simplify:

$$\frac{2^{x+2} + 2^x}{5} = \boxed{2^x}$$

Answer: **Method: Factorization**

$$\begin{aligned} \frac{2^{x+2} + 2^x}{5} &= \frac{2^x \cdot 2^2 + 2^x}{5} && \text{(Using the exponent law } a^{m+n} = a^m a^n) \\ &= \frac{4 \cdot 2^x + 2^x}{5} && \text{(Simplifying the power)} \\ &= \frac{2^x(4+1)}{5} && \text{(Factor out the common term } 2^x) \\ &= \frac{2^x \cdot 5}{5} && \text{(Simplify the expression in brackets)} \\ &= 2^x && \text{(Cancel the common factor)} \end{aligned}$$

Ex 174: Simplify:

$$3^x(n+1) - 3^x = \boxed{n \cdot 3^x}$$

Answer: **Method: Factorization**

$$\begin{aligned} 3^x(n+1) - 3^x &= 3^x \cdot (n+1) - 3^x \cdot 1 && \text{(Rewrite the second term)} \\ &= 3^x((n+1) - 1) && \text{(Factor out the common term } 3^x) \\ &= 3^x(n) && \text{(Simplify the expression in brackets)} \\ &= n \cdot 3^x \end{aligned}$$

Ex 175: Simplify:

$$\frac{4^x - 2^x}{2^x} = \boxed{2^x - 1}$$

Answer:

- **Method 1: Splitting the Fraction**

$$\begin{aligned} \frac{4^x - 2^x}{2^x} &= \frac{4^x}{2^x} - \frac{2^x}{2^x} && \text{(Splitting the fraction)} \\ &= \left(\frac{4}{2}\right)^x - 1 && \text{(Using the exponent law and simplifying)} \\ &= 2^x - 1 && \text{(Simplifying the base)} \end{aligned}$$

- **Method 2: Factorization**

$$\begin{aligned}\frac{4^x-2^x}{2^x} &= \frac{2^x\cdot 2^x-2^x}{2^x} && \text{(Rewrite the first term)}\\ &= \frac{2^x(2^x-1)}{2^x} && \text{(Factor out the common term } 2^x\text{)}\\ &= 2^x-1 && \text{(Cancel the common factor)}\end{aligned}$$

L.3 EXPANDING AND SIMPLIFYING EXPONENTIAL EXPRESSIONS

Ex 176: Expand and simplify:

$$(2^x-1)(2^x+1) = \boxed{4^x-1}$$

Answer:

- **Method 1: Using the Difference of Squares Identity**
  This expression is in the form $(a-b)(a+b)$, where $a=2^x$ and $b=1$. The identity is $(a-b)(a+b)=a^2-b^2$.

$$\begin{aligned}(2^x-1)(2^x+1) &= (2^x)^2-1^2 && \text{(Apply the identity)}\\ &= 2^{2x}-1 && \text{(Use exponent law } (a^m)^n=a^{mn}\text{)}\\ &= (2^2)^x-1 && \text{(Rewrite the first term)}\\ &= 4^x-1\end{aligned}$$

- **Method 2: Using the Distributive Property**

$$\begin{aligned}&(2^x-1)(2^x+1)\\ =&2^x(2^x+1)-1(2^x+1) && \text{(Distribute)}\\ =&(2^x\cdot 2^x)+(2^x\cdot 1)-(1\cdot 2^x)-(1\cdot 1) && \text{(Expand)}\\ =&2^{2x}+2^x-2^x-1 && \text{(Simplify)}\\ =&2^{2x}-1\\ =&4^x-1\end{aligned}$$

Ex 177: Expand and simplify:

$$(2^x-1)^2 = \boxed{4^x-2^{x+1}+1}$$

Answer:

- **Method 1: Using the Perfect Square Trinomial Identity**
  This expression is in the form $(a-b)^2=a^2-2ab+b^2$, where $a=2^x$ and $b=1$.

$$\begin{aligned}(2^x-1)^2 &= (2^x)^2-2(2^x)(1)+1^2 && \text{(Apply the identity)}\\ &= 2^{2x}-2(2^x)+1 && \text{(Simplify)}\\ &= 4^x-2^{x+1}+1 && \text{(Use exponent laws)}\end{aligned}$$

- **Method 2: Using the Distributive Property**

$$\begin{aligned}&(2^x-1)^2\\ =&(2^x-1)(2^x-1) && \text{(Expand the square)}\\ =&(2^x\cdot 2^x)-(2^x\cdot 1)-(1\cdot 2^x)+(-1\cdot -1) && \text{(Distribute)}\\ =&2^{2x}-2^x-2^x+1 && \text{(Simplify)}\\ =&2^{2x}-2(2^x)+1 && \text{(Combine like terms)}\\ =&4^x-2^{x+1}+1\end{aligned}$$

Ex 178: Expand and simplify:

$$(3^x+3^{-x})^2 = \boxed{9^x+2+9^{-x}}$$

Answer:

- **Method 1: Using the Perfect Square Trinomial Identity**
  This expression is in the form $(a+b)^2=a^2+2ab+b^2$, where $a=3^x$ and $b=3^{-x}$.

$$\begin{aligned}(3^x+3^{-x})^2 &= (3^x)^2+2(3^x)(3^{-x})+(3^{-x})^2 && \text{(Apply the identity)}\\ &= 3^{2x}+2(3^{x-x})+3^{-2x} && \text{(Use exponent laws)}\\ &= 3^{2x}+2(3^0)+3^{-2x} && \text{(Simplify exponent)}\\ &= 3^{2x}+2(1)+3^{-2x} && \text{(Since } 3^0=1\text{)}\\ &= 9^x+2+9^{-x} && \text{(Rewrite the terms)}\end{aligned}$$

- **Method 2: Using the Distributive Property**

$$\begin{aligned}&(3^x+3^{-x})^2\\ =&(3^x+3^{-x})(3^x+3^{-x}) && \text{(Expand the square)}\\ =&(3^x\cdot 3^x)+(3^x\cdot 3^{-x})+(3^{-x}\cdot 3^x)+(3^{-x}\cdot 3^{-x}) && \text{(Distribute)}\\ =&3^{2x}+3^0+3^0+3^{-2x} && \text{(Simplify)}\\ =&9^x+1+1+9^{-x} && \text{(Rewrite terms)}\\ =&9^x+2+9^{-x}\end{aligned}$$

L.4 FACTORIZING EXPONENTIAL EXPRESSIONS

Ex 179: Factorize:

$$2^{2x}-2^x = \boxed{2^x(2^x-1)}$$

Answer: **Method: Finding the Common Factor**

$$\begin{aligned}2^{2x}-2^x &= 2^x\cdot 2^x-2^x && \text{(Rewrite the first term)}\\ &= 2^x(2^x-1) && \text{(Factor out the common term } 2^x\text{)}\end{aligned}$$

Ex 180: Factorize:

$$3^{2x}-2\cdot 3^x+1 = \boxed{(3^x-1)^2}$$

Answer: **Method 1: Recognizing a Perfect Square Trinomial**
The expression is in the form $a^2-2ab+b^2$, which factorizes to $(a-b)^2$.
Let $a=3^x$ and $b=1$.

$$\begin{aligned}3^{2x}-2\cdot 3^x+1 &= (3^x)^2-2(3^x)(1)+1^2 && \text{(Identify the pattern)}\\ &= (3^x-1)^2 && \text{(Apply the identity)}\end{aligned}$$

Ex 181: Factorize:

$$(x+1)2^x-2^{x+1} = \boxed{2^x(x-1)}$$

Answer: **Method: Factorization**

$$\begin{aligned}(x+1)2^x-2^{x+1} &= (x+1)2^x-2^x\cdot 2^1 && \text{(Rewrite the second term)}\\ &= 2^x((x+1)-2) && \text{(Factor out the common term } 2^x\text{)}\\ &= 2^x(x-1) && \text{(Simplify the expression in brackets)}\end{aligned}$$

Ex 182: Factorize:

$$4^x - 3\cdot 2^x + 2 = \boxed{(2^x-1)(2^x-2)}$$

Answer:

$$\begin{aligned}4^x - 3\cdot 2^x + 2 &= \left(2^2\right)^x - 3\cdot 2^x + 2\\ &= \left(2^x\right)^2 - 3\cdot 2^x + 2\\ &= \left(2^x-1\right)\left(2^x-2\right)\end{aligned}$$

We know that the quadratic $X^2 - 3X + 2$ factorizes as $(X-1)(X-2)$ with $X = 2^x$.

M THE EXPONENTIAL NUMBER e

M.1 SIMPLIFYING USING EXPONENT LAWS

Ex 183: Simplify:

$$e^{x-1} \times e^{x+1} = \boxed{e^{2x}}$$

Answer: **Method: Using the Product Rule for Exponents**

$$\begin{aligned}e^{x-1} \times e^{x+1} &= e^{(x-1)+(x+1)} && \text{(Apply the product rule } a^m\cdot a^n = a^{m+n}\text{)}\\ &= e^{2x} && \text{(Simplify the exponent)}\end{aligned}$$

Ex 184: Simplify:

$$\frac{e^{x+2}}{e} = \boxed{e^{x+1}}$$

Answer: **Method: Using Exponent Laws**

$$\begin{aligned}\frac{e^{x+2}}{e} &= \frac{e^{x+2}}{e^1} && \text{(Rewrite the denominator)}\\ &= e^{(x+2)-1} && \text{(Apply the quotient rule } \tfrac{a^m}{a^n} = a^{m-n}\text{)}\\ &= e^{x+1} && \text{(Simplify the exponent)}\end{aligned}$$

Ex 185: Simplify:

$$\frac{\left(e^2\right)^{x+1}}{e^x} = \boxed{e^{x+2}}$$

Answer: **Method: Using Exponent Laws**

$$\begin{aligned}\frac{\left(e^2\right)^{x+1}}{e^x} &= \frac{e^{2(x+1)}}{e^x} && \text{(Apply the power rule } (a^m)^n = a^{mn}\text{)}\\ &= \frac{e^{2x+2}}{e^x} && \text{(Distribute in the exponent)}\\ &= e^{(2x+2)-x} && \text{(Apply the quotient rule } \tfrac{a^m}{a^n} = a^{m-n}\text{)}\\ &= e^{x+2}\end{aligned}$$

Ex 186: Simplify:

$$\left(e^x \cdot e^{2x}\right)^3 = \boxed{e^{9x}}$$

Answer: **Method: Using Exponent Laws**

$$\begin{aligned}\left(e^x \cdot e^{2x}\right)^3 &= \left(e^{x+2x}\right)^3 && \text{(Apply the product rule inside the brackets)}\\ &= \left(e^{3x}\right)^3 && \text{(Simplify the exponent)}\\ &= e^{3x\cdot 3} && \text{(Apply the power rule } (a^m)^n = a^{mn}\text{)}\\ &= e^{9x}\end{aligned}$$

M.2 SIMPLIFYING EXPONENTIAL EXPRESSIONS

Ex 187: Simplify:

$$\frac{e^{2x}+e^x}{e^x} = \boxed{e^x+1}$$

Answer:

- **Method 1: Splitting the Fraction**

$$\begin{aligned}\frac{e^{2x}+e^x}{e^x} &= \frac{e^{2x}}{e^x} + \frac{e^x}{e^x}\\ &= e^{2x-x}+1\\ &= e^x+1\end{aligned}$$

- **Method 2: Factorization**

$$\begin{aligned}\frac{e^{2x}+e^x}{e^x} &= \frac{e^x(e^x+1)}{e^x}\\ &= e^x+1\end{aligned}$$

Ex 188: Simplify:

$$\frac{e^{x+1}-e^x}{e^x} = \boxed{e-1}$$

Answer:

- **Method 1: Splitting the Fraction**

$$\begin{aligned}\frac{e^{x+1}-e^x}{e^x} &= \frac{e^{x+1}}{e^x} - \frac{e^x}{e^x}\\ &= e^{(x+1)-x}-1\\ &= e^1-1\\ &= e-1\end{aligned}$$

- **Method 2: Factorization**

$$\begin{aligned}\frac{e^{x+1}-e^x}{e^x} &= \frac{e^x\cdot e^1-e^x}{e^x}\\ &= \frac{e^x(e-1)}{e^x}\\ &= e-1\end{aligned}$$

Ex 189: Simplify:

$$\frac{e^{2x}-1}{e^x-1} = \boxed{e^x+1}$$

Answer: **Method: Factorizing the Numerator**

The numerator $e^{2x}-1$ is a difference of squares, since $e^{2x} = (e^x)^2$. It can be factorized as $(e^x-1)(e^x+1)$.

$$\begin{aligned}\frac{e^{2x}-1}{e^x-1} &= \frac{(e^x-1)(e^x+1)}{e^x-1}\\ &= e^x+1\end{aligned}$$

M.3 EXPANDING AND SIMPLIFYING EXPONENTIAL EXPRESSIONS

Ex 190: Expand and simplify:

$$(e^x-1)(e^x+1)=\boxed{e^{2x}-1}$$

Answer: Using the identity $(a-b)(a+b)=a^2-b^2$:

$$\begin{aligned}(e^x-1)(e^x+1)&=(e^x)^2-1^2\\&=e^{2x}-1\end{aligned}$$

Ex 191: Expand and simplify:

$$(e^x+e^{-x})^2=\boxed{e^{2x}+2+e^{-2x}}$$

Answer: Using the identity $(a+b)^2=a^2+2ab+b^2$:

$$\begin{aligned}(e^x+e^{-x})^2&=(e^x)^2+2(e^x)(e^{-x})+(e^{-x})^2\\&=e^{2x}+2e^{x-x}+e^{-2x}\\&=e^{2x}+2e^0+e^{-2x}\\&=e^{2x}+2+e^{-2x}\end{aligned}$$

Ex 192: Expand and simplify:

$$(e^x-e^{-x})^2=\boxed{e^{2x}-2+e^{-2x}}$$

Answer: Using the identity $(a-b)^2=a^2-2ab+b^2$:

$$\begin{aligned}(e^x-e^{-x})^2&=(e^x)^2-2(e^x)(e^{-x})+(e^{-x})^2\\&=e^{2x}-2e^{x-x}+e^{-2x}\\&=e^{2x}-2e^0+e^{-2x}\\&=e^{2x}-2+e^{-2x}\end{aligned}$$

Ex 193: Expand and simplify:

$$(e^x+2)(e^x-3)=\boxed{e^{2x}-e^x-6}$$

Answer: Using the distributive property:

$$\begin{aligned}(e^x+2)(e^x-3)&=(e^x)(e^x)-3(e^x)+2(e^x)-(2)(3)\\&=e^{2x}-3e^x+2e^x-6\\&=e^{2x}-e^x-6\end{aligned}$$

M.4 FACTORIZING EXPONENTIAL EXPRESSIONS

Ex 194: Factorize:

$$e^{2x}-e^x=\boxed{e^x(e^x-1)}$$

Answer: **Method: Finding the Common Factor**

$$\begin{aligned}e^{2x}-e^x&=e^x\cdot e^x-e^x &&\text{(Rewrite the first term)}\\&=e^x(e^x-1) &&\text{(Factor out the common term } e^x\text{)}\end{aligned}$$

Ex 195: Factorize:

$$e^{2x}-2e^x+1=\boxed{(e^x-1)^2}$$

Answer: **Method: Recognizing a Perfect Square Trinomial**
The expression is in the form $a^2-2ab+b^2$, which factorizes to $(a-b)^2$.
Let $a=e^x$ and $b=1$.

$$\begin{aligned}e^{2x}-2e^x+1&=(e^x)^2-2(e^x)(1)+1^2 &&\text{(Identify the pattern)}\\&=(e^x-1)^2 &&\text{(Apply the identity)}\end{aligned}$$

Ex 196: Factorize:

$$xe^x-e^{x+1}=\boxed{e^x(x-e)}$$

Answer: **Method: Factorization**

$$\begin{aligned}xe^x-e^{x+1}&=xe^x-e^x\cdot e^1 &&\text{(Rewrite the second term)}\\&=e^x(x-e) &&\text{(Factor out the common term } e^x\text{)}\end{aligned}$$

Ex 197: Factorize:

$$e^{2x}-3e^x+2=\boxed{(e^x-1)(e^x-2)}$$

Answer: **Method: Factorizing a Quadratic in e^x**
The expression is a quadratic in terms of e^x. We can factor it directly.

$$\begin{aligned}e^{2x}-3e^x+2&=(e^x)^2-3(e^x)+2 &&\text{(Rewrite the first term)}\\&=(e^x-1)(e^x-2) &&\text{(Factorize } X^2-3X+2=(X-1)(X-2)\text{)}\end{aligned}$$

N EXPONENTIAL EQUATIONS

N.1 SOLVING BY EQUATING INDICES: LEVEL 1

Ex 198: Solve for x: $2^x=16$

Answer:

$$\begin{aligned}&2^x=16\\\Leftrightarrow\ &2^x=2^4\\\Leftrightarrow\ &x=4\end{aligned}$$

Ex 199: Solve for x: $3^x=27$

Answer:

$$\begin{aligned}&3^x=27\\\Leftrightarrow\ &3^x=3^3\\\Leftrightarrow\ &x=3\end{aligned}$$

Ex 200: Solve for x: $2^x=\frac{1}{4}$

Answer:

$$\begin{aligned}&2^x=\frac{1}{4}\\\Leftrightarrow\ &2^x=2^{-2}\\\Leftrightarrow\ &x=-2\end{aligned}$$

Ex 201: Solve for x: $5^x=\sqrt{5}$

Answer:

$$\begin{aligned}&5^x=\sqrt{5}\\\Leftrightarrow\ &5^x=5^{\frac{1}{2}}\\\Leftrightarrow\ &x=\frac{1}{2}\end{aligned}$$

Ex 202: Solve for x: $e^x=1$

Answer:

$$\begin{aligned}&e^x=1\\\Leftrightarrow\ &e^x=e^0\\\Leftrightarrow\ &x=0\end{aligned}$$

N.2 SOLVING BY EQUATING INDICES: LEVEL 2

Ex 203: Solve for x: $3^{x-2} = 81$

Answer:

$$\begin{aligned} & 3^{x-2} = 81 \\ \Leftrightarrow\ & 3^{x-2} = 3^4 \\ \Leftrightarrow\ & x - 2 = 4 \\ \Leftrightarrow\ & x = 6 \end{aligned}$$

Ex 204: Solve for x: $5 \cdot 2^x = 40$

Answer:

$$\begin{aligned} & 5 \cdot 2^x = 40 \\ \Leftrightarrow\ & 2^x = 8 \\ \Leftrightarrow\ & 2^x = 2^3 \\ \Leftrightarrow\ & x = 3 \end{aligned}$$

Ex 205: Solve for x: $4^x = 32$

Answer:

$$\begin{aligned} & 4^x = 32 \\ \Leftrightarrow\ & (2^2)^x = 2^5 \\ \Leftrightarrow\ & 2^{2x} = 2^5 \\ \Leftrightarrow\ & 2x = 5 \\ \Leftrightarrow\ & x = \frac{5}{2} \end{aligned}$$

Ex 206: Solve for x: $e^{2x} = e^x$

Answer:

$$\begin{aligned} & e^{2x} = e^x \\ \Leftrightarrow\ & 2x = x \\ \Leftrightarrow\ & 2x - x = 0 \\ \Leftrightarrow\ & x = 0 \end{aligned}$$

N.3 SOLVING BY EQUATING INDICES: LEVEL 3

Ex 207: Solve for x: $4^{x+1} = 8^{2x-2}$

Answer:

$$\begin{aligned} & 4^{x+1} = 8^{2x-2} \\ \Leftrightarrow\ & (2^2)^{x+1} = (2^3)^{2x-2} \\ \Leftrightarrow\ & 2^{2(x+1)} = 2^{3(2x-2)} \\ \Leftrightarrow\ & 2(x+1) = 3(2x-2) \\ \Leftrightarrow\ & 2x + 2 = 6x - 6 \\ \Leftrightarrow\ & 8 = 4x \\ \Leftrightarrow\ & x = 2 \end{aligned}$$

Ex 208: Solve for x: $3^{2x+1} = 27 \cdot 3^{x-1}$

Answer:

$$\begin{aligned} & 3^{2x+1} = 27 \cdot 3^{x-1} \\ \Leftrightarrow\ & 3^{2x+1} = 3^3 \cdot 3^{x-1} \\ \Leftrightarrow\ & 3^{2x+1} = 3^{3+(x-1)} \\ \Leftrightarrow\ & 3^{2x+1} = 3^{x+2} \\ \Leftrightarrow\ & 2x + 1 = x + 2 \\ \Leftrightarrow\ & x = 1 \end{aligned}$$

Ex 209: Solve for x: $2^{x^2} = 4^x$

Answer:

$$\begin{aligned} & 2^{x^2} = 4^x \\ \Leftrightarrow\ & 2^{x^2} = (2^2)^x \\ \Leftrightarrow\ & 2^{x^2} = 2^{2x} \\ \Leftrightarrow\ & x^2 = 2x \\ \Leftrightarrow\ & x^2 - 2x = 0 \\ \Leftrightarrow\ & x(x-2) = 0 \end{aligned}$$

This gives two possible solutions: $x = 0$ or $x = 2$.

N.4 SOLVING EQUATIONS IN QUADRATIC FORM

Ex 210: Solve for x: $4^x + 2^x - 20 = 0$

Answer:

$$\begin{aligned} & 4^x + 2^x - 20 = 0 \\ \Leftrightarrow & (2^x)^2 + 2^x - 20 = 0 && \text{(let } u=2^x\text{, we have } u^2+u-20=0) \\ \Leftrightarrow & (2^x - 4)(2^x + 5) = 0 && \text{(since } u^2+u-20=(u-4)(u+5)) \\ \Leftrightarrow & 2^x = 4 \quad \text{or} \quad 2^x = -5 \\ \Leftrightarrow & 2^x = 2^2 && \text{(since } 2^x>0\text{, so } 2^x=-5 \text{ is impossible)} \\ \Leftrightarrow & x = 2 \end{aligned}$$

Ex 211: Solve for x: $e^{2x} - 2e^x + 1 = 0$

Answer:

$$\begin{aligned} & e^{2x} - 2e^x + 1 = 0 \\ \Leftrightarrow & (e^x)^2 - 2(e^x) + 1 = 0 && \text{(let } u=e^x\text{, we have } u^2-2u+1=0) \\ \Leftrightarrow & (e^x - 1)^2 = 0 && \text{(since } u^2-2u+1=(u-1)^2) \\ \Leftrightarrow & e^x - 1 = 0 \\ \Leftrightarrow & e^x = 1 \\ \Leftrightarrow & e^x = e^0 && \text{(since } e^0=1) \\ \Leftrightarrow & x = 0 \end{aligned}$$

Ex 212: Solve for x: $e^{2x} + e^x - 2 = 0$

Answer:

$$\begin{aligned} & e^{2x} + e^x - 2 = 0 \\ \Leftrightarrow & (e^x)^2 + e^x - 2 = 0 && \text{(let } u=e^x\text{, we have } u^2+u-2=0) \\ \Leftrightarrow & (e^x + 2)(e^x - 1) = 0 && \text{(since } u^2+u-2=(u+2)(u-1)) \\ \Leftrightarrow & e^x = -2 \quad \text{or} \quad e^x = 1 \\ \Leftrightarrow & e^x = e^0 && \text{(since } e^x>0\text{, so } e^x=-2 \text{ is impossible)} \\ \Leftrightarrow & x = 0 \end{aligned}$$