EXPANSION OF ALGEBRAIC EXPRESSIONS

In algebra, expressions can be written in different forms. A **factored form** represents an expression as a product, like $a(b+c)$. An **expanded form** represents it as a sum of terms, like $ab + ac$.
Expanding is the algebraic process of converting a factored form into an expanded form. In practice, this means *removing the brackets by multiplying* the factor outside by each term inside. This is a fundamental skill used for simplifying expressions and solving equations.

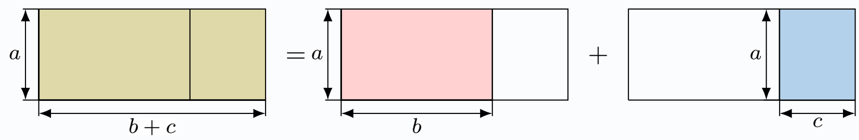
A DISTRIBUTIVE LAW 1

Proposition Distributive Law

Multiplication is distributive over addition and subtraction:

- **Addition:**

$$a(b+c) = ab + ac$$



- **Subtraction:**

$$a(b-c) = ab - ac$$

Ex: Show that $2(\ell + L) = 2\ell + 2L$.

Answer:

$$\begin{aligned} 2(\ell + L) &= 2\times\ell + 2\times L \\ &= 2\ell + 2L \end{aligned}$$

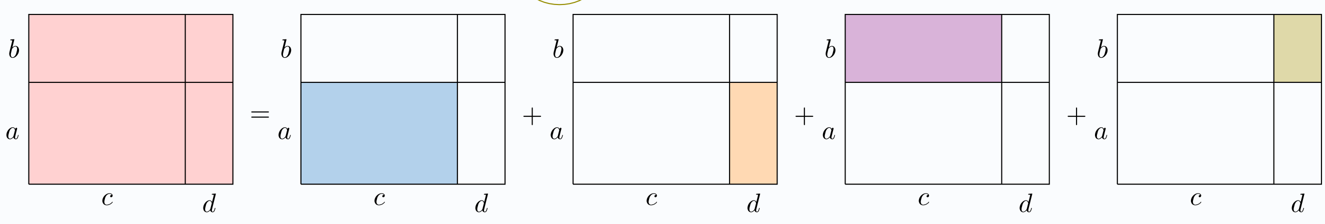
So $2(\ell + L) = 2\ell + 2L$.

B DISTRIBUTIVE LAW 2

Proposition Distributive Law 2

Each term in the first bracket multiplies each term in the second bracket.

$$(a+b)\cdot(c+d) = ac + ad + bc + bd$$



$$(a+b)(c+d) = ac + ad + bc + bd$$

Ex: Expand and simplify $(x+4)(2x+2)$.

Answer:

$$\begin{aligned} (x+4)\cdot(2x+2) &= x\times 2x + x\times 2 + 4\times 2x + 4\times 2 \\ &= 2x^2 + 2x + 8x + 8 \\ &= 2x^2 + 10x + 8 \end{aligned}$$

So $(x+4)(2x+2) = 2x^2 + 10x + 8$.

C DIFFERENCE OF TWO SQUARES

Proposition Difference of Two Squares

$$(a-b)(a+b)=a^2-b^2$$

This identity is called the **difference of two squares**.

Proof

$$\begin{aligned}(a-b)(a+b) &= a(a+b)-b(a+b) && \text{(distributive law)}\\ &= a^2+ab-ab-b^2 && \text{(expanding)}\\ &= a^2+\cancel{ab}-\cancel{ab}-b^2\\ &= a^2-b^2.\end{aligned}$$

Ex: Expand and simplify: $(x-3)(x+3)$.

Answer:

$$\begin{aligned}(x-3)(x+3) &= x^2-3^2\\ &= x^2-9.\end{aligned}$$

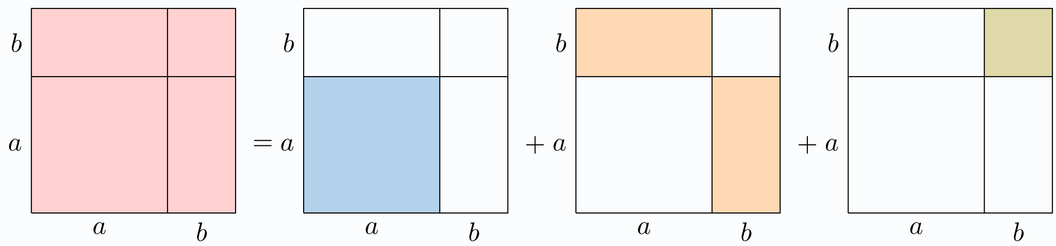
So $(x-3)(x+3)=x^2-9$.

D PERFECT SQUARES EXPANSION

Proposition Perfect Squares Expansion

The square of a sum and the square of a difference can be written as:

$$(a+b)^2=a^2+2ab+b^2 \quad \text{and} \quad (a-b)^2=a^2-2ab+b^2.$$



$(a+b)^2 \quad = \quad a^2 \quad + \quad 2ab \quad + \quad b^2$

Proof

$$\begin{aligned}(a+b)^2 &= (a+b)(a+b) && \text{(definition of a square)}\\ &= a(a+b)+b(a+b) && \text{(distributive law)}\\ &= a^2+ab+ab+b^2 && \text{(expanding)}\\ &= a^2+2ab+b^2 && \text{(combining like terms).}\end{aligned}$$

Similarly,

$$\begin{aligned}(a-b)^2 &= (a-b)(a-b) && \text{(definition of a square)}\\ &= a(a-b)-b(a-b) && \text{(distributive law)}\\ &= a^2-ab-ab+b^2 && \text{(expanding)}\\ &= a^2-2ab+b^2 && \text{(combining like terms).}\end{aligned}$$

Ex: Expand and simplify $(x+2)^2$.

Answer: Using the formula $(a+b)^2=a^2+2ab+b^2$ with $a=x$ and $b=2$:

$$\begin{aligned}(x+2)^2 &= x^2+2\times x\times 2+2^2\\ &= x^2+4x+4.\end{aligned}$$

So $(x+2)^2=x^2+4x+4$.