

# ELEMENTS OF GEOMETRY

## A POINT

### Definition Point

A **point** is a single location in space, represented by a dot.



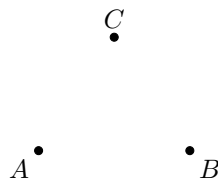
### Definition Point Notation

A point is named using a capital letter, written as **A**.



Points have no size, shape, or dimension. They simply mark a position.

**Ex:** The diagram below shows three points labeled  $A$ ,  $B$ , and  $C$ :



## B LINES, SEGMENTS AND RAYS

### Definition Line

A **line** is a straight collection of points that extends infinitely in both directions.

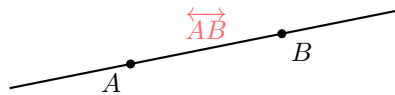


### Definition Line Notation

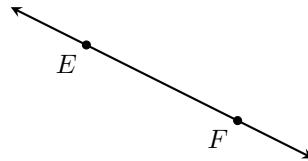
- A line can be named with a lowercase letter, written as  $\overleftrightarrow{l}$ .



- A line is named using two points on it, written as  $\overleftrightarrow{AB}$ .



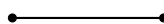
**Ex:** Name the line shown below:



**Answer:** The line is  $\overleftrightarrow{EF}$ .

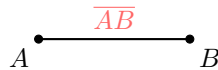
### Definition Line Segment

A **line segment** is a part of a line with two endpoints. It has a definite length.



### Definition Line Segment Notation

A line segment is named by its endpoints, written as  $\overline{AB}$ .



**Ex:** Name the segment shown below:



*Answer:* The segment is  $\overline{EF}$ .

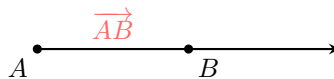
### Definition Ray

A **ray** is a part of a line that starts at one endpoint and extends infinitely in one direction.

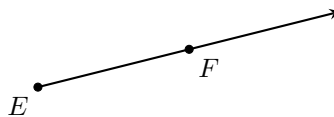


### Definition Ray Notation

A ray is named by its endpoint and another point on it, written as  $\overrightarrow{AB}$ .



**Ex:** Name the ray shown below:

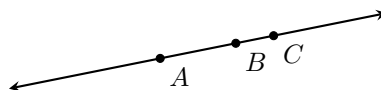


*Answer:* The ray is  $\overrightarrow{EF}$ .

### Definition Collinear Points

**Collinear points** are points that all lie on the same straight line.

**Ex:** The points  $A$ ,  $B$  and  $C$  are collinear points.

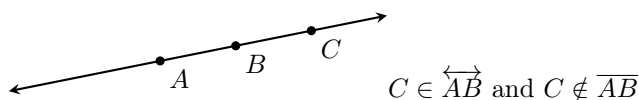


## C ELEMENT RELATION

### Definition Element Relation

The relation **is a point of** (or "is an element of") is used to show that a point lies on a geometric figure, such as a line or segment. It is denoted by the symbol  $\in$ .

**Ex:**



In this figure, point  $C$  lies on the line through points  $A$  and  $B$ , so  $C \in \overleftrightarrow{AB}$ . However,  $C$  does not lie on the segment between  $A$  and  $B$ , so  $C \notin \overline{AB}$ .

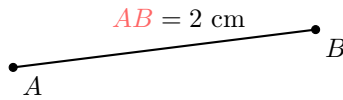
## D LENGTH

### Definition Length of a Line Segment

The **length** of a line segment is the distance between its two endpoints.

### Definition Length Notation

The length of a line segment is denoted by its endpoints, written as  $AB$ .

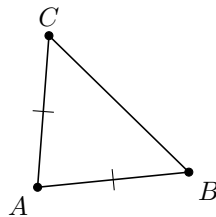


### Definition Equal Lengths

Line segments are **equal in length** if they have the same length. We use **tick marks** to show they are equal.



**Ex:** Identify two segments that have the same length.

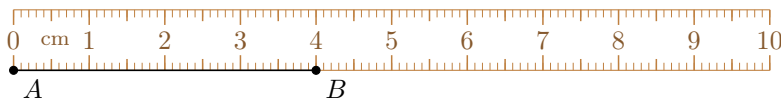


*Answer:* Segments  $\overline{AB}$  and  $\overline{AC}$  have the same length, as shown by their identical tick marks. Therefore,  $AB = AC$ .

### Method Measuring Length

We measure the length of a segment using a ruler by aligning one endpoint with the zero mark and reading the measurement at the other endpoint.

**Ex:** Measure the length of segment  $\overline{AB}$ .

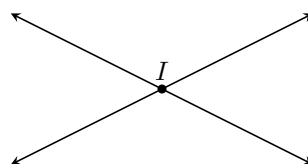


*Answer:* By aligning a ruler with segment  $\overline{AB}$ , the length is measured as  $AB = 4$  cm.

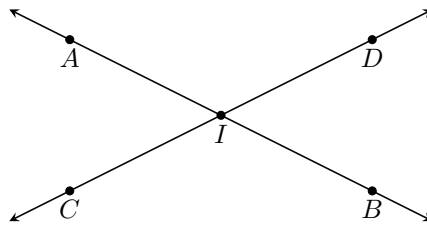
## E INTERSECTION POINT

### Definition Intersection Point

An **intersection point** is a point where two or more geometric objects, such as lines or segments, meet.



**Ex:** Find the intersection point of the lines  $\overleftrightarrow{AB}$  and  $\overleftrightarrow{CD}$ .

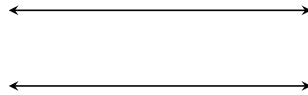


*Answer:* The intersection point is  $I$ .

## F PARALLEL LINES

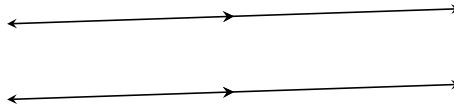
### Definition Parallel Lines

Two **parallel lines** are lines that never intersect, no matter how far they extend.



### Definition Parallel Line Notation

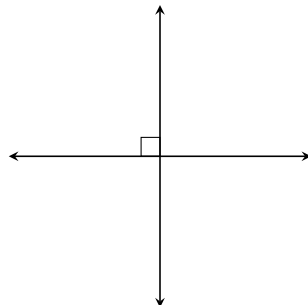
Parallel lines are indicated using matching arrowheads on each line.



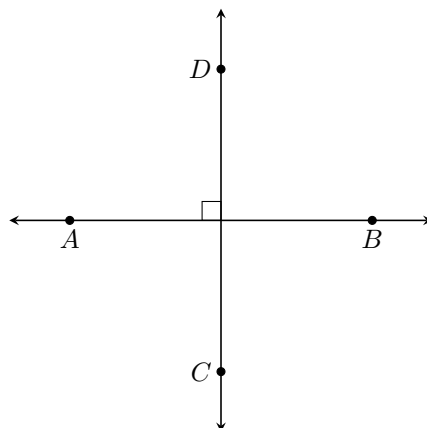
## G PERPENDICULAR LINES

### Definition Perpendicular Lines

Two **perpendicular lines** are lines that intersect at a right angle (90 degrees).



**Ex:** Identify the perpendicular lines in the figure below:

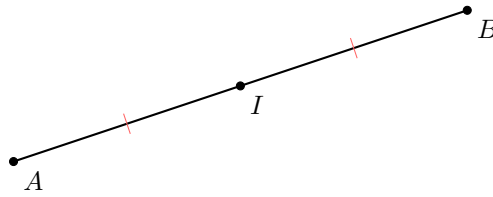


*Answer:* The lines  $\overleftrightarrow{AB}$  and  $\overleftrightarrow{CD}$  are perpendicular, as they intersect forming a right angle, indicated by the right-angle mark.

## H MIDPOINT AND PERPENDICULAR BISECTOR

### Definition Midpoint of a Line Segment

The **midpoint** of a line segment is a point that lies on the segment and divides it into two segments of equal length.

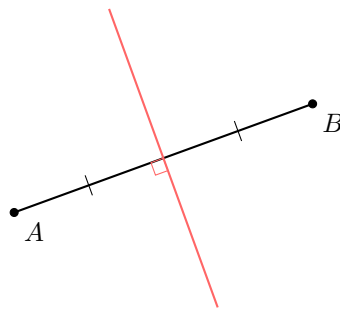


### Proposition Midpoint Length Property

If point  $I$  is the midpoint of segment  $\overline{AB}$ , then  $AB = 2 \times AI$  and  $AI = \frac{AB}{2}$ .

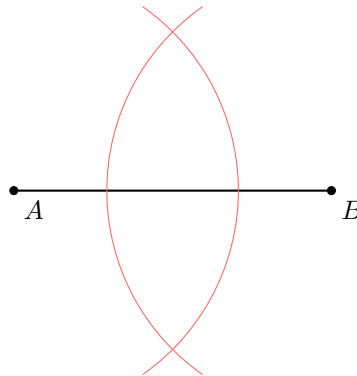
### Definition Perpendicular bisector

The **perpendicular bisector** of a line segment is a line which meets the segment at its midpoint perpendicularly.

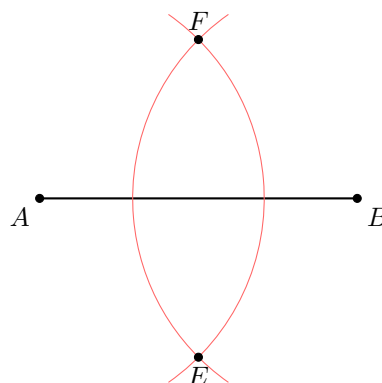


### Method Constructing the Perpendicular Bisector of $\overline{AB}$

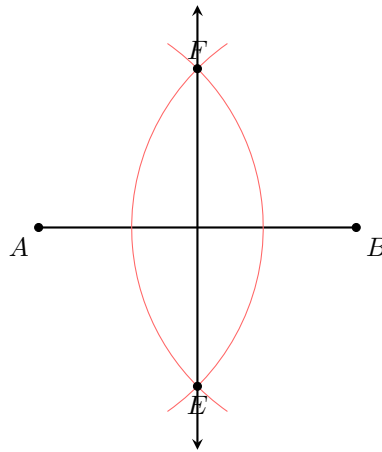
- Construct two arcs of circles with the same radius and centers at  $A$  and  $B$ .



- The arcs intersect at points  $E$  and  $F$ .



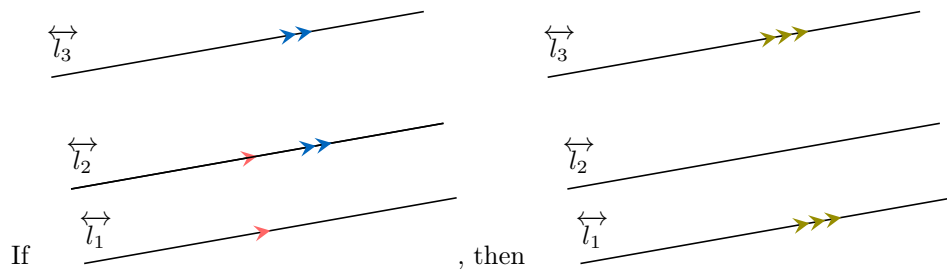
- The perpendicular bisector of  $\overline{AB}$  is the line  $\overleftrightarrow{EF}$ .



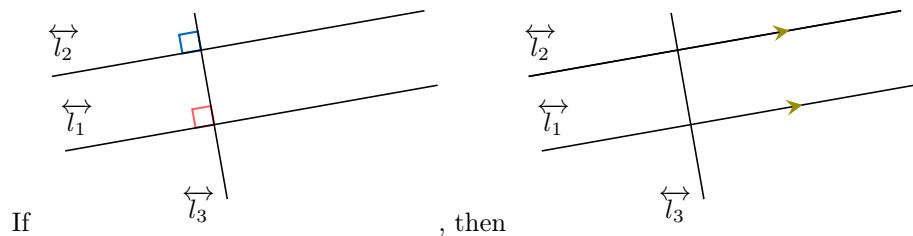
## I PROPERTIES OF PARALLEL LINES

### Proposition Properties of Parallel Lines

- If line  $\overleftrightarrow{l_1}$  is parallel to line  $\overleftrightarrow{l_2}$ , and line  $\overleftrightarrow{l_2}$  is parallel to line  $\overleftrightarrow{l_3}$ , then line  $\overleftrightarrow{l_1}$  is parallel to line  $\overleftrightarrow{l_3}$ .



- If line  $\overleftrightarrow{l_1}$  is perpendicular to line  $\overleftrightarrow{l_3}$ , and line  $\overleftrightarrow{l_2}$  is perpendicular to line  $\overleftrightarrow{l_3}$ , then line  $\overleftrightarrow{l_1}$  is parallel to line  $\overleftrightarrow{l_2}$ .



- If line  $\overleftrightarrow{l_1}$  is parallel to line  $\overleftrightarrow{l_2}$ , and line  $\overleftrightarrow{l_1}$  is perpendicular to line  $\overleftrightarrow{l_3}$ , then line  $\overleftrightarrow{l_2}$  is perpendicular to line  $\overleftrightarrow{l_3}$ .

