ELEMENTS OF GEOMETRY

A POINT

A.1 COUNTING NUMBER OF POINTS

Ex 1: Count the points in the figure.

$\bullet\, B$

$A\,\bullet$

$\overset{\bullet}{C}$
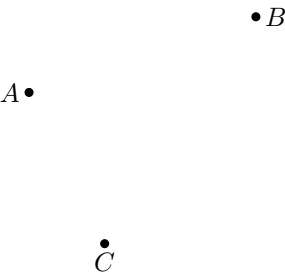
$\boxed{3}$ points

Answer: The figure has 3 points: A, B, and C.

Ex 2: Count the points in the figure.

$\bullet\, F$

$E\,\bullet$

$\boxed{2}$ points

Answer: The figure has 2 points: E and F.

Ex 3: Count the points in the figure.

$\overset{G}{\bullet}$

$E\,\bullet$ $\qquad\qquad$ $\bullet\, F$

$\overset{\bullet}{H}$
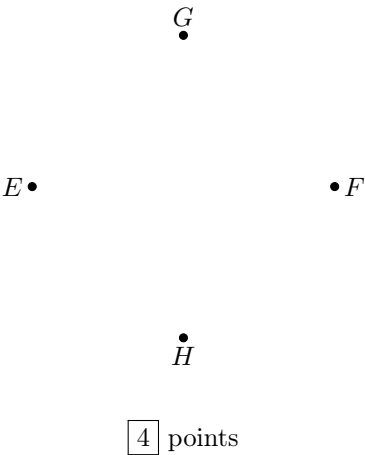
$\boxed{4}$ points

Answer: The figure has 4 points: E, F, G, and H.

Ex 4: Count the points in the figure.

$I\,\bullet$
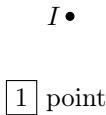
$\boxed{1}$ point

Answer: The figure has 1 point: I.

A.2 DRAWING POINTS

Ex 5: Using a pencil, draw a point and label it A.

Answer:

$A\,\bullet$

The point is labeled A.

Ex 6: Using a pencil, draw two points and label them A and B.

Answer:

$\bullet\, B$

$A\,\bullet$
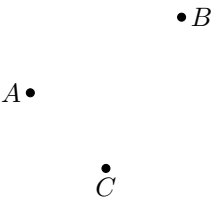
The two points are labeled A and B.

Ex 7: Using a pencil, draw three points and label them A, B, and C.

Answer:

$\bullet\, B$

$A\,\bullet$

$\overset{\bullet}{C}$

The three points are labeled A, B, and C.

B LINES, SEGMENTS AND RAYS

B.1 RECOGNIZING

MCQ 8: Which term describes this figure?

Choose one answer:

☒ Line

☐ Line segment

☐ Ray

Answer: A line extends infinitely in both directions. This figure is a line.

MCQ 9: Which term describes this figure?

Choose one answer:

☐ Line

☐ Line segment

☒ Ray

Answer: A ray starts at one point and extends infinitely in one direction. This figure is a ray.

MCQ 10: Which term describes this figure?

Choose one answer:

☐ Line

☒ Line segment

☐ Ray

Answer: A line segment connects two points with a definite length. This figure is a line segment.
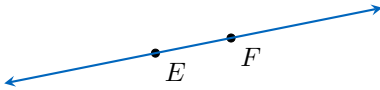
MCQ 11: Which term describes this figure?

Choose one answer:

☐ Line

☐ Line segment

☒ Ray

Answer: A ray starts at one point and extends infinitely in one direction. This figure is a ray.

B.2 NAMING

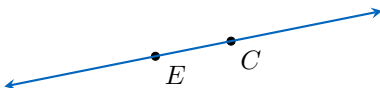**MCQ 12:** Name the line represented in this figure:

Choose one answer:

☐ $\overline{EF}$

☒ $\overleftrightarrow{EF}$

☐ $\overrightarrow{EF}$

Answer: The correct name is $\overleftrightarrow{EF}$ because the figure shows an infinitely straight path extending in both directions through points E and F.

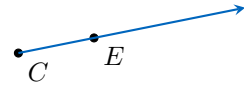**MCQ 13:** Name the line represented in this figure:

Choose one answer:

☐ $\overline{CE}$

☒ $\overleftrightarrow{CE}$

☐ $\overrightarrow{CE}$

Answer: The correct name is $\overleftrightarrow{CE}$ because the arrows at both ends indicate the line continues infinitely in both directions through points C and E.

MCQ 14: Name the ray represented in this figure:

Choose one answer:

☐ $\overline{CE}$

☐ $\overleftrightarrow{CE}$

☒ $\overrightarrow{CE}$

☐ $\overrightarrow{EC}$

Answer: The correct name is $\overrightarrow{CE}$ because the figure shows a ray starting at point C and extending infinitely through point E (indicated by the arrow).

MCQ 15: Name the ray represented in this figure:

Choose one answer:

☐ $\overline{EC}$

☐ $\overleftrightarrow{EC}$

☐ $\overrightarrow{CE}$

☒ $\overrightarrow{EC}$

Answer: The correct name is $\overrightarrow{EC}$ because the figure shows a ray starting at point E and extending infinitely through point C.

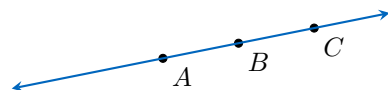**MCQ 16:** Name the segment represented in this figure:

Choose one answer:

☒ $\overline{EC}$

☐ $\overleftrightarrow{EC}$

☐ $\overrightarrow{EC}$

Answer: The correct name is $\overline{EC}$ because the figure shows only the finite portion of the line between points E and C.

MCQ 17: Name the line represented in this figure:
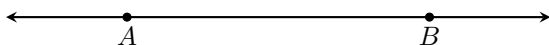

Choose all correct answers:

☒ $\overleftrightarrow{AB}$

☒ $\overleftrightarrow{AC}$

☒ $\overleftrightarrow{BC}$

Answer: All answers are correct: $\overleftrightarrow{AB}$, $\overleftrightarrow{AC}$, and $\overleftrightarrow{BC}$ because they all represent the same infinite straight line passing through all three points.

B.3 DRAWING LINES, SEGMENTS AND RAYS

Ex 18: Using a ruler and pencil, draw a straight line passing through points A and B. Label both points clearly.

Answer:

The straight line passes through points A and B, extends infinitely in both directions with arrows, and both points are labeled clearly.

Ex 19: Using a ruler and pencil, draw a line segment passing through points A and B. Label both points clearly.

Answer:

The line segment passes through points A and B, starts at point A, ends at point B, and both points are labeled clearly.

Ex 20: Using a ruler and pencil, draw a ray passing through points A and B. Label both points clearly.

Answer:

The ray starts at point A, passes through point B, extends infinitely in one direction with an arrow, and both points are labeled clearly.
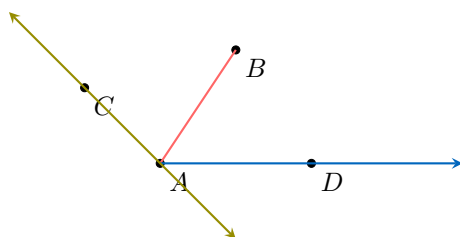
B.4 CHECKING A CONSTRUCTION PROGRAM
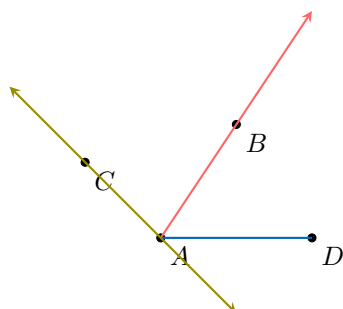
MCQ 21: A teacher gives these construction steps:

1. Draw points A, B, C, and D
2. Draw segment $\overline{AB}$
3. Draw line $\overleftrightarrow{AC}$
4. Draw ray $\overrightarrow{AD}$

Which student followed the instructions correctly?**Select the correct answer:**
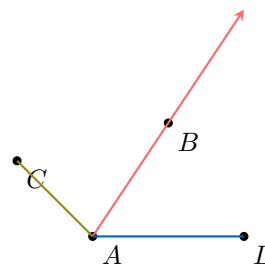
☒ Hugo

☐ Louis

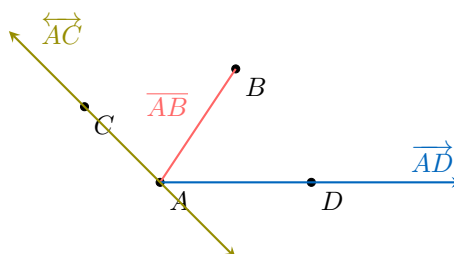

☐ Vincent

Answer: Hugo's construction is correct because:

- The segment $\overline{AB}$ is correctly drawn between points A and B
- The line $\overleftrightarrow{AC}$ extends infinitely in both directions through A and C
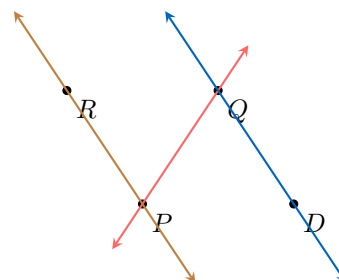- The ray $\overrightarrow{AD}$ starts at point A and extends infinitely through D
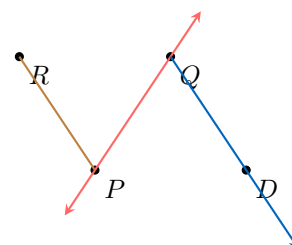

MCQ 22: A teacher gives these construction steps:

1. Draw points P, Q, R, and D
2. Draw segment $\overline{PR}$
3. Draw line $\overleftrightarrow{PQ}$
4. Draw ray $\overrightarrow{QD}$

Which student followed the instructions correctly?**Select the correct answer:**

☐ Hugo

☒ Louis

Answer: Louis's construction is correct because:

- The segment $\overline{PR}$ is correctly drawn between points P and R

- The line $\overleftrightarrow{PQ}$ extends infinitely in both directions through P and Q

- The ray $\overrightarrow{QD}$ starts at point Q and extends infinitely through D

MCQ 23: A teacher gives these construction steps:

1. Draw points X, Y, Z, and W

2. Draw segment $\overline{XZ}$

3. Draw line $\overleftrightarrow{XY}$

4. Draw ray $\overrightarrow{YW}$

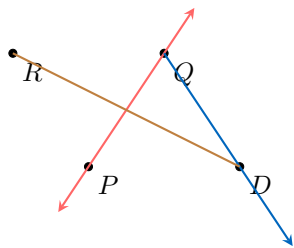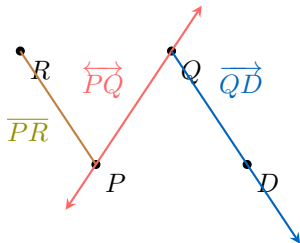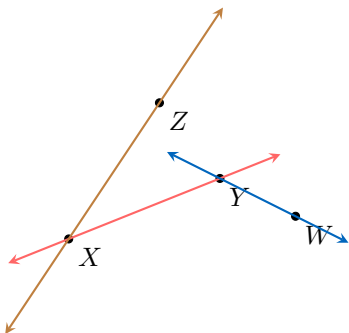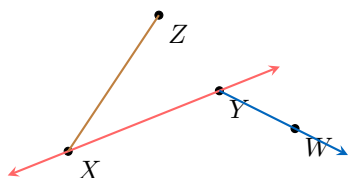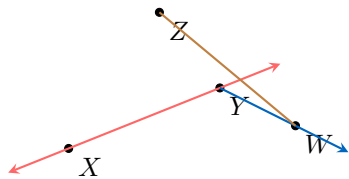Which student followed the instructions correctly? **Select the correct answer:**
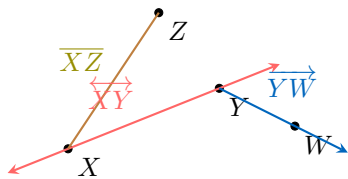
☐ Hugo

⊠ Louis

☐ Vincent

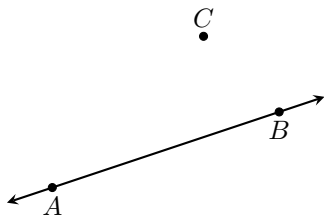
Answer: Louis's construction is correct because:

- The segment $\overline{XZ}$ is correctly drawn between points X and Z

- The line $\overleftrightarrow{XY}$ extends infinitely in both directions through X and Y

- The ray $\overrightarrow{YW}$ starts at point Y and extends infinitely through W

B.5 BUILDING GEOMETRIC FIGURES

Ex 24: Using a ruler and pencil, draw three points A, B, and C, and the straight line $\overleftrightarrow{AB}$.

Answer:

The straight line $\overleftrightarrow{AB}$ passes through points A and B, extends infinitely in both directions with arrows, and all points are labeled clearly.

Ex 25: Using a ruler and pencil, draw three points A, B, and C, and the line segment $\overline{AC}$.
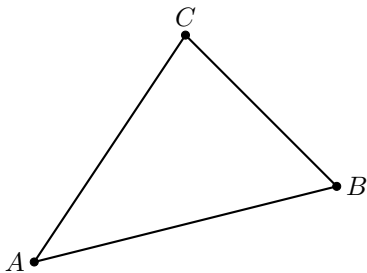
Answer:

The line segment $\overline{AC}$ has endpoints at points A and C, is drawn straight with a ruler, and all points are labeled clearly.

Ex 26: Using a ruler and pencil, draw three points A, B, and C, and the line segments $\overline{AB}$, $\overline{BC}$, and $\overline{CA}$.
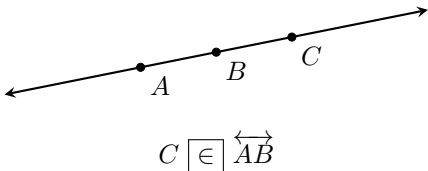
Answer:

The line segments $\overline{AB}$, $\overline{BC}$, and $\overline{CA}$ form a triangle with endpoints at points A, B, and C, are drawn straight with a ruler, and all points are labeled clearly.
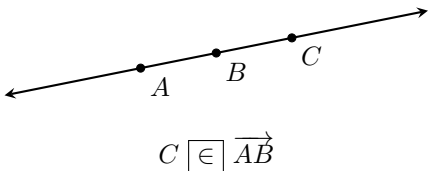
C ELEMENT RELATION

C.1 IDENTIFYING POINTS ON GEOMETRIC FIGURES

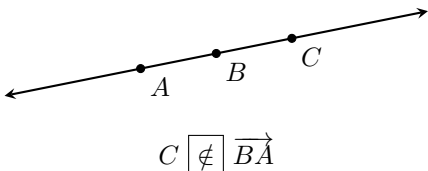**Ex 27:** Does point C lie on the line through points A and B?

$$C \boxed{\in} \overleftrightarrow{AB}$$

Answer: Point C lies on the line through points A and B, denoted $\overleftrightarrow{AB}$. Therefore, $C \in \overleftrightarrow{AB}$.

Ex 28: Does point C lie on the ray from A through B?


$$C \boxed{\in} \overrightarrow{AB}$$

Answer: Point C lies on the ray from A through B, denoted $\overrightarrow{AB}$, because it is on the line extending from A through B and beyond. Therefore, $C \in \overrightarrow{AB}$.

Ex 29: Does point C lie on the ray from B through A?

$$C \boxed{\notin} \overrightarrow{BA}$$

Answer: Point C does not lie on the ray from B through A, denoted $\overrightarrow{BA}$, because C is positioned beyond B in the opposite direction from A. Therefore, $C \notin \overrightarrow{BA}$.
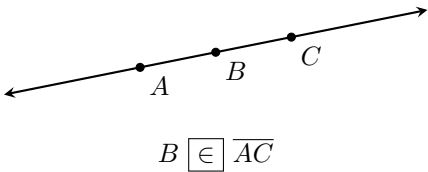
Ex 30: Does point C lie on the line segment between B and A?
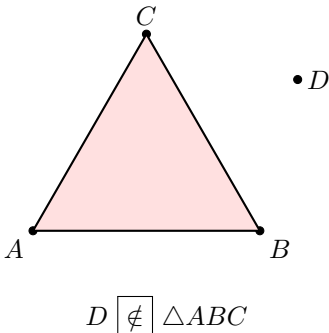

$$C \boxed{\notin} \overline{BA}$$
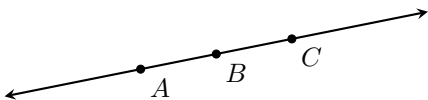
Answer: Point C does not lie on the line segment between B and A, denoted $\overline{BA}$, because C is beyond B and not between B and A. Therefore, $C \notin \overline{BA}$.

Ex 31: Does point B lie on the line segment between A and C?
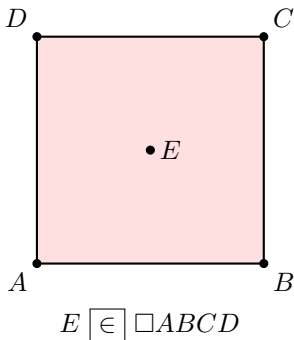

$$B \boxed{\in} \overline{AC}$$

Answer: Point B lies on the line segment between A and C, denoted $\overline{AC}$, because B is positioned between A and C. Therefore, $B \in \overline{AC}$.

Ex 32: Does point D lie on triangle ABC?

$$D \boxed{\notin} \triangle ABC$$

Answer: Point D does not lie on triangle ABC, denoted $\triangle ABC$, because it is outside the triangle. Therefore, $D \notin \triangle ABC$.

Ex 33: Does point E lie on square $ABCD$?
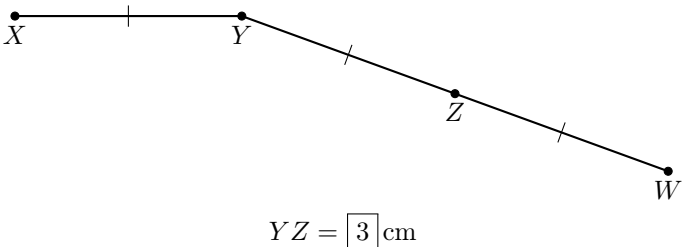

$$E \boxed{\in} \square ABCD$$

Answer: Point E lies on square $ABCD$, denoted $\square ABCD$, because it is inside the square. Therefore, $E \in \square ABCD$.
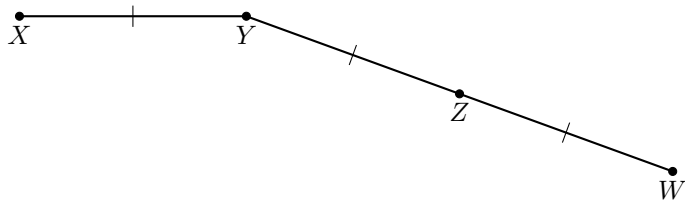
D LENGTH

D.1 USING TICK MARKS TO CALCULATE LENGTHS

Ex 34: The segment $\overline{XY}$ measures 3 cm. Use the tick marks to find the length of segment $\overline{YZ}$.

$$YZ = \boxed{3}\,\text{cm}$$

Answer: The tick marks show that segments $\overline{XY}$ and $\overline{YZ}$ are equal in length. Since $\overline{XY}$ measures 3 cm, $\overline{YZ}$ also measures 3 cm.

Ex 35: The segment $\overline{XY}$ measures 3 cm. Use the tick marks to find the length of segment $\overline{ZW}$.

$$ZW = \boxed{3}\,\text{cm}$$

Answer: The tick marks show that segments $\overline{XY}$ and $\overline{ZW}$ are equal in length. Since $\overline{XY}$ measures 3 cm, $\overline{ZW}$ also measures 3 cm.
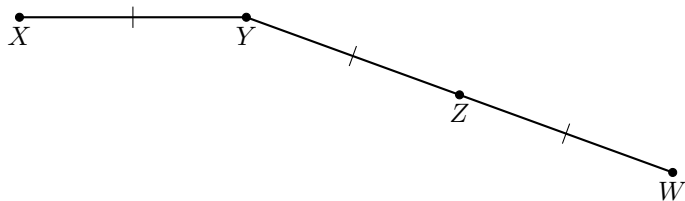
Ex 36: The segment $\overline{XY}$ measures 3 cm. Use the tick marks to find the length of segment $\overline{YW}$.
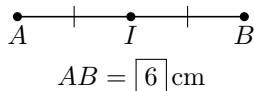

$$YW = \boxed{6}\,\text{cm}$$

Answer: The tick marks show that segments $\overline{XY}$, $\overline{YZ}$, and $\overline{ZW}$ are equal in length. Since $\overline{XY}$ measures 3 cm, both $\overline{YZ}$ and $\overline{ZW}$ also measure 3 cm each. Segment $\overline{YW}$ stretches from Y to W, covering both $\overline{YZ}$ and $\overline{ZW}$. Therefore, the length of $\overline{YW}$ is:

$$\begin{aligned} YW &= YZ + ZW \\ &= XY + XY \\ &= 3\,\text{cm} + 3\,\text{cm} \\ &= 6\,\text{cm}. \end{aligned}$$

D.2 CALCULATE LENGTHS USING A MIDPOINT

Ex 37: The segment $\overline{AI}$ measures 3 cm. Use the tick marks to find the length of segment $\overline{AB}$.
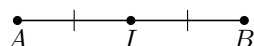

$$AB = \boxed{6}\,\text{cm}$$

Answer: The tick marks show that segments $\overline{AI}$ and $\overline{IB}$ are equal in length. Since $\overline{AI}$ measures 3 cm, $\overline{IB}$ also measures 3 cm. The length of $\overline{AB}$ is:

$$\begin{aligned} AB &= AI + IB \\ &= AI + AI \\ &= 3\,\text{cm} + 3\,\text{cm} \\ &= 6\,\text{cm}. \end{aligned}$$

Ex 38: The segment $\overline{IB}$ measures 10 cm. Use the tick marks to find the length of segment $\overline{AB}$.

$$AB = \boxed{20}\,\text{cm}$$

Answer: The tick marks show that segments $\overline{AI}$ and $\overline{IB}$ are equal in length. Since $\overline{IB}$ measures 10 cm, $\overline{AI}$ also measures 10 cm. The length of $\overline{AB}$ is:

$$\begin{aligned} AB &= AI + IB \\ &= IB + IB \\ &= 10\,\text{cm} + 10\,\text{cm} \\ &= 20\,\text{cm}. \end{aligned}$$

Ex 39: The segment $\overline{AB}$ measures 10 cm. Use the tick marks to find the length of segment $\overline{AI}$.

$$AI = \boxed{5}\,\text{cm}$$

Answer: The tick marks show that segments $\overline{AI}$ and $\overline{IB}$ are equal in length. Since $\overline{AB}$ measures 10 cm and is divided into two equal segments by point I, the length of $\overline{AI}$ is half of $\overline{AB}$:

$$\begin{aligned} AB &= AI + IB \\ 10\,\text{cm} &= AI + AI \\ 10\,\text{cm} &= 2 \times AI \\ AI &= 10\,\text{cm} \div 2 \\ &= 5\,\text{cm}. \end{aligned}$$

Ex 40: The segment $\overline{AB}$ measures 20 cm. Use the tick marks to find the length of segment $\overline{AI}$.

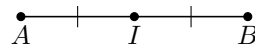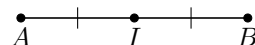
$$AI = \boxed{10}\,\text{cm}$$

Answer: The tick marks show that segments $\overline{AI}$ and $\overline{IB}$ are equal in length. Since $\overline{AB}$ measures 20 cm and is divided into two equal segments by point I, the length of $\overline{AI}$ is half of $\overline{AB}$:

$$\begin{aligned} AB &= AI + IB \\ 20\,\text{cm} &= AI + AI \\ 20\,\text{cm} &= 2 \times AI \\ AI &= 20\,\text{cm} \div 2 \\ &= 10\,\text{cm}. \end{aligned}$$

D.3 USING TICK MARKS TO FIND PERIMETER

Ex 41: The segment $\overline{AB}$ measures 3 cm and segment $\overline{BC}$ measures 2 cm. Use the tick marks to find the perimeter of rectangle $ABCD$.
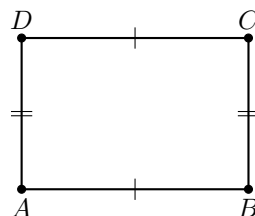

Perimeter of the rectangle $ABCD = \boxed{10}\,\text{cm}$

Answer: The tick marks show that segments $\overline{AB}$ and $\overline{CD}$ are equal in length, and segments $\overline{BC}$ and $\overline{DA}$ are equal in length. Since $\overline{AB}$ measures 3 cm, $\overline{CD}$ also measures 3 cm. Since $\overline{BC}$ measures 2 cm, $\overline{DA}$ also measures 2 cm. The perimeter of rectangle $ABCD$ is the sum of all its sides:

$$\begin{aligned} \text{Perimeter} &= AB + BC + CD + DA \\ &= 3\,\text{cm} + 2\,\text{cm} + 3\,\text{cm} + 2\,\text{cm} \\ &= 10\,\text{cm}. \end{aligned}$$

Ex 42: The segment $\overline{AB}$ measures 3 cm. Use the tick marks to find the perimeter of triangle ABC.
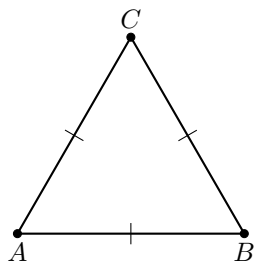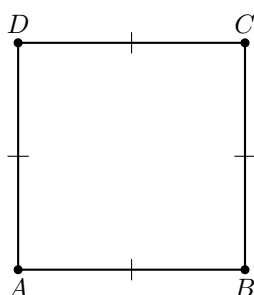

Perimeter of $\triangle ABC =$ $\boxed{9}$ cm

Answer: The tick marks show that segments $\overline{AB}$, $\overline{BC}$, and $\overline{CA}$ are equal in length. Since $\overline{AB}$ measures 3 cm, $\overline{BC}$ and $\overline{CA}$ also measure 3 cm each. The perimeter of triangle ABC is the sum of all its sides:

$$\begin{aligned} \text{Perimeter} &= AB + BC + CA \\ &= 3\,\text{cm} + 3\,\text{cm} + 3\,\text{cm} \\ &= 9\,\text{cm}. \end{aligned}$$

Ex 43: The segment $\overline{AB}$ measures 3 cm. Use the tick marks to find the perimeter of square $ABCD$.
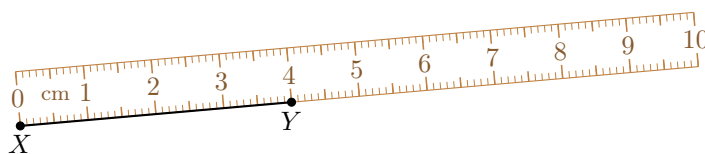

Perimeter of $\square ABCD =$ $\boxed{12}$ cm

Answer: The tick marks show that segments $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, and $\overline{DA}$ are equal in length. Since $\overline{AB}$ measures 3 cm, $\overline{BC}$, $\overline{CD}$, and $\overline{DA}$ also measure 3 cm each. The perimeter of square $ABCD$ is the sum of all its sides:

$$\begin{aligned} \text{Perimeter} &= AB + BC + CD + DA \\ &= 3\,\text{cm} + 3\,\text{cm} + 3\,\text{cm} + 3\,\text{cm} \\ &= 12\,\text{cm}. \end{aligned}$$

D.4 MEASURING WITH A RULER

Ex 44: Measure the length of segment $\overline{XY}$ using the ruler shown.

$XY =$ $\boxed{4}$ cm

Answer: By aligning the ruler's zero mark with point X, segment $\overline{XY}$ measures $XY = 4$ cm.
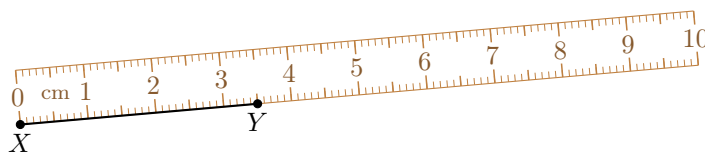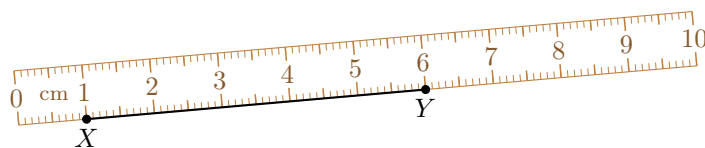
Ex 45: Measure the length of segment $\overline{XY}$ using the ruler shown.

$XY =$ $\boxed{3.5}$ cm

Answer: By aligning the ruler's zero mark with point X, segment $\overline{XY}$ measures $XY = 3.5$ cm.

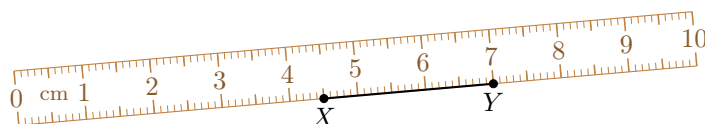**Ex 46:** Measure the length of segment $\overline{XY}$ using the ruler shown.

$XY =$ $\boxed{5}$ cm

Answer: To measure segment $\overline{XY}$, notice that point X is at the 1 cm mark and point Y is at the 6 cm mark on the ruler. The length of the segment is the distance between these points, so we subtract the starting mark from the ending mark:

$$\begin{aligned} XY &= 6\,\text{cm} - 1\,\text{cm} \\ &= 5\,\text{cm}. \end{aligned}$$

Ex 47: Measure the length of segment $\overline{XY}$ using the ruler shown.


$XY =$ $\boxed{2.5}$ cm

Answer: To measure segment $\overline{XY}$, notice that point X is at the 4.5 cm mark and point Y is at the 7 cm mark on the ruler. The length of the segment is the distance between these points, so we subtract the starting mark from the ending mark:

$$\begin{aligned} XY &= 7\,\text{cm} - 4.5\,\text{cm} \\ &= 2.5\,\text{cm}. \end{aligned}$$

E INTERSECTION POINT

E.1 PICKING THE INTERSECTION POINTS
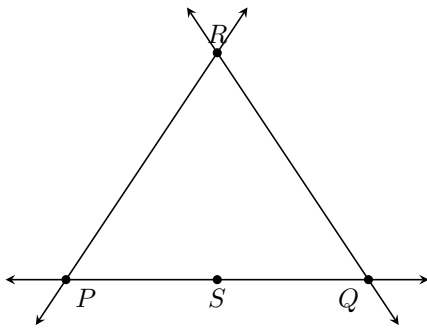
MCQ 48: Pick the point where the lines intersect.
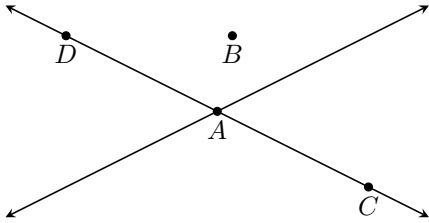
Choose one point:

☒ A

☐ B

☐ C

☐ D

Answer: The lines intersect at the point labeled A.

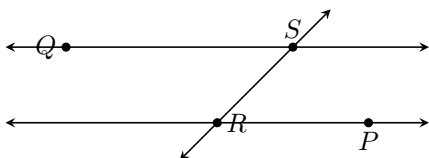**MCQ 49:** Pick the points where the lines intersect.

Choose all correct points:

☒ P

☒ Q

☒ R

☐ S

Answer: The lines intersect at the points labeled P, Q, and R.

MCQ 50: Pick the points where the lines intersect.

Choose all correct points:

☐ P

☐ Q

☒ R

☒ S

Answer: The lines intersect at the points labeled R and S.

F PARALLEL LINES

F.1 IDENTIFYING PARALLEL LINES

MCQ 51:
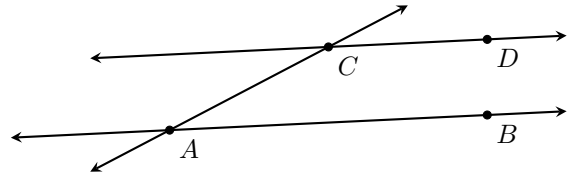
Choose the true statement:

☐ $\overleftrightarrow{AB}$ is parallel to $\overleftrightarrow{AC}$.

☐ $\overleftrightarrow{CD}$ is parallel to $\overleftrightarrow{AC}$.

☒ $\overleftrightarrow{CD}$ is parallel to $\overleftrightarrow{AB}$.

Answer:

Lines $\overleftrightarrow{CD}$ and $\overleftrightarrow{AB}$ go in the same direction, so they are parallel.

MCQ 52:

Choose the true statement:

☐ $\overleftrightarrow{AB}$ is parallel to $\overleftrightarrow{DC}$.

☒ $\overleftrightarrow{DC}$ is parallel to $\overleftrightarrow{FE}$.

☐ $\overleftrightarrow{AB}$ is parallel to $\overleftrightarrow{FE}$.

Answer:

Lines $\overleftrightarrow{DC}$ and $\overleftrightarrow{FE}$ go in the same direction, so they are parallel.

MCQ 53:

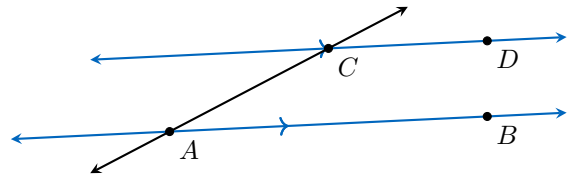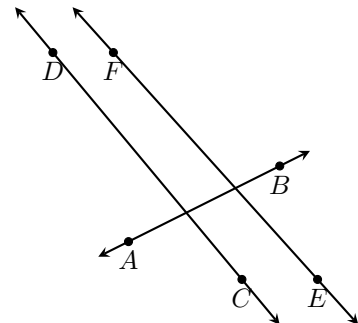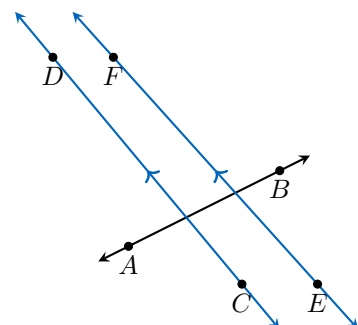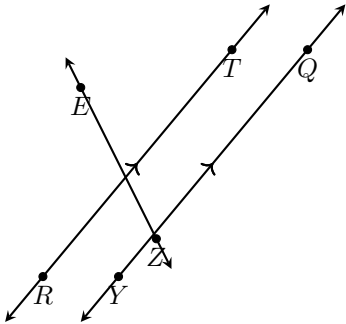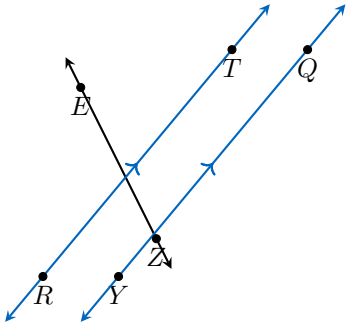**Choose the true statement:**
- ☐ $\overleftrightarrow{ZE}$ is parallel to $\overleftrightarrow{RT}$.
- ☐ $\overleftrightarrow{ZE}$ is parallel to $\overleftrightarrow{YQ}$.
- ☒ $\overleftrightarrow{RT}$ is parallel to $\overleftrightarrow{YQ}$.

Answer:

Lines $\overleftrightarrow{RT}$ and $\overleftrightarrow{YQ}$ go in the same direction, so they are parallel.

F.2 COUNTING POSSIBLE LINES

MCQ 54: Can you find a line that passes through points A and B ? How many such lines are possible?
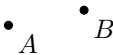
- ☐ 0
- ☒ 1
- ☐ Infinite

Answer: Only one line can pass through two points.
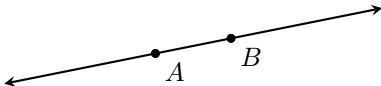
MCQ 55: Can you find a line that passes through points A, B, and C together? How many such lines are possible?
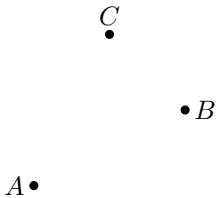
- ☒ 0
- ☐ 1
- ☐ Infinite

Answer: No line can pass through three points that are not in a straight line.
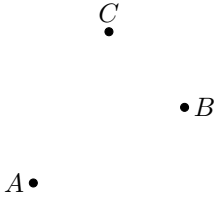
MCQ 56: Can you find a line that passes through point A? How many such lines are possible?

- ☐ 0
- ☐ 1
- ☒ Infinite

Answer: Infinitely many different lines can pass through one point because they can go in any direction.
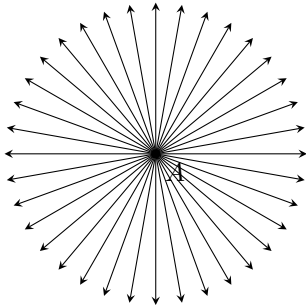
MCQ 57: Are the blue thick lines parallel?
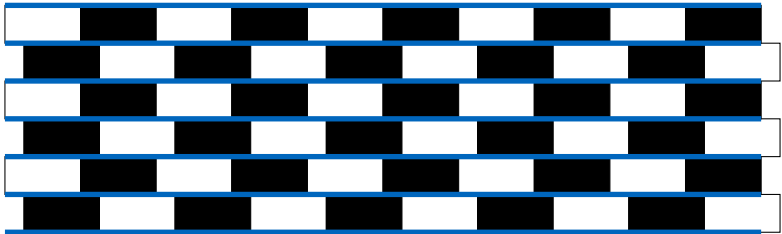
- ☒ Yes
- ☐ No

Answer: The thick lines are parallel, but they look different because of the rectangles. This is the famous "Café wall illusion," where parallel lines seem not parallel.

G PERPENDICULAR LINES

G.1 IDENTIFYING PERPENDICULAR LINES

MCQ 58:

Choose the true statement:

☐ $\overleftrightarrow{CD}$ is parallel to $\overleftrightarrow{AB}$.

☐ $\overleftrightarrow{AB}$ is parallel to $\overleftrightarrow{CD}$.

☒ $\overleftrightarrow{CD}$ is perpendicular to $\overleftrightarrow{AB}$.

Answer: $\overleftrightarrow{CD}$ is perpendicular to $\overleftrightarrow{AB}$, as indicated by the right-angle mark at their intersection, showing they form a 90-degree angle.

MCQ 59:

Choose the true statement:

☐ $\overleftrightarrow{AB}$ is perpendicular to $\overleftrightarrow{AC}$.

☐ $\overleftrightarrow{AB}$ is perpendicular to $\overleftrightarrow{BC}$.

☒ $\overleftrightarrow{AC}$ is perpendicular to $\overleftrightarrow{BC}$.

Answer: $\overleftrightarrow{AC}$ is perpendicular to $\overleftrightarrow{BC}$, as indicated by the right-angle mark at vertex C, showing they form a 90-degree angle in the right triangle ABC.

MCQ 60:

Choose all true statements:

☒ $\overleftrightarrow{AB}$ is perpendicular to $\overleftrightarrow{AD}$.

☒ $\overleftrightarrow{AB}$ is perpendicular to $\overleftrightarrow{BC}$.

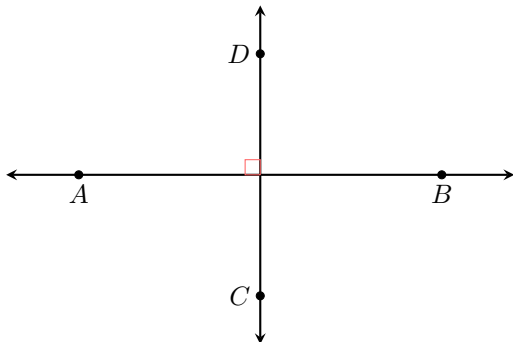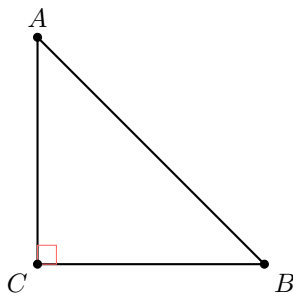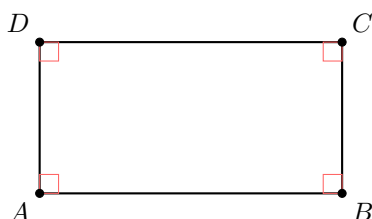☒ $\overleftrightarrow{BC}$ is perpendicular to $\overleftrightarrow{CD}$.

Answer: All statements are true. In rectangle $ABCD$, all angles are 90 degrees, so adjacent sides are perpendicular:

- $\overleftrightarrow{AB}$ is perpendicular to $\overleftrightarrow{AD}$, as shown by the right-angle mark at vertex A.
- $\overleftrightarrow{AB}$ is perpendicular to $\overleftrightarrow{BC}$, as shown by the right-angle mark at vertex B.
- $\overleftrightarrow{BC}$ is perpendicular to $\overleftrightarrow{CD}$, as shown by the right-angle mark at vertex C.

MCQ 61:
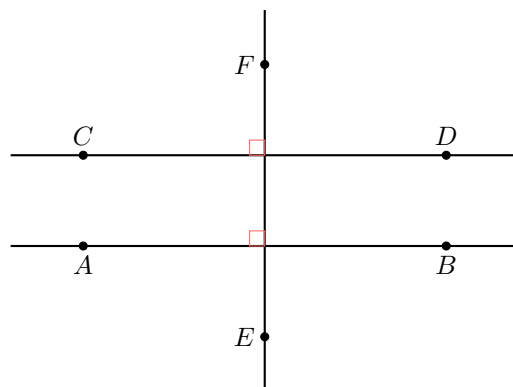
Choose the true statements:

☐ $\overleftrightarrow{CD}$ is perpendicular to $\overleftrightarrow{AB}$.

☒ $\overleftrightarrow{EF}$ is perpendicular to $\overleftrightarrow{CD}$.

☒ $\overleftrightarrow{EF}$ is perpendicular to $\overleftrightarrow{AB}$.

Answer:

- $\overleftrightarrow{EF}$ is perpendicular to $\overleftrightarrow{CD}$, as indicated by the right-angle mark at their intersection.
- $\overleftrightarrow{EF}$ is perpendicular to $\overleftrightarrow{AB}$, as indicated by the right-angle mark at their intersection.

G.2 BUILDING GEOMETRIC FIGURES

Ex 62: Using a ruler, pencil, and set square, draw line $\overleftrightarrow{AB}$ and a perpendicular line through a point P on $\overleftrightarrow{AB}$.

Answer:

The line $\overleftrightarrow{AB}$ is a straight line passing through points A and B, extending infinitely in both directions. Point P is marked on $\overleftrightarrow{AB}$, and a perpendicular line is drawn through P using a set square, forming a right angle with $\overleftrightarrow{AB}$, as indicated by the right-angle mark.

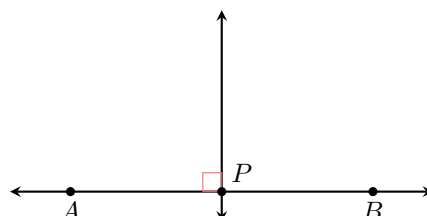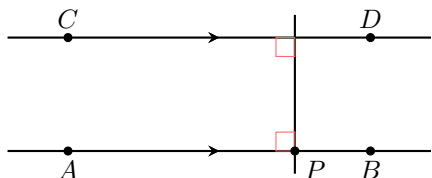**Ex 63:** Using a ruler, pencil, and set square, draw two parallel lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$, and a line perpendicular to $\overleftrightarrow{AB}$ through a point P on $\overleftrightarrow{AB}$.
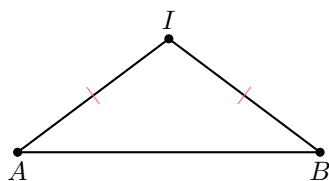
Answer:

The lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are drawn parallel, with arrows indicating they extend infinitely and matching arrowheads showing parallelism. Point P is marked on $\overleftrightarrow{AB}$, and a perpendicular line is drawn through P using a set square, forming a right angle with $\overleftrightarrow{AB}$, as indicated by the right-angle mark. Since $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are parallel, the perpendicular line through P is also perpendicular to $\overleftrightarrow{CD}$, as shown by the second right-angle mark.

H MIDPOINT AND PERPENDICULAR BISECTOR

H.1 IDENTIFYING MIDPOINTS AND PERPENDICULAR BISECTORS

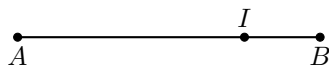**MCQ 64:** Point I is the midpoint of segment $\overline{AB}$.

Is the statement true or false?

☐ True

☒ False

Answer: The statement is false. For point I to be the midpoint of segment $\overline{AB}$, it must lie on the segment $\overline{AB}$ and divide it into two equal segments $\overline{AI}$ and $\overline{IB}$. In the diagram, point I is not on the straight line segment $\overline{AB}$, so it cannot be the midpoint.

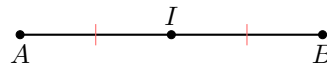**MCQ 65:** Point I is the midpoint of segment $\overline{AB}$.

Is the statement true or false?

☐ True

☒ False

Answer: The statement is false. For point I to be the midpoint of segment $\overline{AB}$, it must lie on the segment and divide it into two equal segments $\overline{AI}$ and $\overline{IB}$. In the diagram, point I is on $\overline{AB}$, but it is closer to B than to A, so the lengths of $\overline{AI}$ and $\overline{IB}$ are not equal. Therefore, I is not the midpoint.

MCQ 66: Point I is the midpoint of segment $\overline{AB}$.

Is the statement true or false?

☒ True

☐ False

Answer: The statement is true. For point I to be the midpoint of segment $\overline{AB}$, it must lie on the segment and divide it into two equal segments $\overline{AI}$ and $\overline{IB}$. In the diagram, point I is on $\overline{AB}$, and the equal segment marks indicate that $\overline{AI}$ and $\overline{IB}$ have the same length. Therefore, I is the midpoint.

MCQ 67: Line $\overleftrightarrow{EF}$ is the perpendicular bisector of segment $\overline{AB}$.
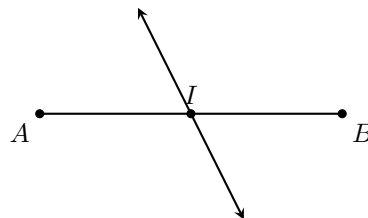
Is the statement true or false?

☐ True

☒ False

Answer: The statement is false. A perpendicular bisector of segment $\overline{AB}$ must be perpendicular to $\overline{AB}$ and pass through its midpoint. In the diagram, $\overleftrightarrow{EF}$ passes through the midpoint I of $\overline{AB}$, but it is not perpendicular to $\overline{AB}$, as the angle at I is not a right angle. Therefore, $\overleftrightarrow{EF}$ is not the perpendicular bisector.

MCQ 68: Line $\overleftrightarrow{EF}$ is the perpendicular bisector of segment $\overline{AB}$.

Is the statement true or false?

☐ True

☒ False

Answer: The statement is false. A perpendicular bisector of segment $\overline{AB}$ must be perpendicular to $\overline{AB}$ and pass through its midpoint. In the diagram, $\overleftrightarrow{EF}$ is perpendicular to $\overline{AB}$, as shown by the right-angle mark, but it passes through point I, which is not the midpoint of $\overline{AB}$. Therefore, $\overleftrightarrow{EF}$ is not the perpendicular bisector.

MCQ 69: Line $\overleftrightarrow{EF}$ is the perpendicular bisector of segment $\overline{AB}$.

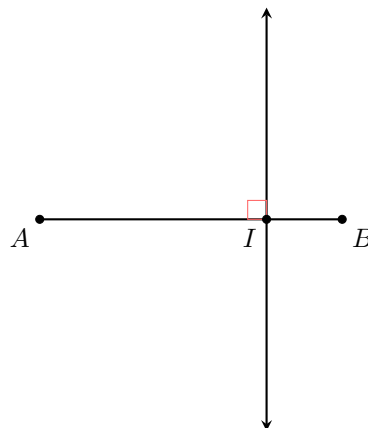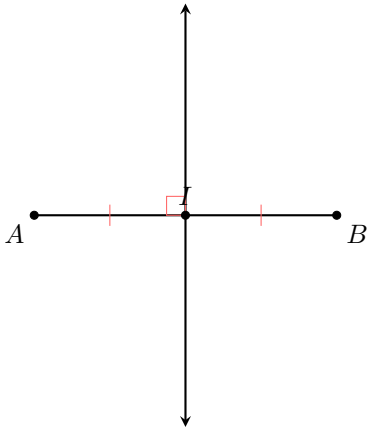**Is the statement true or false?**

☒ True

☐ False

Answer: The statement is true. A perpendicular bisector of segment $\overline{AB}$ must be perpendicular to $\overline{AB}$ and pass through its midpoint. In the diagram, $\overleftrightarrow{EF}$ is perpendicular to $\overline{AB}$, as shown by the right-angle mark, and it passes through point I, which is the midpoint of $\overline{AB}$, as indicated by the equal segment marks showing $\overline{AI} = \overline{IB}$. Therefore, $\overleftrightarrow{EF}$ is the perpendicular bisector.

H.2 BUILDING GEOMETRIC FIGURES

Ex 70: Using a ruler and pencil, draw segment $\overline{AB}$ and its midpoint I.

Answer:
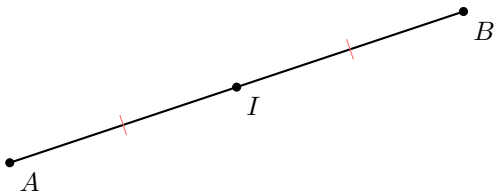
Ex 71: Using a ruler and pencil, draw segment $\overline{AB}$, its midpoint I, and the perpendicular bisector of $\overline{AB}$.

Answer:
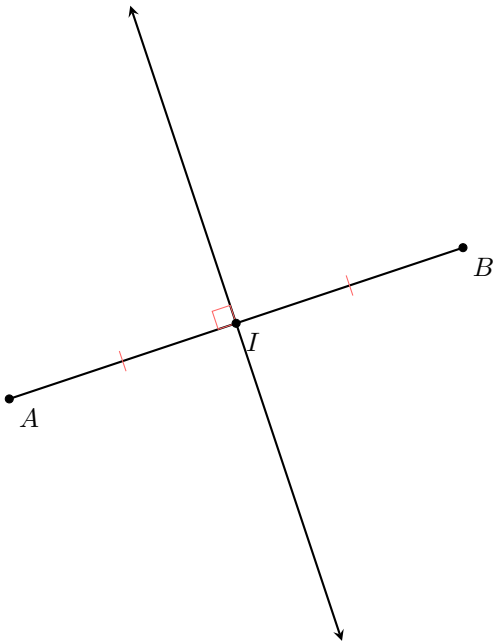
Ex 72: Using a ruler, compass, and pencil, draw triangle ABC and construct the perpendicular bisectors of its three sides $\overline{AB}$, $\overline{BC}$, and $\overline{CA}$ following the method for constructing a perpendicular bisector. Observe where the perpendicular bisectors intersect.

Answer:
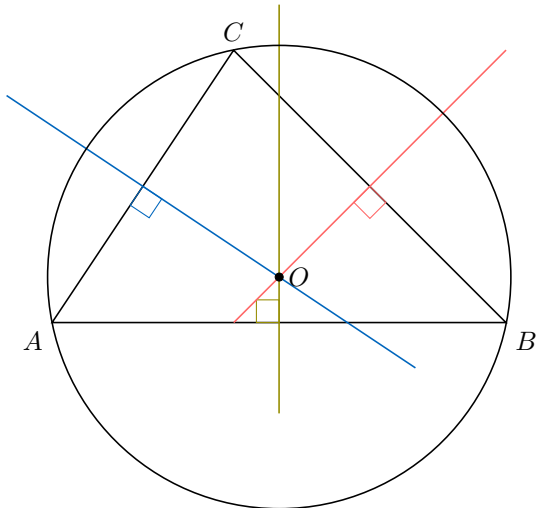
The triangle ABC is drawn with vertices A, B, and C. The perpendicular bisector of $\overline{AB}$ is constructed by drawing arcs of equal radius from A and B intersecting at two points, and drawing the line through them. Similarly, the perpendicular bisectors of $\overline{BC}$ and $\overline{CA}$ are constructed. The three perpendicular bisectors intersect at a single point O, the circumcenter. A circle centered at O with radius equal to the distance from O to A (or B or C) is drawn, passing through all three vertices A, B, and C, forming the circumcircle of the triangle.

I PROPERTIES OF PARALLEL LINES

I.1 INVESTIGATING LINE RELATIONSHIPS

MCQ 73: Given that $\overleftrightarrow{l_1}$ is perpendicular to $\overleftrightarrow{l_3}$ and $\overleftrightarrow{l_2}$ is perpendicular to $\overleftrightarrow{l_3}$, what is the relationship between $\overleftrightarrow{l_1}$ and $\overleftrightarrow{l_2}$?

☒ Parallel

☐ Perpendicular

Answer:

If ⟷ , then ⟷ .

$\overleftrightarrow{l_1}$ and $\overleftrightarrow{l_2}$ are parallel. By the geometric property, if two lines are each perpendicular to the same line, they are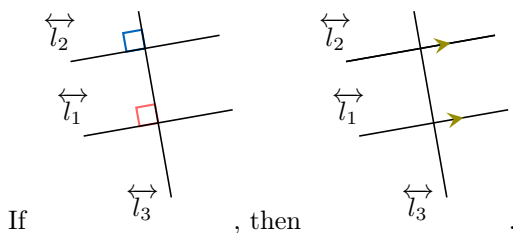 parallel. The diagram shows $\overleftrightarrow{l_1}$ perpendicular to $\overleftrightarrow{l_3}$ (right-angle mark at their intersection) and $\overleftrightarrow{l_2}$ perpendicular to $\overleftrightarrow{l_3}$ (right-angle mark at their intersection). Thus, $\overleftrightarrow{l_1}$ and $\overleftrightarrow{l_2}$ are parallel, as shown by their matching arrowheads.

MCQ 74: Given that $\overleftrightarrow{l_1}$ is perpendicular to $\overleftrightarrow{l_3}$ and $\overleftrightarrow{l_1}$ is parallel to $\overleftrightarrow{l_2}$, what is the relationship between $\overleftrightarrow{l_2}$ and $\overleftrightarrow{l_3}$?

☒ Perpendicular

☐ Parallel

Answer:

If , then .

$\overleftrightarrow{l_2}$ is perpendicular to $\overleftrightarrow{l_3}$. By the geometric property, if $\overleftrightarrow{l_1}$ is perpendicular to $\overleftrightarrow{l_3}$ and $\overleftrightarrow{l_1}$ is parallel to $\overleftrightarrow{l_2}$, then $\overleftrightarrow{l_2}$ must also be perpendicular to $\overleftrightarrow{l_3}$. In the first diagram, $\overleftrightarrow{l_1}$ is perpendicular to $\overleftrightarrow{l_3}$ (right-angle mark at their intersection), and $\overleftrightarrow{l_1}$ is parallel to $\overleftrightarrow{l_2}$. The second diagram confirms that $\overleftrightarrow{l_2}$ is perpendicular to $\overleftrightarrow{l_3}$ (right-angle mark at their intersection), and the matching arrowheads show that $\overleftrightarrow{l_1}$ and $\overleftrightarrow{l_2}$ are parallel.

MCQ 75: Given that $\overleftrightarrow{l_1}$ is parallel to $\overleftrightarrow{l_2}$ and $\overleftrightarrow{l_2}$ is parallel to $\overleftrightarrow{l_3}$, what is the relationship between $\overleftrightarrow{l_1}$ and $\overleftrightarrow{l_3}$?
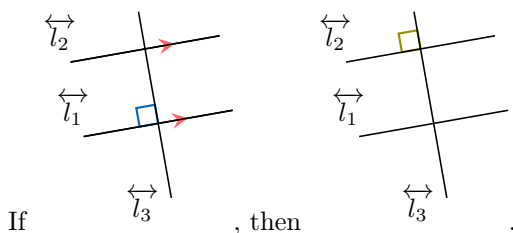
☒ Parallel

☐ Perpendicular

Answer:

If , then .
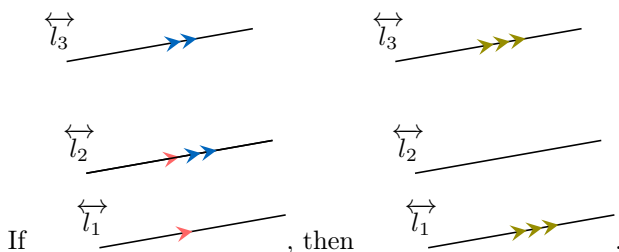
$\overleftrightarrow{l_1}$ and $\overleftrightarrow{l_3}$ are parallel. By the geometric property of parallel lines, if $\overleftrightarrow{l_1}$ is parallel to $\overleftrightarrow{l_2}$ and $\overleftrightarrow{l_2}$ is parallel to $\overleftrightarrow{l_3}$, then $\overleftrightarrow{l_1}$ is parallel to $\overleftrightarrow{l_3}$. In the first diagram, $\overleftrightarrow{l_1} \parallel \overleftrightarrow{l_2}$ (matching arrowheads) and $\overleftrightarrow{l_2} \parallel \overleftrightarrow{l_3}$ (matching arrowheads). The second diagram confirms that $\overleftrightarrow{l_1} \parallel \overleftrightarrow{l_3}$, as shown by their matching arrowheads.

MCQ 76:
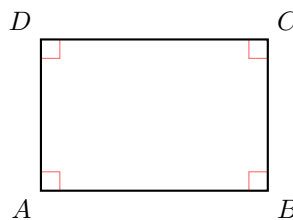
Which student correctly explains why $\overleftrightarrow{AB}$ and $\overleftrightarrow{DC}$ are parallel in rectangle $ABCD$?

☐ Su: "I see that $\overleftrightarrow{AB}$ and $\overleftrightarrow{DC}$ are parallel."

☒ Louis: "Since $\overleftrightarrow{AB}$ and $\overleftrightarrow{AD}$ are perpendicular and $\overleftrightarrow{DC}$ and $\overleftrightarrow{AD}$ are perpendicular, $\overleftrightarrow{AB}$ and $\overleftrightarrow{DC}$ are parallel."

☐ Hugo: "Since $\overleftrightarrow{AB}$ and $\overleftrightarrow{BC}$ are parallel and $\overleftrightarrow{DC}$ and $\overleftrightarrow{DA}$ are parallel, $\overleftrightarrow{AB}$ and $\overleftrightarrow{DC}$ are parallel."

Answer: Louis provides the correct explanation. In rectangle $ABCD$, as shown in the diagram, $\overleftrightarrow{AB}$ is perpendicular to $\overleftrightarrow{AD}$ (right-angle mark at A), and $\overleftrightarrow{DC}$ is perpendicular to $\overleftrightarrow{AD}$ (right-angle mark at D). By the geometric property, if two lines are perpendicular to the same line, they are parallel. Thus, $\overleftrightarrow{AB}$ and $\overleftrightarrow{DC}$ are parallel. Su's statement is true but lacks justification, making it incomplete. Hugo's explanation is incorrect because $\overleftrightarrow{AB}$ and $\overleftrightarrow{BC}$ are perpendicular, not parallel, and $\overleftrightarrow{DC}$ and $\overleftrightarrow{DA}$ are also perpendicular, not parallel.