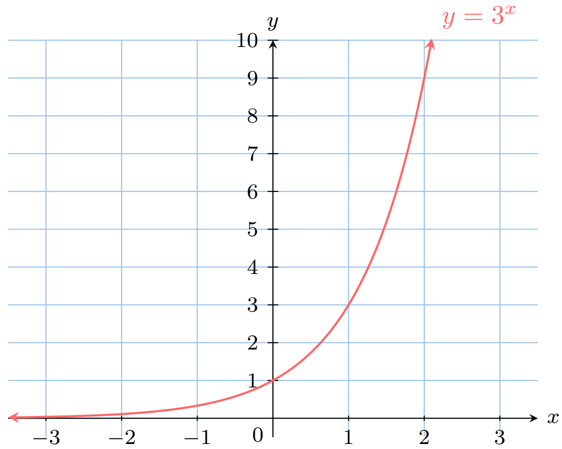
EXPONENTIAL FUNCTIONS

A EXPONENTIAL FUNCTIONS

A.1 READING AND SKETCHING EXPONENTIAL FUNCTIONS

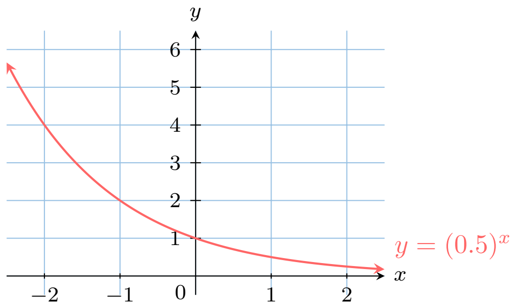
Ex 1:



By reading from the graph of $y = 3^x$, complete the following inequalities with consecutive integers:

1. $\square \leq 3^{1.5} < \square$
2. $\square \leq 3^{0.5} < \square$

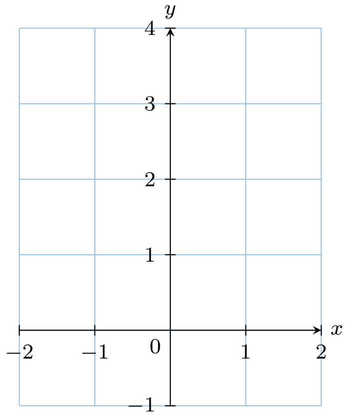
Ex 2:



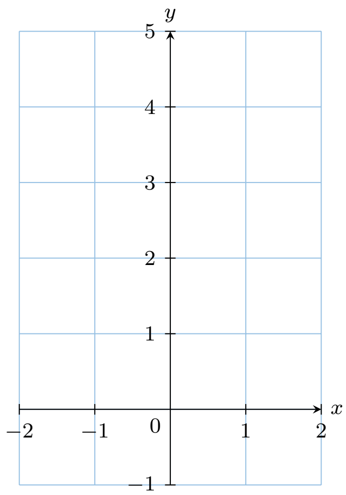
By reading from the graph of $y = (0.5)^x$, complete the following inequalities with consecutive integers:

1. $\square \leq (0.5)^{-0.5} < \square$
2. $\square \leq (0.5)^{-1.5} < \square$

Ex 3: Sketch the graph of the function $f(x) = 2^x$. Use a table of values for integer values of x from -2 to 2 to help you plot key points.



Ex 4: Sketch the graph of the function $f(x) = \left(\frac{1}{2}\right)^x$. Use a table of values for integer values of x from -2 to 2 to help you plot key points.



A.2 EVALUATING EXPONENTIAL FUNCTIONS

Ex 5: For $f(x) = 3^x$, evaluate:

1. $f(2) = \square$
2. $f(0) = \square$
3. $f(-1) = \square$

Ex 6: For $f(x) = 10^x$, evaluate:

1. $f(2) = \square$
2. $f(0) = \square$
3. $f(-1) = \square$

Ex 7: For $f(x) = \left(\frac{1}{2}\right)^x$, evaluate:

1. $f(-2) = \square$
2. $f(-1) = \square$
3. $f(0) = \square$
4. $f(1) = \square$

B NATURAL EXPONENTIAL FUNCTION e^x

B.1 CALCULATING WITH THE NATURAL EXPONENTIAL FUNCTION

Ex 8: Using your calculator, evaluate the following values of $f(x) = e^x$. Round your answers to 2 decimal places.

1. $e^1 \approx$ ☐
2. $e^2 \approx$ ☐
3. $e^{-1} \approx$ ☐
4. $\sqrt{e} \approx$ ☐

Ex 9: Simplify the following expressions using the laws of exponents.

1. $e^x \cdot e^2 =$ ☐
2. $\frac{e^{3x}}{e^x} =$ ☐
3. $(e^2)^x =$ ☐

Ex 10: Solve the following equations for x without using a calculator.

1. $e^x = 1 \implies x =$ ☐
2. $e^x = e^5 \implies x =$ ☐
3. $e^x = -3 \implies$ ☐ $x = 0$ ☐ $x = -3$ ☐ No solution

B.2 GRAPHING AND PROPERTIES OF THE NATURAL EXPONENTIAL FUNCTION

Ex 11: Based on the properties of the natural exponential function $y = e^x$:

1. What are the coordinates of the y-intercept?

   (☐;☐)

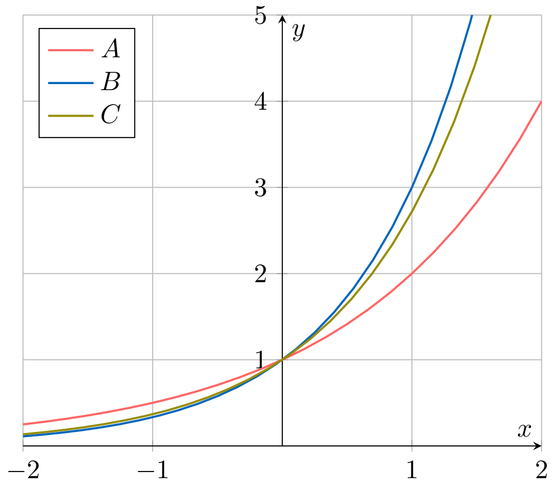
2. What is the equation of the horizontal asymptote?

   $y =$ ☐

3. Is the function strictly increasing or decreasing?

   ☐ Increasing
   ☐ Decreasing

Ex 12: The graphs of $y = 2^x$, $y = 3^x$, and $y = e^x$ are plotted below. Identify which curve corresponds to $y = e^x$.



The curve representing $y = e^x$ is: ☐ A ☐ B ☐ C

Ex 13: Consider the function $g(x) = e^x - 2$.

1. Calculate the y-intercept of the graph of g.

   $y =$ ☐

2. Determine the equation of the horizontal asymptote of g.

   $y =$ ☐

C TRANSFORMATIONS OF EXPONENTIAL FUNCTIONS

C.1 CALCULATING $f(x)$

Ex 14: For $f : x \mapsto 3 \cdot 2^x$, find in simplest form:

1. $f(0) =$ ☐
2. $f(2) =$ ☐
3. $f(-1) =$ ☐

Ex 15: For $f : x \mapsto 5 \cdot e^x$, find in simplest form:

1. $f(0) =$ ☐
2. $f(1) =$ ☐
3. $f(\ln 2) =$ ☐

Ex 16: For $f : x \mapsto \left(\frac{1}{3}\right)^x + 1$, find in simplest form:

1. $f(0) =$ ☐
2. $f(2) =$ ☐
3. $f(-2) =$ ☐

C.2 FINDING $f(g(x))$

Ex 17: For the function $f(x) = x + 1$ and $g(x) = 4^x$, find and simplify:

$$(f \circ g)(x) = \square$$

Ex 18: For the function $f(x) = x + 1$ and $g(x) = 4^x$, find and simplify:

$$(g \circ f)(x) = \square$$

Ex 19: For the function $f(x) = 3x$ and $g(x) = e^x$, find and simplify:

$$(f \circ g)(x) = \square$$

Ex 20: For the function $f(x) = 3x$ and $g(x) = e^x$, find and simplify:

$$(g \circ f)(x) = \square$$

Ex 21: For the function $f(x) = x - 2$ and $g(x) = 5^x$, find and simplify:

$$(f \circ g)(x) = \square$$

Ex 22: For the function $f(x) = x - 2$ and $g(x) = 5^x$, find and simplify:

$$(g \circ f)(x) = \square$$

C.3 DESCRIBING TRANSFORMATIONS OF EXPONENTIAL GRAPHS

Ex 23: Describe the sequence of transformations that maps the graph of $y = 2^x$ to the graph of $f(x) = 2^{x+1} + 4$.

Ex 24: Describe the transformation that maps the graph of $y = 2^x$ to the graph of $f(x) = 2^{-x}$.

Ex 25: Describe the transformation that maps the graph of $y = e^x$ to the graph of $f(x) = e^{2x}$.

Ex 26: Describe the sequence of transformations that maps the graph of $y = 2^x$ to the graph of $f(x) = 3 \cdot 2^x - 1$.

D EXPONENTIAL MODELS

D.1 MODELING REAL-WORLD SITUATIONS WITH EXPONENTIAL FUNCTIONS

Ex 27: A population of bacteria doubles every second. At time $x = 0$, there is a single bacterium.
Find the function to model this growth.

$$P(x) = \square$$

Ex 28: A species of bear is introduced to a large island off Alaska where previously there were no bears. 6 pairs of bears were introduced in 1998. It is expected that the population will increase according to $B(t) = B_0 \times (1.13)^t$ where t is the time, in years, since the introduction.

1. Find B_0.

   $\square$ bears

2. Find the expected bear population in 2018.

   $\square$ bears (round to the nearest integer)

3. Find the expected percentage increase in population from 1998 to 2018.

   $\square$% (round to the nearest ten)

Ex 29: Sarah buys a piece of artwork for \$1500 that is expected to appreciate (increase in value) by 8% each year.

1. Determine a model for A_n, the value of the artwork after n years.

   $A_n = \square$

2. Is this an example of exponential growth?
   ☐ *Yes*
   ☐ *No*
3. Calculate the estimated value of the artwork in 6 years' time.

$ ☐ (round to the nearest integer)

Ex 30: Maxime has an Uncle Scrooge coin worth \$500. Each year, the coin's value increases by 20%.

1. Determine a model for C_n, the value of the coin after n years.

   $C_n =$ ☐

2. Is this an example of exponential growth?
   ☐ *Yes*
   ☐ *No*
3. Calculate the estimated value of the coin in 6 years' time.

   $ ☐ (round to the nearest integer)

Ex 31: A certain radioactive substance loses 12% of its mass each year. Initially, the sample weighs 200 g.

1. Determine a model for M_n, the mass (in grams) remaining after n years.

   $M_n =$ ☐

2. Is this an example of exponential decay?
   ☐ *Yes*
   ☐ *No*
3. Calculate the mass remaining after 10 years.

   ☐ g (round to the nearest integer)

D.2 SOLVING PROBLEMS USING EXPONENTIAL MODELS

Ex 32: The temperature, T, in degrees Celsius (°C), of a cup of coffee t minutes after it is poured is modelled by the function:

$$T(t) = 22 + 70e^{-kt}$$

where k is a positive constant.

1. Find the initial temperature of the coffee.
2. The temperature of the coffee is 65°C after 5 minutes. Find the value of k.
3. Find the temperature of the coffee after 15 minutes.
4. Find the rate at which the temperature of the coffee is decreasing at $t = 10$ minutes.
5. State the temperature of the room, giving a reason for your answer.

Ex 33: The concentration of a drug in a patient's bloodstream, C, in milligrams per litre (mg/L), is modelled by the function:

$$C(t) = 80(e^{-0.2t} - e^{-1.5t})$$

where t is the time in hours after the drug was administered.

1. Find the concentration of the drug in the bloodstream after 2 hours.
2. Use your graphing display calculator to find the maximum concentration of the drug and the time at which it occurs.
3. Find the rate of change of the drug's concentration at $t = 4$ hours. Interpret the meaning of your answer.
4. Determine the long-term concentration of the drug in the bloodstream, justifying your answer.