

EXPANSION

A DISTRIBUTIVE LAW 1

A.1 EXPANDING WITH ADDITION: LEVEL 1

Ex 1: Expand and simplify:

$$5(x + 3) = \boxed{5x + 15}$$

Answer:

$$\begin{array}{l} \textcolor{blue}{5}(\textcolor{blue}{x} + \textcolor{red}{3}) = \textcolor{blue}{5} \times \textcolor{blue}{x} + \textcolor{blue}{5} \times \textcolor{red}{3} \\ = \textcolor{blue}{5x} + \textcolor{red}{15} \end{array}$$

Ex 2: Expand and simplify:

$$2(3 + x) = \boxed{6 + 2x}$$

Answer:

$$\begin{array}{l} 2(\textcolor{blue}{3} + \textcolor{blue}{x}) = \textcolor{blue}{2} \times \textcolor{blue}{3} + \textcolor{blue}{2} \times \textcolor{blue}{x} \\ = \textcolor{blue}{6} + \textcolor{blue}{2x} \end{array}$$

Ex 3: Expand and simplify:

$$3(2x + 2) = \boxed{6x + 6}$$

Answer:

$$\begin{array}{l} 3(\textcolor{blue}{2x} + \textcolor{red}{2}) = \textcolor{blue}{3} \times \textcolor{blue}{2x} + \textcolor{blue}{3} \times \textcolor{red}{2} \\ = \textcolor{blue}{6x} + \textcolor{red}{6} \end{array}$$

Ex 4: Expand and simplify:

$$2(5 + 3x) = \boxed{10 + 6x}$$

Answer:

$$\begin{array}{l} 2(\textcolor{blue}{5} + \textcolor{blue}{3x}) = \textcolor{blue}{2} \times \textcolor{blue}{5} + \textcolor{blue}{2} \times \textcolor{blue}{3x} \\ = \textcolor{blue}{10} + \textcolor{blue}{6x} \end{array}$$

A.2 EXPANDING WITH ADDITION: LEVEL 2

Ex 5: Expand and simplify:

$$x(x + 1) = \boxed{x^2 + x}$$

Answer:

$$\begin{array}{l} \textcolor{blue}{x}(\textcolor{blue}{x} + \textcolor{blue}{1}) = \textcolor{blue}{x} \times \textcolor{blue}{x} + \textcolor{blue}{x} \times \textcolor{blue}{1} \\ = \textcolor{blue}{x^2} + \textcolor{blue}{x} \end{array}$$

Ex 6: Expand and simplify:

$$x(2x + 3) = \boxed{2x^2 + 3x}$$

Answer:

$$\begin{array}{l} \textcolor{blue}{x}(\textcolor{blue}{2x} + \textcolor{blue}{3}) = \textcolor{blue}{x} \times \textcolor{blue}{2x} + \textcolor{blue}{x} \times \textcolor{blue}{3} \\ = \textcolor{blue}{2x^2} + \textcolor{blue}{3x} \end{array}$$

Ex 7: Expand and simplify:

$$2x(x + 2) = \boxed{2x^2 + 4x}$$

Answer:

$$\begin{array}{l} 2\textcolor{blue}{x}(\textcolor{blue}{x} + \textcolor{blue}{2}) = \textcolor{blue}{2x} \times \textcolor{blue}{x} + \textcolor{blue}{2x} \times \textcolor{blue}{2} \\ = \textcolor{blue}{2x^2} + \textcolor{blue}{4x} \end{array}$$

Ex 8: Expand and simplify:

$$3x(2x + 5) = \boxed{6x^2 + 15x}$$

Answer:

$$\begin{array}{l} 3\textcolor{blue}{x}(\textcolor{blue}{2x} + \textcolor{blue}{5}) = \textcolor{blue}{3x} \times \textcolor{blue}{2x} + \textcolor{blue}{3x} \times \textcolor{blue}{5} \\ = \textcolor{blue}{6x^2} + \textcolor{blue}{15x} \end{array}$$

A.3 EXPANDING WITH ADDITION: LEVEL 3

Ex 9: Expand and simplify:

$$2(x + 1) + x = \boxed{3x + 2}$$

Answer:

$$\begin{array}{l} 2(x + 1) + x = \textcolor{red}{2} \times \textcolor{blue}{x} + \textcolor{blue}{2} \times \textcolor{blue}{1} + x \quad (\text{expanding}) \\ = \textcolor{red}{2x} + \textcolor{blue}{2} + x \\ = (2 + 1)x + 2 \quad (\text{combining}) \\ = 3x + 2 \quad (\text{simplifying}) \end{array}$$

Ex 10: Expand and simplify:

$$2(2x + 3) - 3x = \boxed{x + 6}$$

Answer:

$$\begin{array}{l} 2(2x + 3) - 3x = \textcolor{red}{2} \times \textcolor{blue}{2x} + \textcolor{blue}{2} \times \textcolor{blue}{3} - 3x \quad (\text{expanding}) \\ = \textcolor{red}{4x} + \textcolor{blue}{6} - 3x \\ = (4 - 3)x + 6 \quad (\text{combining}) \\ = x + 6 \quad (\text{simplifying}) \end{array}$$

Ex 11: Expand and simplify:

$$x(x + 2) - x^2 = \boxed{2x}$$

Answer:

$$\begin{array}{l} x(x + 2) - x^2 = \textcolor{red}{x} \times \textcolor{blue}{x} + \textcolor{blue}{x} \times \textcolor{blue}{2} - x^2 \quad (\text{expanding}) \\ = \textcolor{red}{x^2} + \textcolor{blue}{2x} - x^2 \\ = 2x \quad (\text{combining}) \end{array}$$

Ex 12: Expand and simplify:

$$2x(3x + 2) - 8x = \boxed{6x^2 - 4x}$$

Answer:

$$\begin{array}{l} 2x(3x + 2) - 8x = \textcolor{red}{2x} \times \textcolor{blue}{3x} + \textcolor{blue}{2x} \times \textcolor{blue}{2} - 8x \quad (\text{expanding}) \\ = \textcolor{red}{6x^2} + \textcolor{blue}{4x} - 8x \\ = 6x^2 + (4 - 8)x \quad (\text{combining}) \\ = 6x^2 - 4x \quad (\text{simplifying}) \end{array}$$

A.4 EXPANDING WITH SUBTRACTION: LEVEL 1

Ex 13: Expand and simplify:

$$2(x - 2) = \boxed{2x - 4}$$

Answer:

$$2(x - 2) = 2 \times x - 2 \times 2 = 2x - 4$$

Ex 14: Expand and simplify:

$$3(5x - 6) = \boxed{15x - 18}$$

Answer:

$$3(5x - 6) = 3 \times 5x - 3 \times 6 = 15x - 18$$

Ex 15: Expand and simplify:

$$2(3 - x) = \boxed{6 - 2x}$$

Answer:

$$2(3 - x) = 2 \times 3 - 2 \times x = 6 - 2x$$

Ex 16: Expand and simplify:

$$4(3 - 5x) = \boxed{12 - 20x}$$

Answer:

$$4(3 - 5x) = 4 \times 3 - 4 \times 5x = 12 - 20x$$

A.5 EXPANDING WITH SUBTRACTION: LEVEL 2

Ex 17: Expand and simplify:

$$x(x - 1) = \boxed{x^2 - x}$$

Answer:

$$x(x - 1) = x \times x - x \times 1 = x^2 - x$$

Ex 18: Expand and simplify:

$$x(2x - 3) = \boxed{2x^2 - 3x}$$

Answer:

$$x(2x - 3) = x \times 2x - x \times 3 = 2x^2 - 3x$$

Ex 19: Expand and simplify:

$$2x(x - 2) = \boxed{2x^2 - 4x}$$

Answer:

$$2x(x - 2) = 2x \times x - 2x \times 2 = 2x^2 - 4x$$

Ex 20: Expand and simplify:

$$3x(2x - 5) = \boxed{6x^2 - 15x}$$

Answer:

$$3x(2x - 5) = 3x \times 2x - 3x \times 5 = 6x^2 - 15x$$

A.6 EXPANDING WITH SUBTRACTION: LEVEL 3

Ex 21: Expand and simplify:

$$2(x - 2) + 4 = \boxed{2x}$$

Answer:

$$\begin{aligned} 2(x - 2) + 4 &= 2 \times x - 2 \times 2 + 4 \quad (\text{expanding}) \\ &= 2x - 4 + 4 \\ &= 2x + 0 \quad (\text{simplifying}) \\ &= 2x \end{aligned}$$

Ex 22: Expand and simplify:

$$4(x - 3) - 5x = \boxed{-x - 12}$$

Answer:

$$\begin{aligned} 4(x - 3) - 5x &= 4 \times x - 4 \times 3 - 5x \quad (\text{expanding}) \\ &= 4x - 12 - 5x \\ &= (4 - 5)x - 12 \quad (\text{combining like terms}) \\ &= -x - 12 \quad (\text{simplifying}) \end{aligned}$$

Ex 23: Expand and simplify:

$$x(x - 2) + 6 = \boxed{x^2 - 2x + 6}$$

Answer:

$$\begin{aligned} x(x - 2) + 6 &= x \times x - x \times 2 + 6 \quad (\text{expanding}) \\ &= x^2 - 2x + 6 \end{aligned}$$

Ex 24: Expand and simplify:

$$2(x - 2) + 3x - 10 = \boxed{5x - 14}$$

Answer:

$$\begin{aligned} 2(x - 2) + 3x - 10 &= 2 \times x - 2 \times 2 + 3x - 10 \quad (\text{expanding}) \\ &= 2x - 4 + 3x - 10 \\ &= (2 + 3)x - 4 - 10 \quad (\text{combining like terms}) \\ &= 5x - 14 \quad (\text{simplifying}) \end{aligned}$$

B DISTRIBUTIVE LAW 2

B.1 EXPANDING WITH ADDITION

Ex 25: Expand and simplify

$$(x+2)(x+1) = \boxed{x^2 + 3x + 2}$$

Answer:

$$\begin{aligned} (x+2) \cdot (x+1) &= x \times x + x \times 1 + 2 \times x + 2 \times 1 \\ &= x^2 + x + 2x + 2 \\ &= x^2 + 3x + 2 \end{aligned}$$

Ex 26: Expand and simplify

$$(x+3)(x+4) = \boxed{x^2 + 7x + 12}$$

Answer:

$$\begin{aligned} (x+3) \cdot (x+4) &= x \times x + x \times 4 + 3 \times x + 3 \times 4 \\ &= x^2 + 4x + 3x + 12 \\ &= x^2 + 7x + 12 \end{aligned}$$

Ex 27: Expand and simplify

$$(x+4)(2x+2) = \boxed{2x^2 + 10x + 8}$$

Answer:

$$\begin{aligned} (x+4) \cdot (2x+2) &= x \times 2x + x \times 2 + 4 \times 2x + 4 \times 2 \\ &= 2x^2 + 2x + 8x + 8 \\ &= 2x^2 + 10x + 8 \end{aligned}$$

Ex 28: Expand and simplify

$$(2x+1)(3x+2) = \boxed{6x^2 + 7x + 2}$$

Answer:

$$\begin{aligned} (2x+1) \cdot (3x+2) &= 2x \times 3x + 2x \times 2 + 1 \times 3x + 1 \times 2 \\ &= 6x^2 + 4x + 3x + 2 \\ &= 6x^2 + 7x + 2 \end{aligned}$$

Ex 29: Expand and simplify

$$(x+1)^2 = \boxed{x^2 + 2x + 1}$$

Answer:

$$\begin{aligned} (x+1)^2 &= (x+1)(x+1) && \text{(square definition)} \\ &= x \times x + x \times 1 + 1 \times x + 1 \times 1 && \text{(expanding)} \\ &= x^2 + 2x + 1 && \text{(combining)} \end{aligned}$$

B.2 EXPANDING WITH SUBTRACTION

Ex 30: Expand and simplify

$$(x+2)(x-1) = \boxed{x^2 + x - 2}$$

Answer:

$$\begin{aligned} (x+2) \cdot (x-1) &= x \times x + x \times (-1) + 2 \times x + 2 \times (-1) \\ &= x^2 - x + 2x - 2 \\ &= x^2 + x - 2 \end{aligned}$$

Ex 31: Expand and simplify

$$(x-1)(x-2) = \boxed{x^2 - 3x + 2}$$

Answer:

$$\begin{aligned} (x-1) \cdot (x-2) &= x \times x + x \times (-2) + (-1) \times x + (-1) \times (-2) \\ &= x^2 - 2x - x + 2 \\ &= x^2 - 3x + 2 \end{aligned}$$

Ex 32: Expand and simplify

$$(x+3)(x-2) = \boxed{x^2 + x - 6}$$

Answer:

$$\begin{aligned} (x+3) \cdot (x-2) &= x \times x + x \times (-2) + 3 \times x + 3 \times (-2) \\ &= x^2 - 2x + 3x - 6 \\ &= x^2 + x - 6 \end{aligned}$$

Ex 33: Expand and simplify

$$(2x+1)(1-2x) = \boxed{1 - 4x^2}$$

Answer:

$$\begin{aligned} (2x+1) \cdot (1-2x) &= 2x \times 1 + 2x \times (-2x) + 1 \times 1 + 1 \times (-2x) \\ &= 2x - 4x^2 + 1 - 2x \\ &= 1 - 4x^2 \end{aligned}$$

Ex 34: Expand and simplify

$$(-1+2x)(1-x) = \boxed{-2x^2 + 3x - 1}$$

Answer:

$$\begin{aligned} (-1+2x) \cdot (1-x) &= (-1) \times 1 + (-1) \times (-x) + 2x \times 1 + 2x \times (-x) \\ &= -1 + x + 2x - 2x^2 \\ &= -2x^2 + 3x - 1 \end{aligned}$$

C DIFFERENCE OF TWO SQUARES

C.1 EXPANDING: LEVEL 1

Ex 35: Expand and simplify

$$(x+1)(x-1) = \boxed{x^2 - 1}$$

Answer:

$$\begin{aligned} (x+1)(x-1) &= x^2 - 1^2 \\ &= x^2 - 1 \end{aligned}$$

Ex 36: Expand and simplify

$$(x-3)(x+3) = \boxed{x^2 - 9}$$

Answer:

$$\begin{aligned} (x-3)(x+3) &= x^2 - 3^2 \\ &= x^2 - 9 \end{aligned}$$

Ex 37: Expand and simplify

$$(4-x)(4+x) = \boxed{16 - x^2}$$

Answer:

$$\begin{aligned} (4-x)(4+x) &= 4^2 - x^2 \\ &= 16 - x^2 \end{aligned}$$

Ex 38: Expand and simplify

$$(5+x)(5-x) = \boxed{25 - x^2}$$

Answer:

$$\begin{aligned} (5+x)(5-x) &= 5^2 - x^2 \\ &= 25 - x^2 \end{aligned}$$

C.2 EXPANDING: LEVEL 2

Ex 39: Expand and simplify

$$(2x-4)(2x+4) = \boxed{4x^2 - 16}$$

Answer:

$$\begin{aligned} (2x-4)(2x+4) &= (2x)^2 - (4)^2 \\ &= 2^2 x^2 - 4^2 \\ &= 4x^2 - 16 \end{aligned}$$

Ex 40: Expand and simplify

$$(x+\sqrt{2})(x-\sqrt{2}) = \boxed{x^2 - 2}$$

Answer:

$$\begin{aligned} (x+\sqrt{2})(x-\sqrt{2}) &= x^2 - (\sqrt{2})^2 \\ &= x^2 - 2 \end{aligned}$$

Ex 41: Expand and simplify

$$\left(\frac{1}{2} - x\right)\left(\frac{1}{2} + x\right) = \boxed{\frac{1}{4} - x^2}$$

Answer:

$$\begin{aligned} \left(\frac{1}{2} - x\right)\left(\frac{1}{2} + x\right) &= \left(\frac{1}{2}\right)^2 - x^2 \\ &= \frac{1^2}{2^2} - x^2 \\ &= \frac{1}{4} - x^2 \end{aligned}$$

Ex 42: Expand and simplify

$$\left(\frac{x}{2} - 1\right)\left(\frac{x}{2} + 1\right) = \boxed{\frac{x^2}{4} - 1}$$

Answer:

$$\begin{aligned} \left(\frac{x}{2} - 1\right)\left(\frac{x}{2} + 1\right) &= \left(\frac{x}{2}\right)^2 - (1)^2 \\ &= \frac{x^2}{2^2} - 1^2 \\ &= \frac{x^2}{4} - 1 \end{aligned}$$

D PERFECT SQUARES EXPANSION

D.1 EXPANDING WITH ADDITION

Ex 43: Expand and simplify

$$(x+2)^2 = \boxed{x^2 + 4x + 4}$$

Answer: In the perfect squares expansion, we substitute $a = x$ and $b = 2$:

$$\begin{aligned} (x+2)^2 &= x^2 + 2 \times x \times 2 + 2^2 \\ &= x^2 + 4x + 4 \end{aligned}$$

Ex 44: Expand and simplify

$$(3+x)^2 = \boxed{9 + 6x + x^2}$$

Answer: In the perfect squares expansion, we substitute $a = 3$ and $b = x$:

$$\begin{aligned} (3+x)^2 &= 3^2 + 2 \times 3 \times x + x^2 \\ &= x^2 + 6x + 9 \end{aligned}$$

Ex 45: Expand and simplify

$$(2x+1)^2 = \boxed{4x^2 + 4x + 1}$$

Answer: In the perfect squares expansion, we substitute $a = 2x$ and $b = 1$:

$$\begin{aligned} (2x+1)^2 &= (2x)^2 + 2 \times 2x \times 1 + (1)^2 \\ &= 2^2 x^2 + 4x + 1 \\ &= 4x^2 + 4x + 1 \end{aligned}$$

Ex 46: Expand and simplify

$$(2+3x)^2 = \boxed{4 + 12x + 9x^2}$$

Answer: In the perfect squares expansion, we substitute $a = 2$ and $b = 3x$:

$$\begin{aligned} (2+3x)^2 &= 2^2 + 2 \times 2 \times 3x + (3x)^2 \\ &= 4 + 12x + 3^2 x^2 \\ &= 9x^2 + 12x + 4 \end{aligned}$$

D.2 EXPANDING WITH SUBTRACTION

Ex 47: Expand and simplify

$$(x - 2)^2 = \boxed{x^2 - 4x + 4}$$

Answer: In the perfect squares expansion, we substitute $a = x$ and $b = 2$:

$$\begin{aligned}(x - 2)^2 &= x^2 - 2 \times x \times 2 + 2^2 \\ &= x^2 - 4x + 4\end{aligned}$$

Ex 48: Expand and simplify

$$(3 - x)^2 = \boxed{9 - 6x + x^2}$$

Answer: In the perfect squares expansion, we substitute $a = 3$ and $b = x$:

$$\begin{aligned}(3 - x)^2 &= 3^2 - 2 \times 3 \times x + x^2 \\ &= x^2 - 6x + 9\end{aligned}$$

Ex 49: Expand and simplify

$$(2x - 1)^2 = \boxed{4x^2 - 4x + 1}$$

Answer: In the perfect squares expansion, we substitute $a = 2x$ and $b = 1$:

$$\begin{aligned}(2x - 1)^2 &= (2x)^2 - 2 \times 2x \times 1 + (1)^2 \\ &= 4x^2 - 4x + 1\end{aligned}$$

Ex 50: Expand and simplify

$$(2 - 3x)^2 = \boxed{4 - 12x + 9x^2}$$

Answer: In the perfect squares expansion, we substitute $a = 2$ and $b = 3x$:

$$\begin{aligned}(2 - 3x)^2 &= 2^2 - 2 \times 2 \times 3x + (3x)^2 \\ &= 9x^2 - 12x + 4\end{aligned}$$