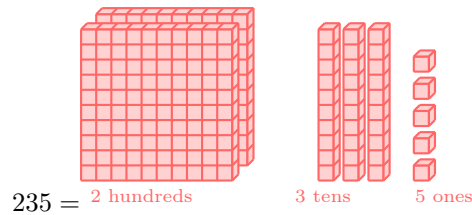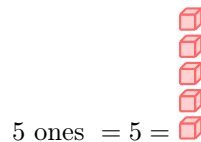# DECIMAL NUMBERS

A DEFINITION
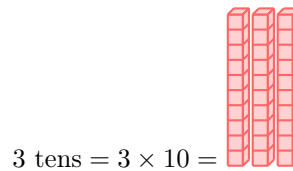
Discover: When we write numbers, the place of each digit is very important. Look at the number:

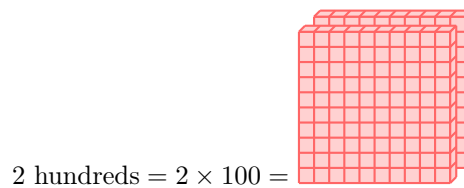$$235 = \quad \text{2 hundreds} \qquad \text{3 tens} \qquad \text{5 ones}$$

- The **5** is in the **Ones** place. That means we have 5 ones:

$$5 \text{ ones } = 5 =$$

- The **3** is in the **Tens** place. That means we have 3 tens:

$$3 \text{ tens} = 3 \times 10 =$$

- The **2** is in the **Hundreds** place. That means we have 2 hundreds:

$$2 \text{ hundreds} = 2 \times 100 =$$

When we move to the left $\leftarrow$, each place is 10 times bigger:

- 10 ones makes 1 ten:

$$= 10 \times 1 \text{ one} = 1 \text{ ten} =$$
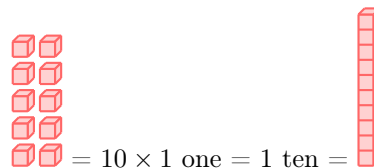
- 10 tens make 1 hundred:
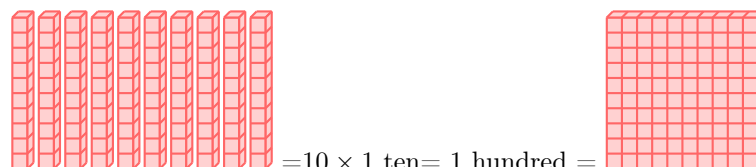
$$= 10 \times 1 \text{ ten} = 1 \text{ hundred} =$$

When we move to the right $\rightarrow$, each place is 10 times smaller:

- 1 hundred $\div 10 = 1$ ten

- 1 ten $\div 10 = 1$ one

What if we continue past the Ones place? If we divide one ☐ by 10, we get a **tenth** which is written as $\dfrac{1}{10}$ or ∥ To show where the Ones place is, we use a decimal point:

$$35.4 = \underbrace{}_{\text{3 tens}} \quad \underbrace{}_{\text{5 ones}} \quad \underbrace{}_{\text{4 tenths}}$$
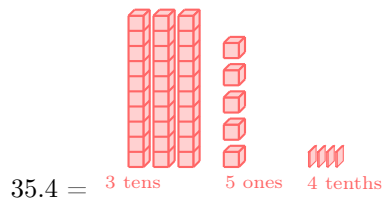
Definition Decimal Number

A **decimal number** is a number that has a decimal point. The decimal point separates the whole number part from the part that is smaller than one. We can write a decimal number in different ways:

- **Numeral form**:

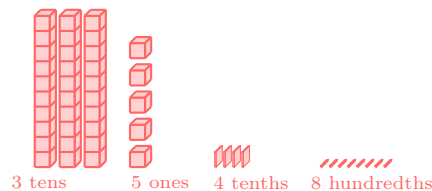$$35.48$$

- **Decimal Fraction**:

$$\frac{3548}{100}$$

- **Expanded form** :

$$\begin{array}{ccc} 3 \text{ tens} + & 5 \text{ ones} + & 4 \text{ tenths} \\ 30+ & 5+ & \dfrac{4}{10} \\ 3 \times 10+ & 5 \times 1+ & 4 \times \dfrac{1}{10} \end{array}$$

- **Table**:

Tens	Ones	.	Tenths	Hundredths
3	5	.	4	8

- **Blocks**:
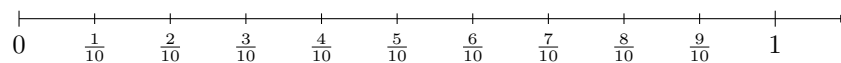
3 tens   5 ones   4 tenths   8 hundredths


B ON THE NUMBER LINE

Discover: Decimal numbers, like fractions, can be shown on a number line. A number line helps us see where the numbers go.

- **Showing Tenths on a Number Line**

  - Imagine we cut the space between 0 and 1 into 10 equal parts. Each part is called one tenth, or $\frac{1}{10}$.

  $$0 \quad \tfrac{1}{10} \quad \tfrac{2}{10} \quad \tfrac{3}{10} \quad \tfrac{4}{10} \quad \tfrac{5}{10} \quad \tfrac{6}{10} \quad \tfrac{7}{10} \quad \tfrac{8}{10} \quad \tfrac{9}{10} \quad 1$$

  - Since $\frac{1}{10} = 0.1$, we can write the number line like this:

  $$0 \quad 0.1 \quad 0.2 \quad 0.3 \quad 0.4 \quad 0.5 \quad 0.6 \quad 0.7 \quad 0.8 \quad 0.9 \quad 1$$
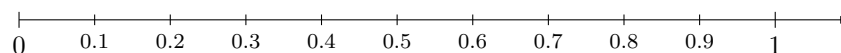
- **Showing Hundredths on a Number Line**
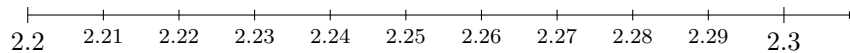 Now, let's zoom in and cut the line between 0 and 0.1 into 10 equal parts. Each tiny part is called one hundredth, or 0.01.
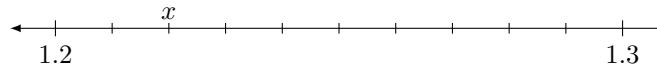
  $$0 \quad 0.01 \quad 0.02 \quad 0.03 \quad 0.04 \quad 0.05 \quad 0.06 \quad 0.07 \quad 0.08 \quad 0.09 \quad 0.1$$
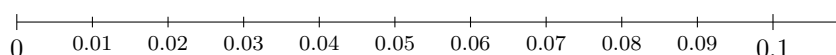
Method **How to Draw a Number Line for a Decimal Number**

Here's how to put a decimal number on a number line:

1. **Choose your range:** If your numbers are between 2.2 and 2.3, your number line starts at 2.2 and ends at 2.3.

2. **Divide into ten equal parts:** On a number line from 2.2 to 2.3, the first mark is 2.21, the second is 2.22, and so on, until you get to 2.30 (which is the same as 2.3).
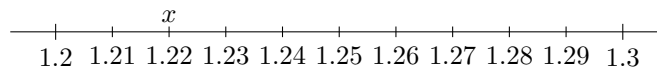
$$\begin{array}{cccccccccc} \vdash & & & & & & & & & \rightarrow \\ 2.2 & 2.21 & 2.22 & 2.23 & 2.24 & 2.25 & 2.26 & 2.27 & 2.28 & 2.29 \quad 2.3 \end{array}$$

Ex: Find the value of x.

$$\overset{x}{\underset{1.2 \qquad\qquad\qquad\qquad\qquad\qquad 1.3}{\longleftrightarrow}}$$

Answer:

- Each small division on the number line shows 0.01.

$$\overset{x}{\underset{1.2 \ \ 1.21 \ 1.22 \ 1.23 \ 1.24 \ 1.25 \ 1.26 \ 1.27 \ 1.28 \ 1.29 \ \ 1.3}{\longrightarrow}}$$

- So, $x = 1.22$

C ORDERING

Method **Comparing by decimal alignment**

1. Align the decimal points between the two numbers and add zeros if necessary.

2. Compare the digits from left to right. The first number to have a larger digit in any place value is the larger number.

Ex: Compare 6.22 and 6.3

Answer:

- Align the decimal points and add zeros:
  6.22
  6.30

- Compare from left to right: Both numbers have a 6 in the units place. Comparing the next digit (2 vs. 3) shows that $6.22 < 6.3$

- 6 ones   2 tenths   2 hundredths $<$ 6 ones   3 tenths

D ROUNDING

Method **Rounding Numbers to a Place Value**

1. Find the digit in the place you are rounding to.

2. Look at the digit to the right of it.

3. If this digit is 5 or more, add 1 to the digit in the place you are rounding to (**round up**).

4. If this digit is 4 or less, keep the digit the same (**round down**).

5. Replace all digits to the right with zeros.

Ex: Round the number 12.346 to the nearest tenth.

Answer:

- Find the digit in the tenths place: 12.346.

- Look at the digit to the right: 12.346.

- Since 4 is less than 5, keep the digit in the tenths place the same.

- Replace all digits to the right with zeros: 12.30.

The rounded number is 12.3.

E MULTIPLYING BY POWERS OF 10

When we multiply a number by 10, each digit shifts one place to the left, making the number ten times larger.

$$10 \times 2.3 = 10 \times (2 \text{ ones} + 3 \text{ tenths})$$
$$= 2 \times 10 \text{ ones} + 3 \times 10 \text{ tenths}$$
$$= 2 \text{ tens} + 3 \text{ ones}$$
$$= 23$$

Method **Multiplying by powers of 10**

- When multiplying by 10, we move the decimal point one place to the right.

- When multiplying by 100, we move the decimal point two places to the right.

- When multiplying by 1000, we move the decimal point three places to the right.

Ex: Calculate 10×5.24

Answer: $10 \times 5.24 = 52.4$

F DIVIDING BY POWERS OF 10

When we divide a number by 10, each digit shifts one place to the right, making the number ten times smaller.

$$23 \div 10 = (2 \text{ tens } + 3 \text{ ones}) \div 10$$
$$= (2 \times \text{tens} \div 10) + (3 \times \text{ones} \div 10)$$
$$= 2 \times \text{ones} + 3 \times \text{tenths}$$
$$= 2.3$$

Method **Dividing by powers of 10**

- When dividing by 10, we move the decimal point one place to the left.

- When dividing by 100, we move the decimal point two places to the left.

- When dividing by 1000, we move the decimal point three places to the left.

Ex: Calculate $23.2 \div 10$

Answer: $23.2 \div 10 = 2.32$