ELEMENTS OF GEOMETRY

A POINT

Definition Point

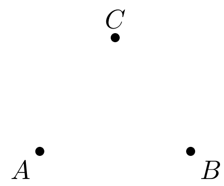
A **point** shows an exact position in space. We draw a point as a small dot.

Definition Point Notation

We usually name a point with a capital letter, such as A.

In mathematics, we imagine that a point has no size or shape. It only marks a position.

Ex: The diagram below shows three points labeled A, B, and C:



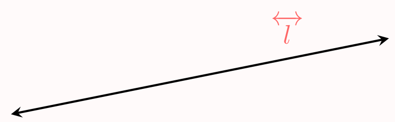
B LINES, SEGMENTS AND RAYS

Definition Line

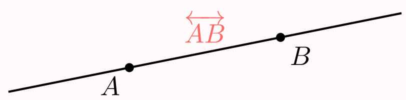
A **line** is a straight path that goes on forever in both directions.

Definition Line Notation

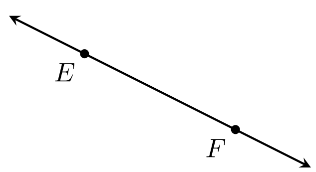
- A line can be named with a lowercase letter, written as $\overleftrightarrow{l}$.



- A line is named using two points on it, written as $\overleftrightarrow{AB}$.



Ex: Name the line shown below:



Answer: The line is $\overleftrightarrow{EF}$.

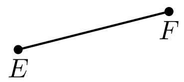
Definition Line Segment

A **line segment** is the part of a line between two endpoints. It has a fixed length.

Definition Line Segment Notation

We name a line segment by its endpoints, written as $\overline{AB}$. We read this as "segment AB".

Ex: Name the segment shown below:



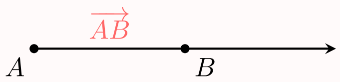
Answer: The segment is $\overline{EF}$.

Definition Ray

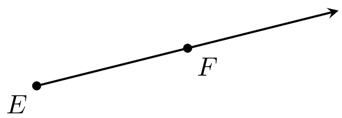
A **ray** is a part of a line that starts at one endpoint and goes on forever in one direction.

Definition Ray Notation

We name a ray by its endpoint first and another point on it, written as $\overrightarrow{AB}$. We read this as "ray AB".



Ex: Name the ray shown below:

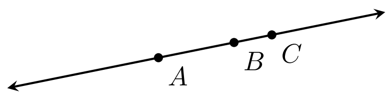


Answer: The ray is $\overrightarrow{EF}$.

Definition Collinear Points

Collinear points are points that all lie on the same straight line.

Ex: The points A, B and C are collinear points.

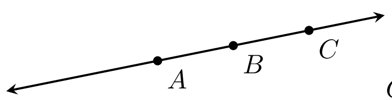


C ELEMENT RELATION

Definition Element Relation

The relation **is a point of** (or "belongs to") is used to show that a point lies on a geometric figure, such as a line or a segment. We write this relation using the symbol $\in$.

Ex:



$C \in \overleftrightarrow{AB}$ and $C \notin \overline{AB}$

In this figure, point C lies on the line through points A and B, so we write $C \in \overleftrightarrow{AB}$ and say that C is a point of the line $\overleftrightarrow{AB}$. However, C does not lie on the segment between A and B, so $C \notin \overline{AB}$.

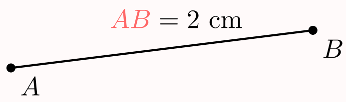
D LENGTH

Definition Length of a Line Segment

The **length** of a line segment is the distance between its two endpoints, measured in units such as centimeters (cm) or meters (m).

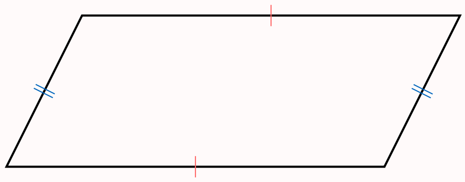
Definition Length Notation

If $\overline{AB}$ is a segment, its length is denoted by AB (without the bar). In diagrams, we may also write AB for the length of segment AB.

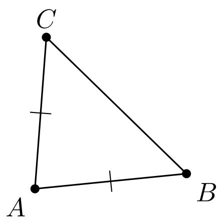


Definition Equal Lengths

Line segments are **equal in length** if they have the same length. We use **tick marks** on the segments to show that they are equal: segments with the same number of tick marks have the same length.



Ex: Identify two segments that have the same length.

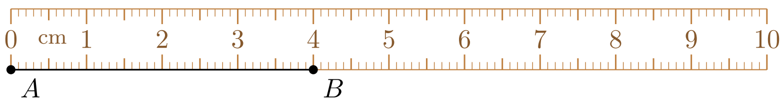


Answer: Segments $\overline{AB}$ and $\overline{AC}$ have the same length, as shown by the identical tick marks on each of them. Therefore, $AB = AC$.

Method Measuring Length

We measure the length of a segment with a ruler. Place one endpoint on the 0 mark, then read the number at the other endpoint: that number is the length of the segment.

Ex: Measure the length of segment $\overline{AB}$.

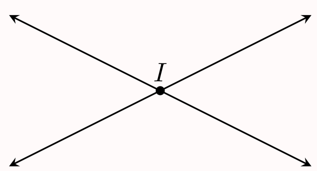


Answer: By aligning a ruler with segment $\overline{AB}$, we measure the length as $AB = 4\,\text{cm}$. So the length of segment AB is 4 cm.

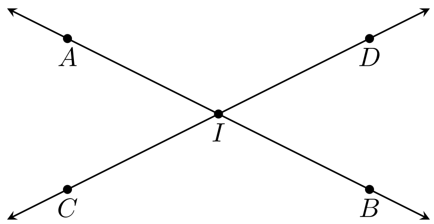
E INTERSECTION POINT

Definition Intersection Point

An **intersection point** is a point where two or more lines, segments, or rays cross each other.



Ex: Find the intersection point of the lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$.

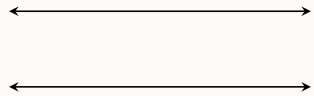


Answer: The intersection point is I.

F PARALLEL LINES

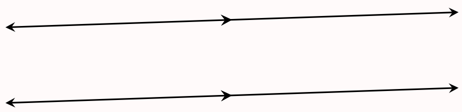
Definition Parallel Lines

Two **parallel lines** are lines that are always the same distance apart and never meet, even if you extend them.



Definition Parallel Line Notation

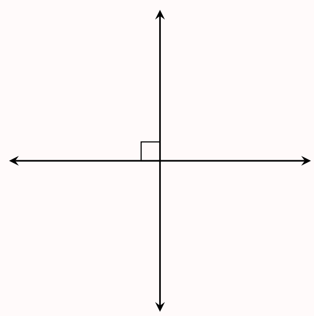
On a diagram, parallel lines are shown using matching little arrows on each line.



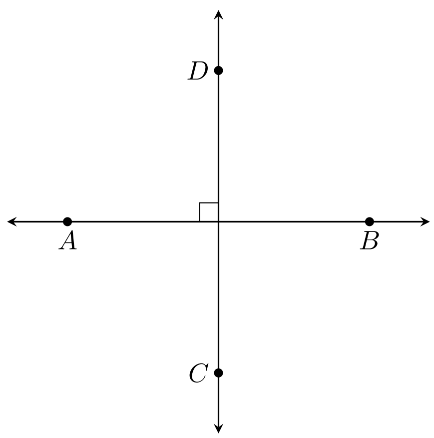
G PERPENDICULAR LINES

Definition Perpendicular Lines

Two **perpendicular lines** are lines that intersect at a right angle (90 degrees). We write $\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$ to show that line $\overleftrightarrow{AB}$ is perpendicular to line $\overleftrightarrow{CD}$.



Ex: Identify the pair of perpendicular lines in the figure below.



Answer: The lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are perpendicular, as they intersect, forming a right angle, indicated by the right-angle mark.

H MIDPOINT AND PERPENDICULAR BISECTOR

Definition Midpoint of a Line Segment

The **midpoint** of a line segment is a point that lies on the segment and divides it into two segments of equal length.For example, if I is the midpoint of segment $\overline{AB}$, then $I \in \overline{AB}$ and $AI = IB$.

Proposition Midpoint Length Property

If point I is the midpoint of segment $\overline{AB}$, then $AB = 2 \times AI$ and $AI = \frac{AB}{2}$.

Proof

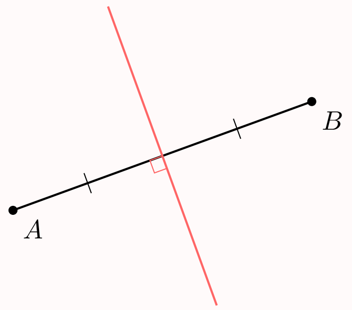
$$\begin{aligned} AB &= AI + IB \quad (I \text{ is the midpoint of } \overline{AB}) \\ &= AI + AI \\ &= 2 \times AI \end{aligned}$$

Thus, $AB = 2 \times AI$. To find AI in terms of AB, we rearrange the equation:

$$AI = \frac{AB}{2}.$$

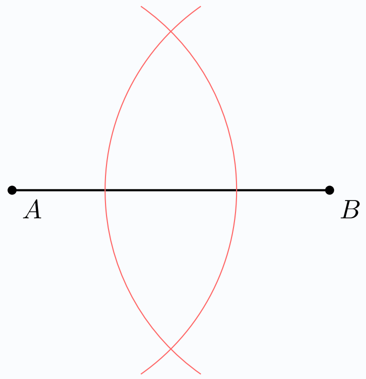
Definition Perpendicular bisector

The **perpendicular bisector** of a line segment is the line that passes through the midpoint of the segment and is perpendicular to the segment.

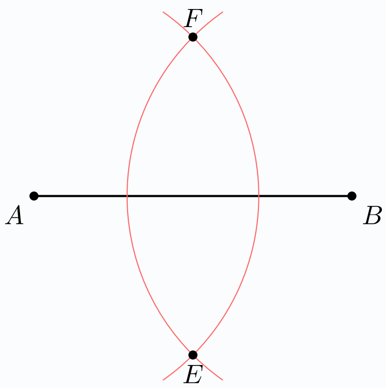


Method Constructing the Perpendicular Bisector of $\overline{AB}$

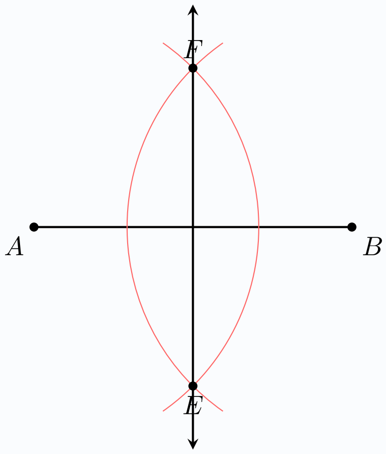
- Using a compass, construct two arcs with the same radius and with centers at A and B.



- The arcs intersect at points E and F.



- The perpendicular bisector of $\overline{AB}$ is the line $\overleftrightarrow{EF}$.

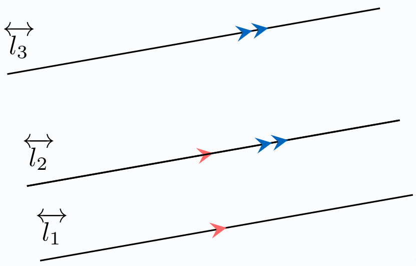


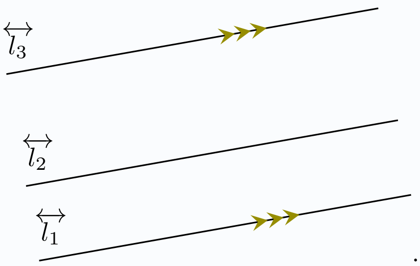
I PROPERTIES OF PARALLEL LINES

Proposition Properties of Parallel Lines

These properties help us decide when two lines are parallel or perpendicular.

- If line $\overleftrightarrow{l_1}$ is parallel to line $\overleftrightarrow{l_2}$ and line $\overleftrightarrow{l_2}$ is parallel to line $\overleftrightarrow{l_3}$, then line $\overleftrightarrow{l_1}$ is parallel to line $\overleftrightarrow{l_3}$.

If 

, then 

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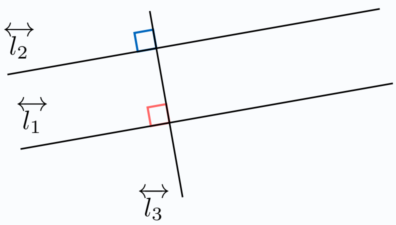
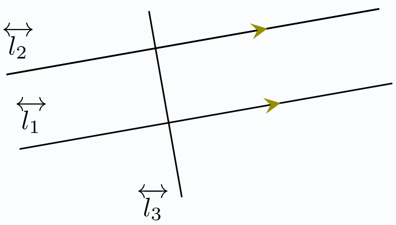
- If line $\overleftrightarrow{l_1}$ is perpendicular to line $\overleftrightarrow{l_3}$ and line $\overleftrightarrow{l_2}$ is perpendicular to line $\overleftrightarrow{l_3}$, then line $\overleftrightarrow{l_1}$ is parallel to line $\overleftrightarrow{l_2}$.

If 

, then 

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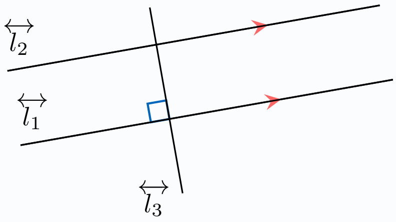
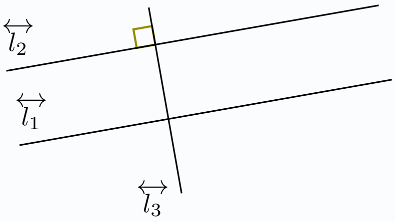
- If line $\overleftrightarrow{l_1}$ is parallel to line $\overleftrightarrow{l_2}$ and line $\overleftrightarrow{l_1}$ is perpendicular to line $\overleftrightarrow{l_3}$, then line $\overleftrightarrow{l_2}$ is perpendicular to line $\overleftrightarrow{l_3}$.



$\overleftrightarrow{l_2}$
$\overleftrightarrow{l_1}$
$\overleftrightarrow{l_3}$



$\overleftrightarrow{l_2}$
$\overleftrightarrow{l_1}$
$\overleftrightarrow{l_3}$

If , then .