DIVISION WITH REMAINDERS

In mathematics, division is used for equal sharing or grouping. Sometimes, a number cannot be shared perfectly into equal groups. The amount that is left over after sharing is called the **remainder**.

A DIVISION WITHOUT REMAINDERS

Definition **Division**

Division is the **inverse operation** of multiplication. It is the process of determining how many times one number is contained within another.
The components of a division expression are formally named:

- The **dividend**: the number that is being divided.
- The **divisor**: the number by which the dividend is divided.
- The **quotient**: the result of the division.

The operation is denoted by the **division symbol** ($\div$).

$$\text{Dividend} \div \text{Divisor} = \text{Quotient}$$

For example, the multiplication fact $3 \times 2 = 6$ corresponds to:

$$\underbrace{6}_{\text{Dividend}} \div \underbrace{3}_{\text{Divisor}} = \underbrace{2}_{\text{Quotient}} .$$

Division can be represented in several ways:

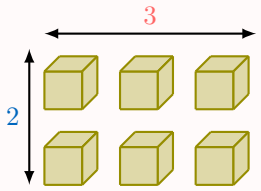
- **Numerical Form**:

$$6 \div 3 = 2$$

- **Word Form**:

Six divided by **three** equals **two**.

- **Grid Model**:



B DIVISION WITH REMAINDERS

Definition **Euclidean Division**

Euclidean Division is the process of dividing one integer (the dividend) by another (the divisor) when the division is not exact. This process yields an integer quotient and a remainder.
The components of a Euclidean division expression are formally named:

- The **dividend**: the number that is being divided.
- The **divisor**: the number by which the dividend is divided.
- The **quotient**: the whole number of times the divisor fits into the dividend.
- The **remainder**: the amount left over after the division.

This relationship is defined by the identity:

$$\text{Dividend} = (\text{Divisor} \times \text{Quotient}) + \text{Remainder}$$

Important rules:

- The remainder is always **smaller** than the divisor. (If it isn't, you can still make another group!)
- If the remainder is 0, the division is **exact** (no remainder).

Euclidean division can be represented in several ways:

- **Word Form**:

  Seven divided by **three** equals **two**, with a remainder of one

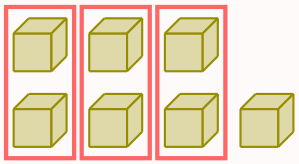
- **Division Sentence**:

$$\underbrace{7}_{\text{Dividend}} \div \underbrace{3}_{\text{Divisor}} = \underbrace{2}_{\text{Quotient}} \text{ R } \underbrace{1}_{\text{Remainder}}$$

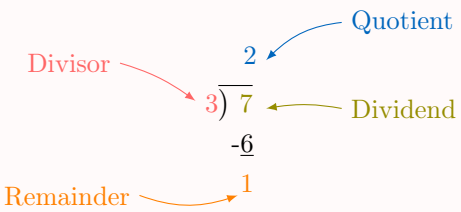
- **Euclidean Identity**:

$$\underbrace{7}_{\text{Dividend}} = (\underbrace{3}_{\text{Divisor}} \times \underbrace{2}_{\text{Quotient}}) + \underbrace{1}_{\text{Remainder}}$$

- **Group Model**:



- **Long Division Algorithm**:



C LONG DIVISION

Discover: Long division is an organized method for solving division problems. The main idea is to find how many times one number fits into another.

- **Case 1: An Exact Fit**
  To solve $12 \div 4$, we ask: "How many times does 4 fit into 12?"
  By knowing our multiplication facts, we know that $4 \times 3 = 12$. It fits exactly 3 times.
  The answer is 3.

- **Case 2: A Fit with a Remainder**

To solve $13 \div 4$, we ask: "How many times does 4 fit into 13 without going over?"

- $4 \times 3 = 12$ (This fits)
- $4 \times 3 = 16$ (This is too large)

So, 4 fits into 13 a total of 3 times. The amount left over is the remainder: $13 - 12 = 1$.
The answer is 3 with a remainder of 1.

Method **The Long Division Algorithm: Single-Step**

To divide with a remainder, like $13 \div 4$, follow these steps:

- **Set up:** Write the dividend (13) inside the division bracket and the divisor (4) on the outside.

$$4\overline{)13}$$

- **Divide:** Ask "How many times does 4 go into 13?" $4 \times 3 = \boxed{12}$ $(\leqslant 13)$, $4 \times 4 = \cancel{16}$ (> 13).
  The answer is 3. Write 3 above the line and 12 under 13.

$$\begin{array}{r} 3 \\ 4\overline{)\,13} \\ -12 \end{array}$$

- **Subtract:** Subtract 12 from 13 to find the remainder. $13 - 12 = 1$.

$$\begin{array}{r} 3 \\ 4\overline{)\,13} \\ \underline{12} \\ 1 \end{array}$$

- **Final answer:** $13 \div 4 = 3\text{R}1$, and $13 = 4 \times 3 + 1$.

Method **The Long Division Algorithm: Multi-Steps**

To divide with a remainder, like $130 \div 4$, follow these steps:

- **Set up:** Write the dividend (130) inside the bracket and the divisor (4) outside.

$$4\overline{)130}$$

- **Divide the first part (13):** "How many times does 4 go into 13?"

$$4 \times 3 = \boxed{12}\ (\leq 13), \qquad 4 \times 4 = \cancel{16}\ (> 13).$$

  Write 3 above and 12 under 13; then subtract.

$$\begin{array}{r} 3 \\ 4\overline{)\,130} \\ -12 \end{array}$$

- **Subtract and Bring down the next digit:** $13 - 12 = 1$; bring down 0 to make 10.

$$\begin{array}{r} 3 \\ 4\overline{)\,130} \\ -\underline{12}\downarrow \\ 10 \end{array}$$

- **Divide the new number (10):** "How many times does 4 go into 10?"

$$4 \times 2 = \boxed{8}\ (\leq 10), \qquad 4 \times 3 = \cancel{12}\ (> 10).$$

Write 2 above, put 8 under 10, and subtract to get the remainder.

$$\begin{array}{r} 32 \\ 4\overline{)\,130} \\ -\underline{12}\downarrow \\ 10 \\ -\underline{8} \\ 2 \end{array}$$

- **Final answer:** $130 \div 4 = 32\text{R}2$, and$130 = 4 \times 32 + 2$.

D TWO WAYS TO THINK ABOUT DIVISION

Method **The Two Models of Division**

Division answers two kinds of questions. When the total does not split evenly, we record a **remainder**.

- **Sharing.** The number of groups is known; find the size of each group (and any leftover).

$$\text{total} \div \text{number of groups} = \text{size of each group with a remainder}.$$

Example: 13 cookies are shared among 3 friends.

$$13 \text{ cookies} \div 3 \text{ friends} = 4 \text{ cookies per friend with remainder } 1 \text{ cookie}.$$

- **Grouping.** The size of each group is known; find how many full groups can be made (and what remains).

$$\text{total} \div \text{size of each group} = \text{number of groups with a remainder}.$$

Example: 13 cookies are packed in bags of 4 cookies each.

$$13 \text{ cookies} \div 4 \text{ cookies per bag} = 3 \text{ bags with remainder } 1 \text{ cookie}.$$