ELEMENTS OF GEOMETRY

A POINT

Definition Point

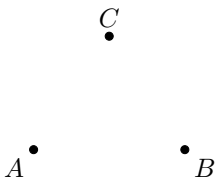
A **point** is a single location in space, represented by a dot.

Definition Point Notation

A point is named using a capital letter, written as A.

Points have no size, shape, or dimension. They simply mark a position.

Ex: The diagram below shows three points labeled A, B, and C:



B LINES, SEGMENTS AND RAYS

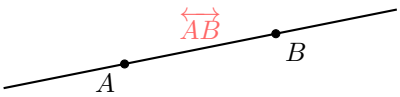
Definition Line

A **line** is a straight collection of points that extends infinitely in both directions.

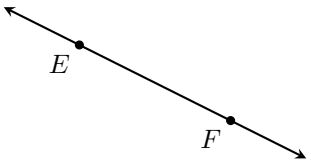
Definition Line Notation

- A line can be named with a lowercase letter, written as $\overleftrightarrow{l}$.

- A line is named using two points on it, written as $\overleftrightarrow{AB}$.



Ex: Name the line shown below:



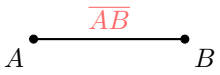
Answer: The line is $\overleftrightarrow{EF}$.

Definition Line Segment

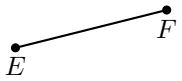
A **line segment** is a part of a line with two endpoints. It has a definite length.

Definition Line Segment Notation

A line segment is named by its endpoints, written as $\overline{AB}$.



Ex: Name the segment shown below:



Answer: The segment is $\overline{EF}$.

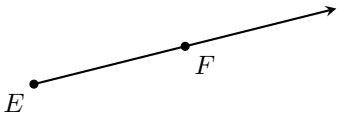
Definition Ray

A **ray** is a part of a line that starts at one endpoint and extends infinitely in one direction.

Definition Ray Notation

A ray is named by its endpoint and another point on it, written as $\overrightarrow{AB}$.

Ex: Name the ray shown below:

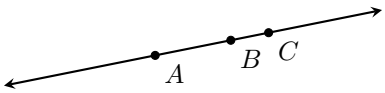


Answer: The ray is $\overrightarrow{EF}$.

Definition Collinear Points

Collinear points are points that all lie on the same straight line.

Ex: The points A, B and C are collinear points.

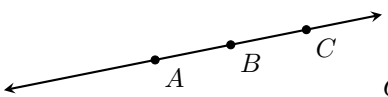


C ELEMENT RELATION

Definition Element Relation

The relation **is a point of** (or "is an element of") is used to show that a point lies on a geometric figure, such as a line or segment. It is denoted by the symbol $\in$.

Ex:



$C \in \overleftrightarrow{AB}$ and $C \notin \overline{AB}$

In this figure, point C lies on the line through points A and B, so $C \in \overleftrightarrow{AB}$. However, C does not lie on the segment between A and B, so $C \notin \overline{AB}$.

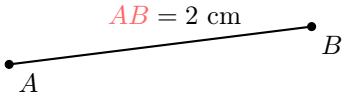
D LENGTH

Definition Length of a Line Segment

The **length** of a line segment is the distance between its two endpoints.

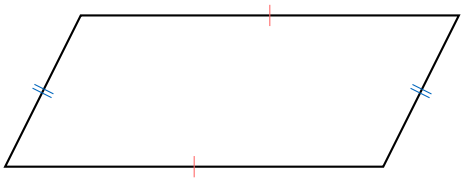
Definition Length Notation

The length of a line segment is denoted by its endpoints, written as AB.

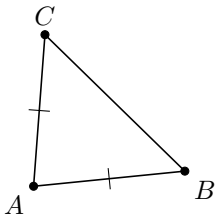


Definition Equal Lengths

Line segments are **equal in length** if they have the same length. We use **tick marks** to show they are equal.



Ex: Identify two segments that have the same length.

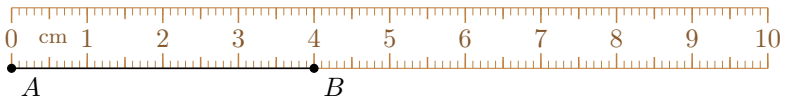


Answer: Segments $\overline{AB}$ and $\overline{AC}$ have the same length, as shown by their identical tick marks. Therefore, $AB = AC$.

Method Measuring Length

We measure the length of a segment using a ruler by aligning one endpoint with the zero mark and reading the measurement at the other endpoint.

Ex: Measure the length of segment $\overline{AB}$.

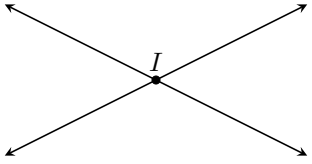


Answer: By aligning a ruler with segment $\overline{AB}$, the length is measured as $AB = 4\,\text{cm}$.

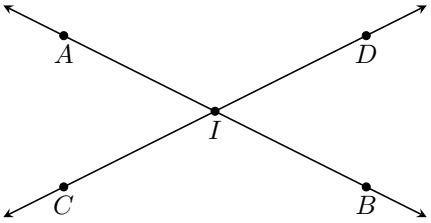
E INTERSECTION POINT

Definition Intersection Point

An **intersection point** is a point where two or more geometric objects, such as lines or segments, meet.



Ex: Find the intersection point of the lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$.

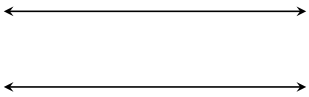


Answer: The intersection point is I.

F PARALLEL LINES

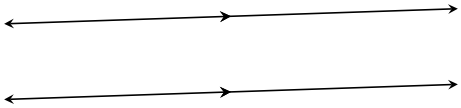
Definition Parallel Lines

Two **parallel lines** are lines that never intersect, no matter how far they extend.



Definition Parallel Line Notation

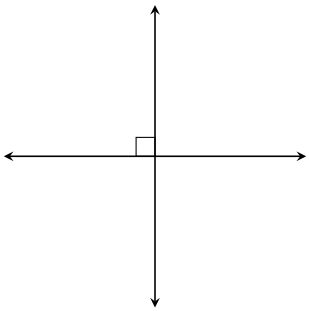
Parallel lines are indicated using matching arrowheads on each line.



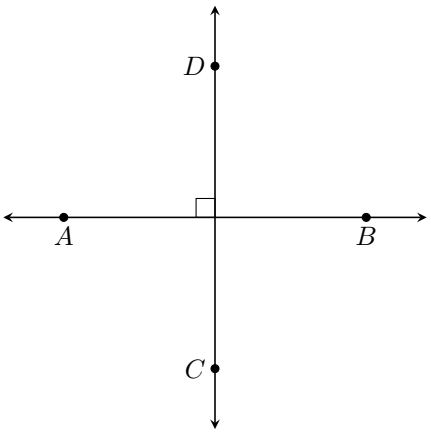
G PERPENDICULAR LINES

Definition Perpendicular Lines

Two **perpendicular lines** are lines that intersect at a right angle (90 degrees).



Ex: Identify the perpendicular lines in the figure below:

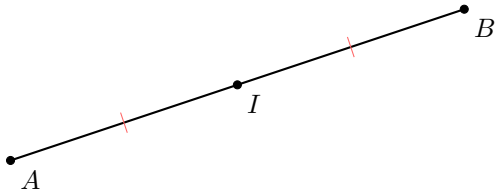


Answer: The lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are perpendicular, as they intersect forming a right angle, indicated by the right-angle mark.

H MIDPOINT AND PERPENDICULAR BISECTOR

Definition Midpoint of a Line Segment

The **midpoint** of a line segment is a point that lies on the segment and divides it into two segments of equal length.



Proposition Midpoint Length Property

If point I is the midpoint of segment $\overline{AB}$, then $AB = 2 \times AI$ and $AI = \frac{AB}{2}$.

Proof

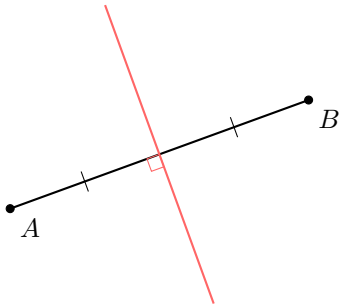
$$\begin{aligned} AB &= AI + IB \quad (I \text{ is the midpoint of } \overline{AB}) \\ &= AI + AI \\ &= 2 \times AI \end{aligned}$$

Thus, $AB = 2 \times AI$. To find AI in terms of AB, we rearrange the equation:

$$AI = \frac{AB}{2}.$$

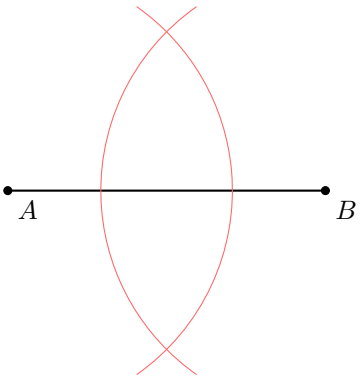
Definition Perpendicular bisector

The **perpendicular bisector** of a line segment is a line which meets the segment at its midpoint perpendicularly.

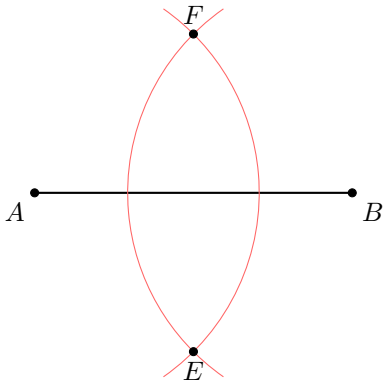


Method Constructing the Perpendicular Bisector of $\overline{AB}$

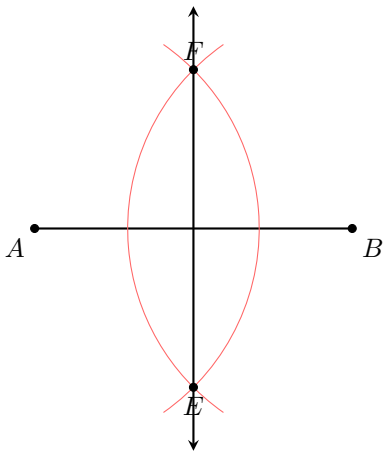
- Construct two arcs of circles with the same radius and centers at A and B.



- The arcs intersect at points E and F.



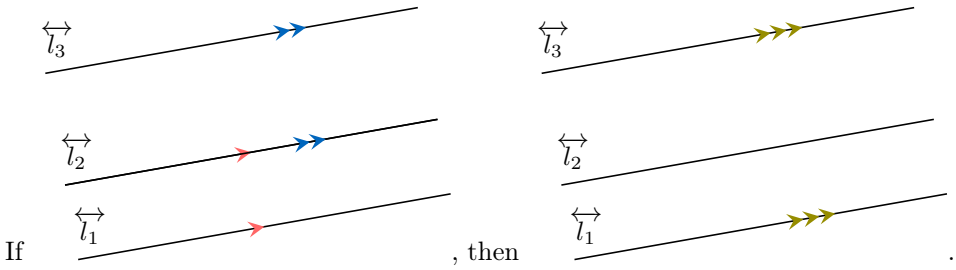
- The perpendicular bisector of $\overline{AB}$ is the line $\overleftrightarrow{EF}$.



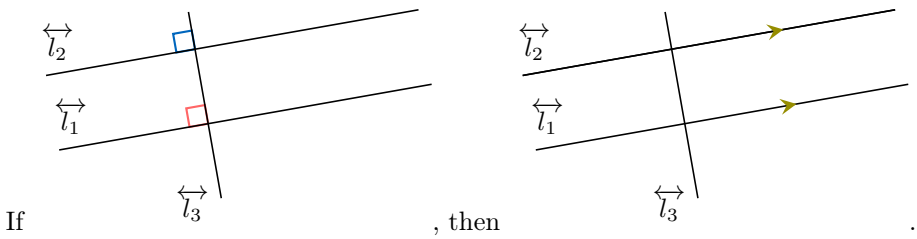
I PROPERTIES OF PARALLEL LINES

Proposition **Properties of Parallel Lines**

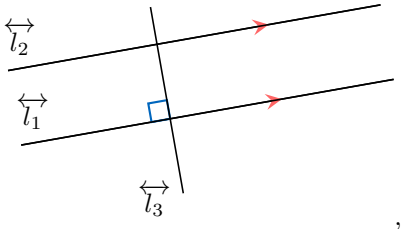
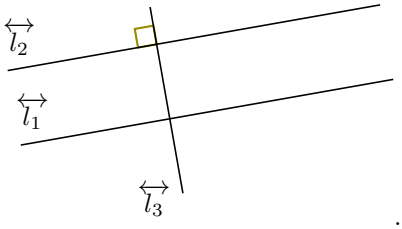
- If line $\overleftrightarrow{l_1}$ is parallel to line $\overleftrightarrow{l_2}$, and line $\overleftrightarrow{l_2}$ is parallel to line $\overleftrightarrow{l_3}$, then line $\overleftrightarrow{l_1}$ is parallel to line $\overleftrightarrow{l_3}$.



- If line $\overleftrightarrow{l_1}$ is perpendicular to line $\overleftrightarrow{l_3}$, and line $\overleftrightarrow{l_2}$ is perpendicular to line $\overleftrightarrow{l_3}$, then line $\overleftrightarrow{l_1}$ is parallel to line $\overleftrightarrow{l_2}$.



- If line $\overleftrightarrow{l_1}$ is parallel to line $\overleftrightarrow{l_2}$, and line $\overleftrightarrow{l_1}$ is perpendicular to line $\overleftrightarrow{l_3}$, then line $\overleftrightarrow{l_2}$ is perpendicular to line $\overleftrightarrow{l_3}$.

If 
$\overleftrightarrow{l_2}$
$\overleftrightarrow{l_1}$
$\overleftrightarrow{l_3}$
, then 
$\overleftrightarrow{l_2}$
$\overleftrightarrow{l_1}$
$\overleftrightarrow{l_3}$
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