CURVES

A TANGENTS AND NORMALS

A.1 EQUATION OF THE TANGENT

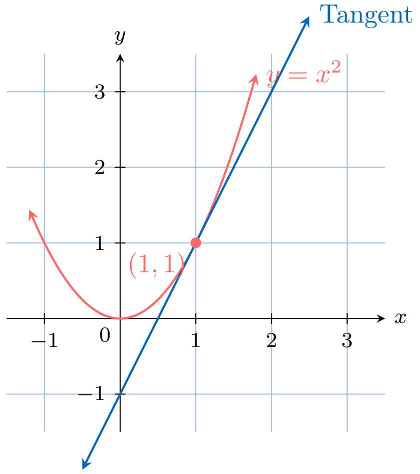
A.1.1 FINDING THE EQUATION OF THE TANGENT

Ex 1: Find the equation of the tangent to $f(x) = x^2$ at $x = 1$.

$$y = \boxed{2x - 1}$$

Answer:

- **Step 1: Find the derivative.**
  Using the power rule:
  $$\begin{aligned} f'(x) &= \frac{d}{dx}\left(x^2\right) \\ &= 2x^{2-1} \\ &= 2x \end{aligned}$$
- **Step 2: Find the coordinates of the point.**
  At $x = 1$, the y-coordinate is $f(1) = 1^2 = 1$. The point is $(1, 1)$.
- **Step 3: Find the slope of the tangent.**
  The slope is the value of the derivative at $x = 1$:
  $$m = f'(1) = 2(1) = 2$$
- **Step 4: Write the equation of the line.**
  Using $y = f'(a)(x - a) + f(a)$:
  $$\begin{aligned} y &= f'(1)(x - 1) + f(1) \\ y &= 2(x - 1) + 1 \\ y &= 2x - 2 + 1 \\ y &= 2x - 1 \end{aligned}$$

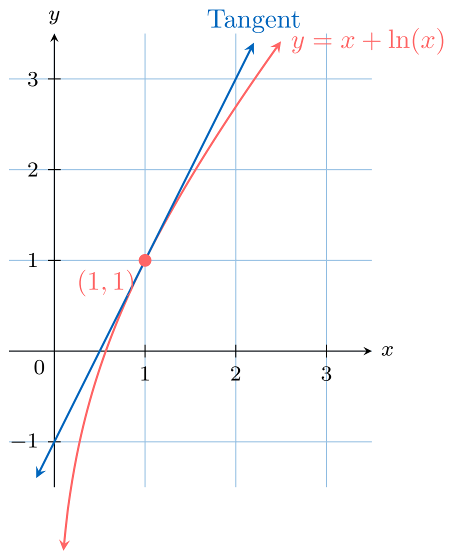


Ex 2: Find the equation of the tangent to $f(x) = x + \ln(x)$ at $x = 1$.

$$y = \boxed{2x - 1}$$

Answer:

- **Step 1: Find the derivative.**
  Using the sum rule and standard derivatives:
  $$\begin{aligned} f'(x) &= \frac{d}{dx}(x + \ln x) \\ &= \frac{d}{dx}(x) + \frac{d}{dx}(\ln x) \\ &= 1 + \frac{1}{x} \end{aligned}$$
- **Step 2: Find the coordinates of the point.**
  At $x = 1$, the y-coordinate is $f(1) = 1 + \ln(1) = 1 + 0 = 1$. The point is $(1, 1)$.
- **Step 3: Find the slope of the tangent.**
  The slope is the value of the derivative at $x = 1$:
  $$m = f'(1) = 1 + \frac{1}{1} = 2$$
- **Step 4: Write the equation of the line.**
  Using $y = f'(a)(x - a) + f(a)$:
  $$\begin{aligned} y &= f'(1)(x - 1) + f(1) \\ y &= 2(x - 1) + 1 \\ y &= 2x - 2 + 1 \\ y &= 2x - 1 \end{aligned}$$



Ex 3: Find the equation of the tangent to $f(x) = \sqrt{x^2 + 5}$ at $x = 2$.

$$y = \boxed{\frac{2}{3}x + \frac{5}{3}}$$

Answer:

- **Step 1: Find the derivative.**
  First, rewrite the function as $f(x) = (x^2 + 5)^{1/2}$. Using the chain rule:
  $$\begin{aligned} f'(x) &= \frac{d}{dx}\left((x^2 + 5)^{1/2}\right) \\ &= \frac{1}{2}(x^2 + 5)^{1/2 - 1}\frac{d}{dx}\left(x^2 + 5\right) \\ &= \frac{1}{2}(x^2 + 5)^{-1/2} \cdot (2x) \\ &= \frac{x}{\sqrt{x^2 + 5}} \end{aligned}$$

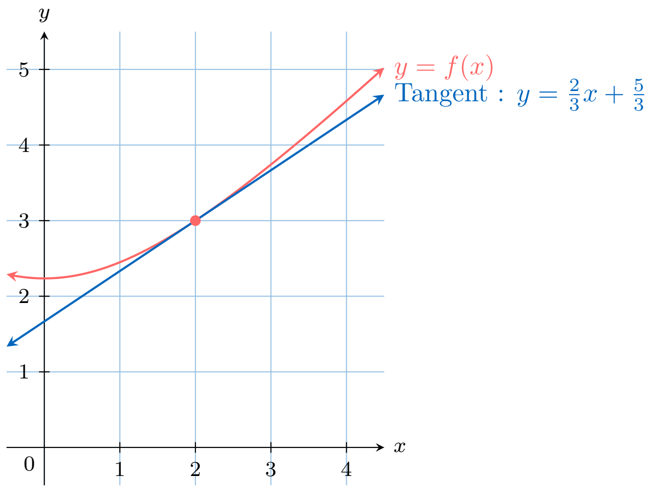
- **Step 2: Find the coordinates of the point.**
  At $x = 2$, the y-coordinate is $f(2) = \sqrt{2^2 + 5} = \sqrt{9} = 3$. The point is $(2, 3)$.

- **Step 3: Find the slope of the tangent.**
  The slope is the value of the derivative at $x = 2$:

$$m = f'(2) = \frac{2}{\sqrt{2^2 + 5}} = \frac{2}{3}$$

- **Step 4: Write the equation of the line.**

$$\begin{aligned} y &= f'(2)(x - 2) + f(2) \\ y &= \frac{2}{3}(x - 2) + 3 \\ y &= \frac{2}{3}x + \frac{5}{3} \end{aligned}$$



Ex 4: Find the equation of the tangent to $f(x) = \frac{1}{x+1}$ at $x = 1$.

$y =$ $\boxed{-1/4 * x + 3/4}$

Answer:

- **Step 1: Find the derivative.**
  Rewrite as $f(x) = (x + 1)^{-1}$. Using the chain rule:

$$\begin{aligned} f'(x) &= \frac{d}{dx}\left((x + 1)^{-1}\right) \\ &= -1(x + 1)^{-1-1} \cdot \frac{d}{dx}(x + 1) \\ &= -1(x + 1)^{-2} \cdot 1 \\ &= -\frac{1}{(x + 1)^2} \end{aligned}$$

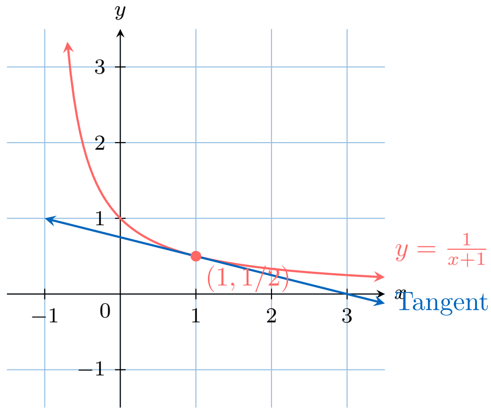
- **Step 2: Find the coordinates of the point.**
  At $x = 1$, the y-coordinate is $f(1) = \frac{1}{1+1} = \frac{1}{2}$. The point is $(1, 1/2)$.

- **Step 3: Find the slope of the tangent.**
  The slope is the value of the derivative at $x = 1$:

$$m = f'(1) = -\frac{1}{(1 + 1)^2} = -\frac{1}{4}$$

- **Step 4: Write the equation of the line.**
  Using $y = f'(a)(x - a) + f(a)$:

$$\begin{aligned} y &= f'(1)(x - 1) + f(1) \\ y &= -\frac{1}{4}(x - 1) + \frac{1}{2} \\ y &= -\frac{1}{4}x + \frac{1}{4} + \frac{1}{2} \\ y &= -\frac{1}{4}x + \frac{3}{4} \end{aligned}$$



A.2 EQUATION OF THE NORMAL

A.2.1 FINDING THE EQUATION OF THE NORMAL

Ex 5: Find the equation of the normal to $f(x) = x^2$ at $x = 1$.

$y =$ $\boxed{-1/2 * x + 3/2}$

Answer:

- **Step 1: Find the derivative.**
  Using the power rule:

$$f'(x) = 2x$$

- **Step 2: Find the coordinates of the point.**
  At $x = 1$, the y-coordinate is $f(1) = 1^2 = 1$. The point is $(1, 1)$.

- **Step 3: Find the slope of the normal.**
  First, find the slope of the tangent at $x = 1$:

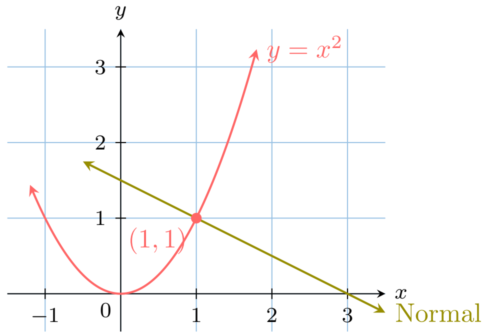
$$m_T = f'(1) = 2(1) = 2$$

  The slope of the normal is the negative reciprocal:

$$m_N = -\frac{1}{m_T} = -\frac{1}{2}$$

- **Step 4: Write the equation of the normal line.**
  Using $y = m_N(x - a) + f(a)$:

$$\begin{aligned} y &= -\frac{1}{2}(x - 1) + 1 \\ y &= -\frac{1}{2}x + \frac{1}{2} + 1 \\ y &= -\frac{1}{2}x + \frac{3}{2} \end{aligned}$$



Ex 6: Find the equation of the normal to $f(x) = x + \ln(x)$ at $x = 1$.

$$y = \boxed{-1/2 * x + 3/2}$$

Answer:

- **Step 1: Find the derivative.**

$$f'(x) = 1 + \frac{1}{x}$$

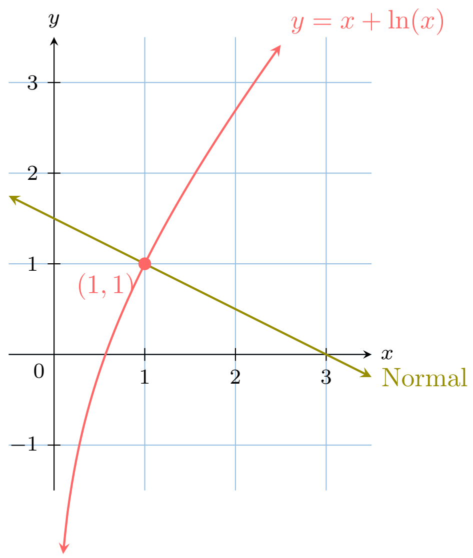
- **Step 2: Find the coordinates of the point.**
At $x = 1$, the y-coordinate is $f(1) = 1 + \ln(1) = 1$. The point is $(1, 1)$.

- **Step 3: Find the slope of the normal.**
The slope of the tangent is $m_T = f'(1) = 1 + \frac{1}{1} = 2$. The slope of the normal is the negative reciprocal:

$$m_N = -\frac{1}{m_T} = -\frac{1}{2}$$

- **Step 4: Write the equation of the normal line.**
Using $y = m_N(x - a) + f(a)$:

$$y = -\frac{1}{2}(x-1) + 1$$
$$y = -\frac{1}{2}x + \frac{1}{2} + 1$$
$$y = -\frac{1}{2}x + \frac{3}{2}$$



Ex 7: Find the equation of the normal to $f(x) = \frac{e^x}{x^2+1}$ at $x = 1$.

$$x = \boxed{1}$$

Answer:

- **Step 1: Find the derivative.**
Using the quotient rule:

$$f'(x) = \frac{e^x(x^2+1) - e^x(2x)}{(x^2+1)^2} = \frac{e^x(x^2-2x+1)}{(x^2+1)^2} = \frac{e^x(x-1)^2}{(x^2+1)^2}$$

- **Step 2: Find the coordinates of the point.**
At $x = 1$, the y-coordinate is $f(1) = \frac{e^1}{1^2+1} = \frac{e}{2}$. The point is $(1, e/2)$.

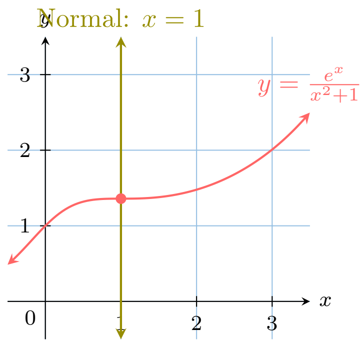
- **Step 3: Find the slope of the normal.**
The slope of the tangent at $x = 1$ is:

$$m_T = f'(1) = \frac{e^1(1-1)^2}{(1^2+1)^2} = \frac{0}{4} = 0$$

A tangent with zero slope is horizontal. The normal line must therefore be a **vertical line**.

- **Step 4: Write the equation of the normal line.**
A vertical line passing through the point $(1, e/2)$ has the equation:

$$x = 1$$



Ex 8: Find the equation of the normal to $f(x) = (x+1)\cos(x)$ at $x = 0$.

$$y = \boxed{-x + 1}$$

Answer:

- **Step 1: Find the derivative.**
Using the product rule:

$$f'(x) = (1)(\cos x) + (x+1)(-\sin x) = \cos x - (x+1)\sin x$$

- **Step 2: Find the coordinates of the point.**
At $x = 0$, the y-coordinate is $f(0) = (0+1)\cos(0) = 1 \cdot 1 = 1$. The point is $(0, 1)$.

- **Step 3: Find the slope of the normal.**
The slope of the tangent at $x = 0$ is:

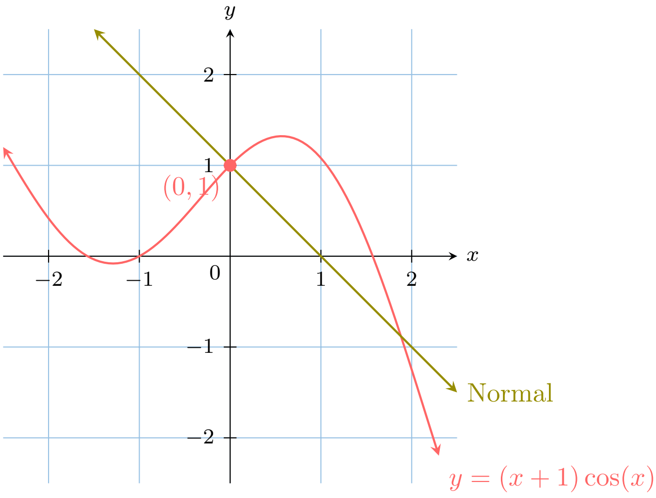
$$m_T = f'(0) = \cos(0) - (0+1)\sin(0) = 1 - 1(0) = 1$$

The slope of the normal is the negative reciprocal:

$$m_N = -\frac{1}{m_T} = -1$$

- **Step 4: Write the equation of the normal line.**
Using $y = m_N(x - a) + f(a)$:

$$y = -1(x - 0) + 1$$
$$y = -x + 1$$

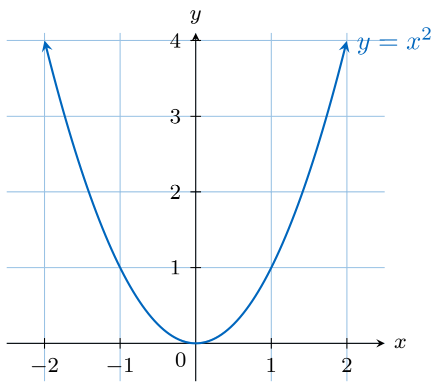


B INCREASING AND DECREASING FUNCTIONS

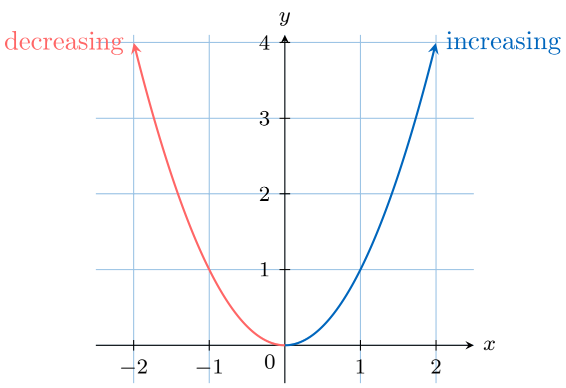
B.1 DEFINITION

B.1.1 DETERMINING VARIATIONS GRAPHICALLY

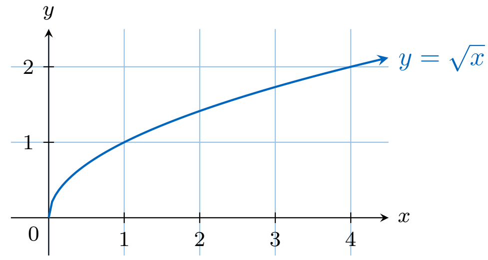
Ex 9: Graphically, find the variations for the function $f(x) = x^2$.



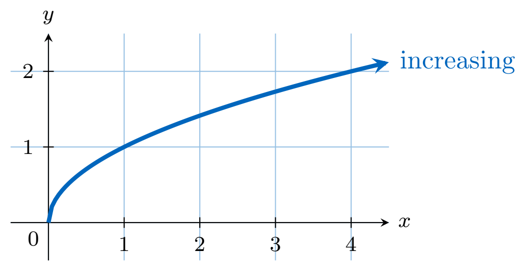
Answer: The function f is **decreasing** on the interval $(-\infty, 0]$ and **increasing** on the interval $[0, +\infty)$.



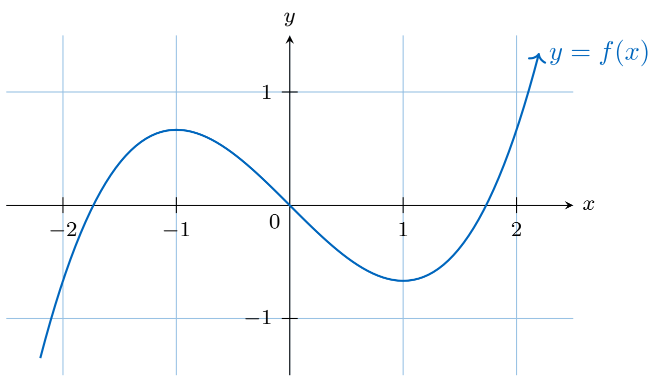
Ex 10: Graphically, find the variations for the function $f(x) = \sqrt{x}$.



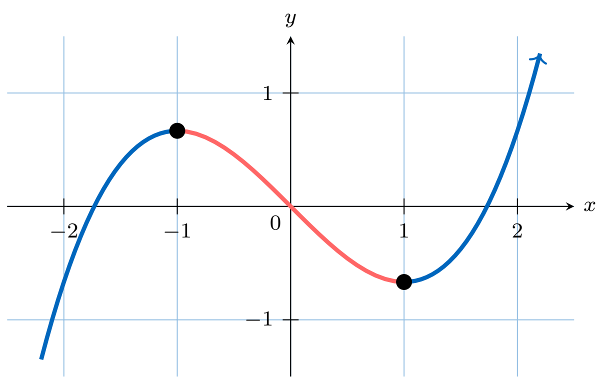
Answer: The function f is **increasing** on its entire domain, $[0, +\infty)$.



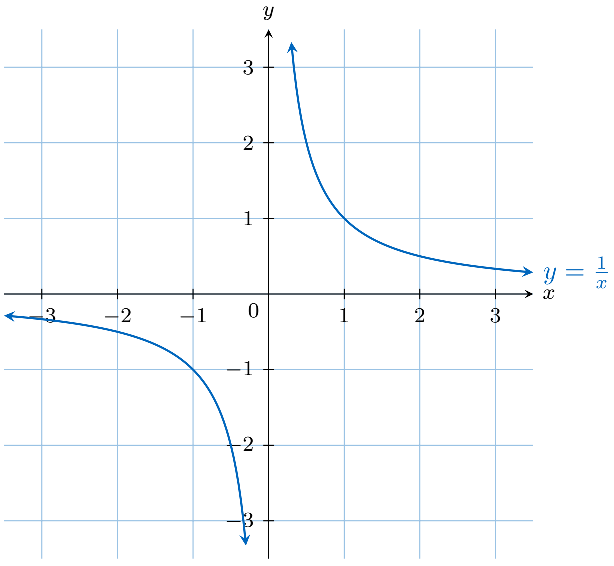
Ex 11: Graphically, find the variations for the function $f(x) = \frac{x^3}{3} - x$.



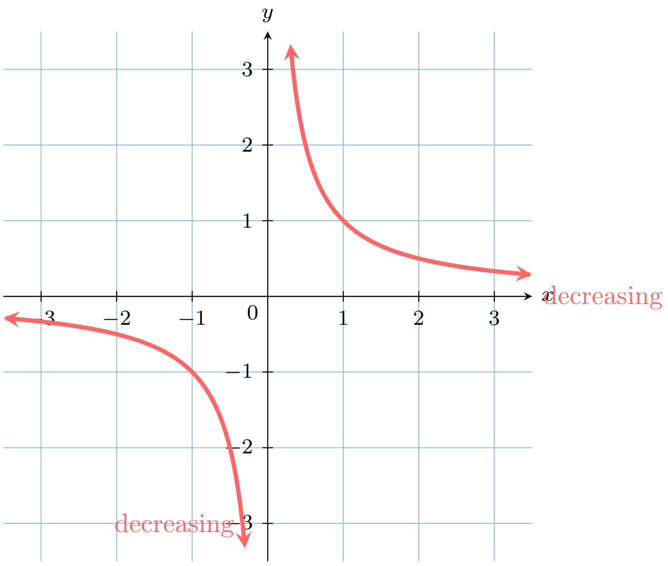
Answer: The function is **increasing** on $(-\infty, -1]$, **decreasing** on $[-1, 1]$, and **increasing** again on $[1, \infty)$.



Ex 12: Graphically, find the variations for the function $f(x) = \frac{1}{x}$.



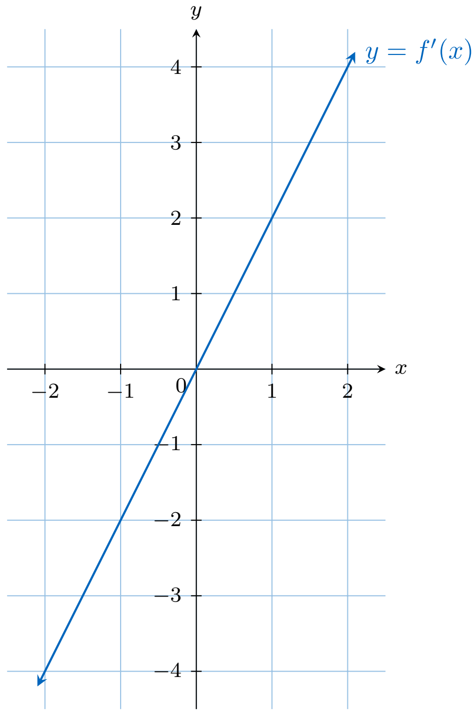
Answer: The function f is **decreasing** on the interval $(-\infty, 0)$ and also **decreasing** on the interval $(0, +\infty)$.



B.2 FIRST DERIVATIVE TEST

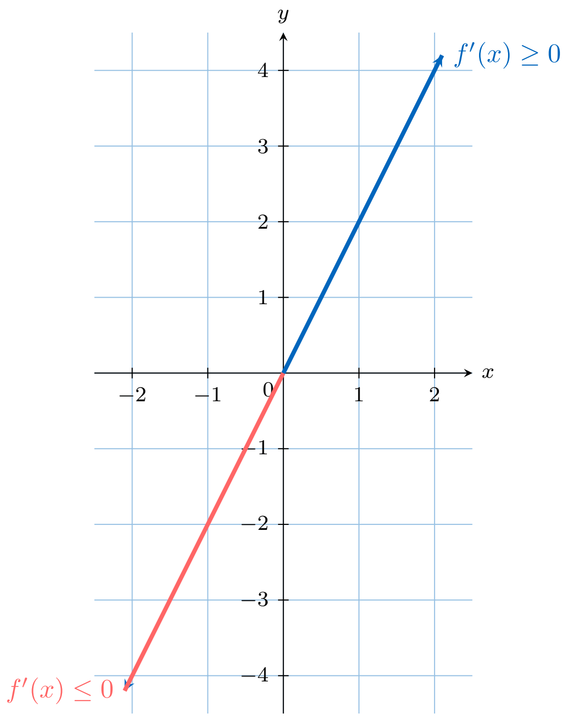
B.2.1 DETERMINING VARIATIONS FROM THE DERIVATIVE GRAPH

Ex 13: The graph of the derivative function, $f'(x) = 2x$, is shown below. Use it to determine the variations of the original function, f.

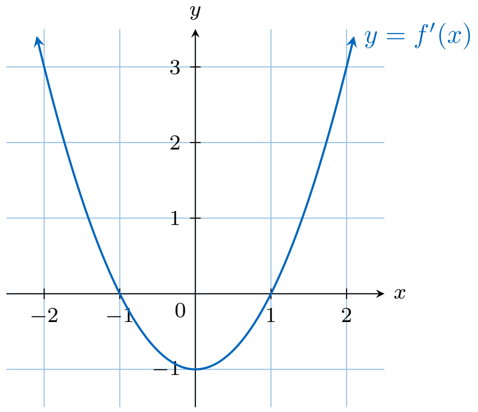


Answer: We analyze the sign of the derivative function $f'(x)$ from its graph.

- The graph of $y = f'(x)$ is below the x-axis for $x < 0$, so $f'(x) \leq 0$ on $(-\infty, 0]$. Therefore, the original function f is **decreasing** on this interval.
- The graph of $y = f'(x)$ is above the x-axis for $x > 0$, so $f'(x) \geq 0$ on $[0, +\infty)$. Therefore, the original function f is **increasing** on this interval.

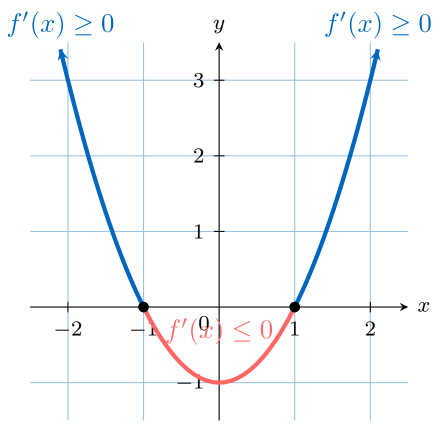


Ex 14: The graph of the derivative function, $f'(x) = x^2 - 1$, is shown below. Use it to determine the variations of the original function, f.

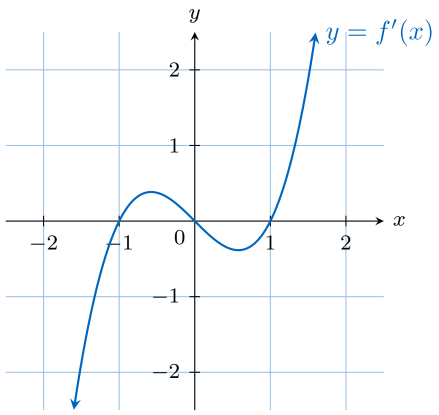


Answer: We analyze the sign of the derivative function $f'(x)$ by observing where its graph is above or below the x-axis. The graph crosses the x-axis at $x = -1$ and $x = 1$.

- For $x < -1$, the graph of $f'(x)$ is above the axis, so $f'(x) \geq 0$ on $(-\infty, -1]$. Therefore, f is **increasing**.
- For $-1 < x < 1$, the graph of $f'(x)$ is below the axis, so $f'(x) \leq 0$ on $[-1, 1]$. Therefore, f is **decreasing**.
- For $x > 1$, the graph of $f'(x)$ is above the axis, so $f'(x) \geq 0$ on $[1, \infty)$. Therefore, f is **increasing**.

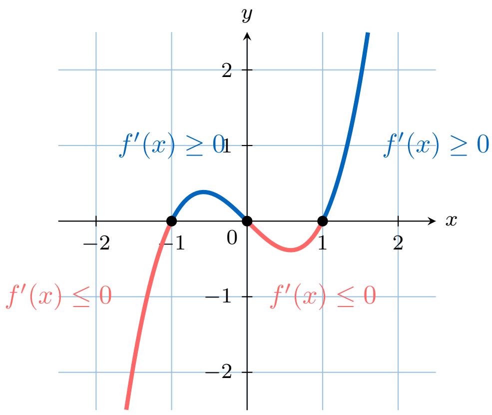


Ex 15: The graph of the derivative function, $f'(x) = x^3 - x$, is shown below. Use it to determine the variations of the original function, f.



Answer: We analyze the sign of the derivative function $f'(x)$ by observing where its graph is above or below the x-axis. The graph crosses the x-axis at $x = -1$, $x = 0$ and $x = 1$.

- For $x < -1$, the graph of $f'(x)$ is below the axis, so $f'(x) \leq 0$ on $(-\infty, -1]$. Therefore, f is **decreasing**.
- For $-1 < x < 0$, the graph of $f'(x)$ is above the axis, so $f'(x) \geq 0$ on $[-1, 0]$. Therefore, f is **increasing**.
- For $0 < x < 1$, the graph of $f'(x)$ is below the axis, so $f'(x) \leq 0$ on $[0, 1]$. Therefore, f is **decreasing**.
- For $x > 1$, the graph of $f'(x)$ is above the axis, so $f'(x) \geq 0$ on $[1, \infty)$. Therefore, f is **increasing**.



B.2.2 STUDYING THE VARIATIONS OF STANDARD FUNCTIONS

Ex 16: Prove that $f(x) = \sqrt{x}$ is an increasing function on its domain.

Answer: The domain of the function $f(x) = \sqrt{x}$ is $[0, \infty)$.

1. **Find the derivative:**

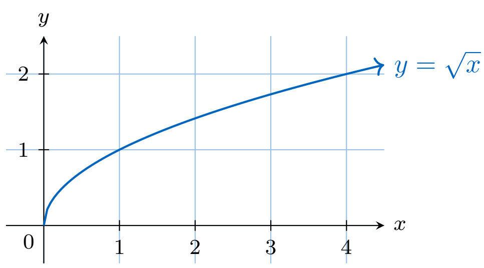
$$f(x) = x^{1/2} \implies f'(x) = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$$

2. **Analyze the sign of the derivative:**
The domain of the derivative f' is $(0, \infty)$.

$$\begin{aligned} x &> 0 \\ 2\sqrt{x} &> 0 \\ \frac{1}{2\sqrt{x}} &> 0 \end{aligned}$$

Therefore, f' is always positive.

3. **Conclusion:**
Since $f'(x) > 0$ for all $x > 0$, $f : x \mapsto \sqrt{x}$ is increasing on its entire domain, $[0, \infty)$.



Ex 17: Prove that $f(x) = \ln(x)$ is an increasing function on its domain.

Answer: The domain of the function $f(x) = \ln(x)$ is $(0, \infty)$.

1. **Find the derivative:**

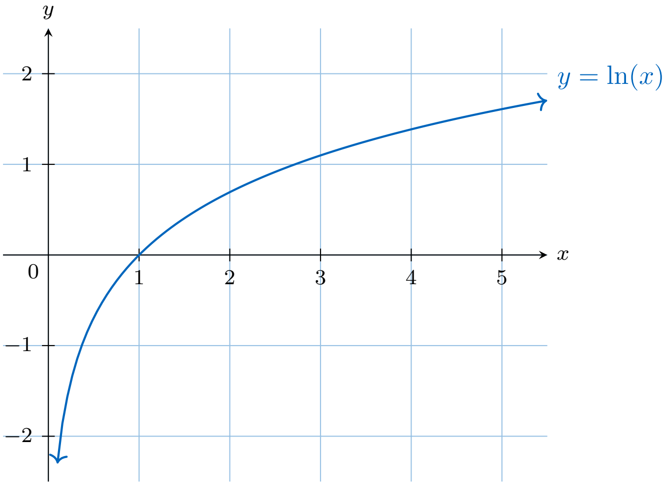
$$f'(x) = \frac{1}{x}$$

2. **Analyze the sign of the derivative:**
The domain of the derivative f' is also $(0, \infty)$.

$$x > 0$$
$$\frac{1}{x} > 0$$

Therefore, f' is always positive on its domain.

3. **Conclusion:**
Since $f'(x) > 0$ for all $x > 0$, the function $f : x \mapsto \ln(x)$ is increasing on its entire domain, $(0, \infty)$.



B.2.3 STUDYING FUNCTION VARIATIONS

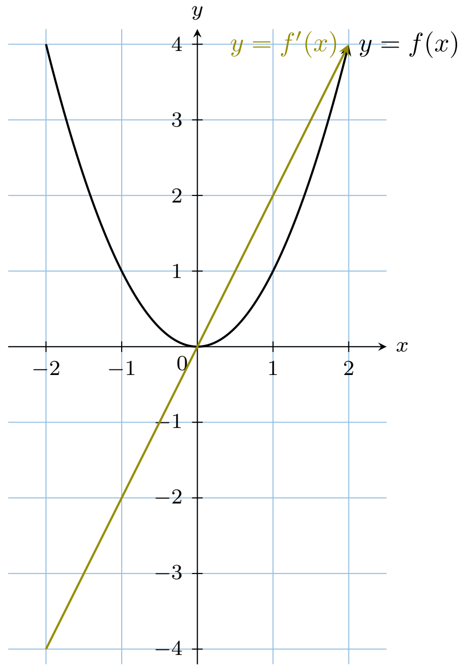
Ex 18: Find the variations of the function $f(x) = x^2$.

Answer: The derivative of the function $f(x) = x^2$ is $f'(x) = 2x$.

- The derivative $f'(x)$ is non-positive on $(-\infty, 0]$, so the function $f(x)$ is decreasing on $(-\infty, 0]$.
- The derivative $f'(x)$ is non-negative on $[0, +\infty)$, so the function $f(x)$ is increasing on $[0, +\infty)$.

This is summarized in the table of variations:

x	$-\infty$		0		$+\infty$
Signs of $f'(x)$		$-$	0	$+$	
Variations of f		↘	0	↗	



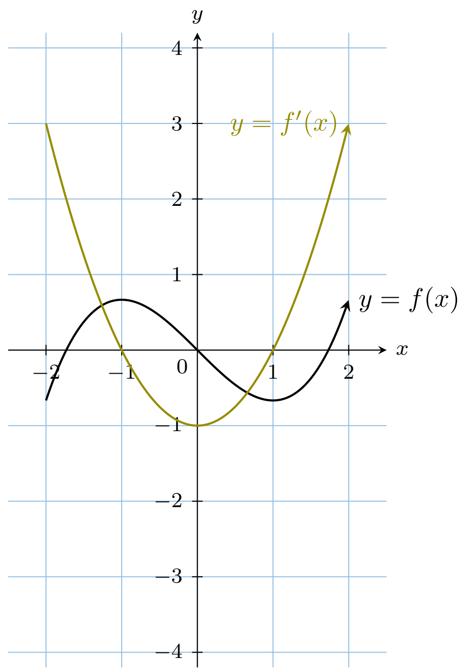
Ex 19: Find the variations of the function $f(x) = \frac{x^3}{3} - x$.

Answer: The derivative of the function $f(x) = \frac{x^3}{3} - x$ is $f'(x) = x^2 - 1$.

- The derivative $f'(x)$ is non-positive on $[-1, 1]$, so the function $f(x)$ is decreasing on $[-1, 1]$.
- The derivative $f'(x)$ is non-negative on $(-\infty, -1] \cup [1, +\infty)$, so the function $f(x)$ is increasing on $(-\infty, -1]$ and $[1, +\infty)$.

This is summarized in the table of variations:

x	$-\infty$		-1		1		$+\infty$
Signes de $f'(x)$		$+$	0	$-$	0	$+$	
Variations de f		↗		↘		↗	



Ex 20: Find the variations of the function $f(x) = \frac{x^3}{3} - \frac{3x^2}{2} + 2x - 1$.

Answer: The derivative of the function is $f'(x) = x^2 - 3x + 2$. To find the stationary points, we solve $f'(x) = 0$:

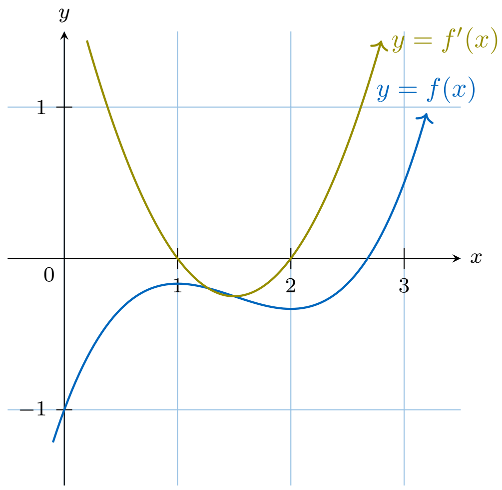
$$x^2 - 3x + 2 = 0 \implies (x-1)(x-2) = 0$$

The roots are $x = 1$ and $x = 2$.

- The derivative $f'(x)$ is non-negative on $(-\infty, 1] \cup [2, \infty)$, so $f(x)$ is increasing on these intervals.
- The derivative $f'(x)$ is non-positive on $[1, 2]$, so $f(x)$ is decreasing on this interval.

This is summarized in the table of variations:

x	$-\infty$		1		2		$+\infty$
Sign of $f'(x)$		$+$	0	$-$	0	$+$	
Variations of f		↗		↘		↗	



B.2.4 STUDYING FUNCTION VARIATIONS: LEVEL 2

Ex 21: Let $f(x) = \ln(x) - \frac{x^2}{2}$.

1. Show that $f'(x) = \frac{(1-x)(1+x)}{x}$.
2. Draw the sign diagram for $f'(x)$.
3. Hence, find the intervals where $y = f(x)$ is increasing or decreasing.

Answer: The domain of $f(x)$ is $x \in (0, \infty)$ because of the natural logarithm.

1. **Show that $f'(x) = \frac{(1-x)(1+x)}{x}$:**
   Differentiate term by term:

$$f'(x) = \frac{d}{dx}(\ln x) - \frac{d}{dx}\left(\frac{x^2}{2}\right) = \frac{1}{x} - \frac{2x}{2} = \frac{1}{x} - x$$

   Combine into a single fraction:

$$f'(x) = \frac{1}{x} - \frac{x^2}{x} = \frac{1 - x^2}{x}$$

   Factor the numerator as a difference of squares:

$$f'(x) = \frac{(1-x)(1+x)}{x}$$

2. **Sign diagram for $f'(x)$:**
   We analyze the sign of $f'(x)$ on the domain $(0, \infty)$.
   - The denominator x is always positive.
   - The numerator $(1-x)(1+x)$ simplifies to $1 - x^2$. As $a = -1 < 0$ it is a downward-opening parabola with roots at $x = -1$ and $x = 1$.

x	0		1		$+\infty$
Sign of $f'(x)$	‖	$+$	0	$-$	

3. **Intervals of increase and decrease:**
   Based on the sign diagram for the domain $x > 0$:
   - $f(x)$ is **increasing** when $f'(x) \geq 0$, which is on the interval $(0, 1]$.
   - $f(x)$ is **decreasing** when $f'(x) \leq 0$, which is on the interval $[1, \infty)$.

Ex 22: Let $f(x) = \dfrac{2-x}{x-1}$.

1. Show that $f'(x) = -\frac{1}{(x-1)^2}$.
2. Draw the sign diagram for $f'(x)$.
3. Hence, find the intervals where $y = f(x)$ is increasing or decreasing.

Answer:

1. **Show that $f'(x) = -\frac{1}{(x-1)^2}$:**
   Using the quotient rule with $u(x) = 2 - x$ and $v(x) = x - 1$:

$$u'(x) = -1, \quad v'(x) = 1$$

$$\begin{aligned} f'(x) &= \frac{u'(x)v(x) - u(x)v'(x)}{[v(x)]^2} \\ &= \frac{(-1)(x-1) - (2-x)(1)}{(x-1)^2} \\ &= \frac{-x + 1 - 2 + x}{(x-1)^2} \\ &= \frac{-1}{(x-1)^2} \end{aligned}$$

2. **Sign diagram for $f'(x)$:**
   The numerator is a constant, -1. The denominator, $(x-1)^2$, is always positive for $x \neq 1$. Therefore, the derivative $f'(x)$ is always negative where it is defined. The derivative is undefined at the vertical asymptote $x = 1$.

x	$-\infty$		1		$+\infty$
Sign of $f'(x)$		$-$	‖	$-$	

3. **Intervals of increase and decrease:**
   Based on the sign diagram:

   - $f(x)$ is **decreasing** when $f'(x) < 0$, which is on its entire domain: $(-\infty, 1)$ and $(1, \infty)$.

Ex 23: Let $f(x) = x + \dfrac{9}{x}$.

1. Show that $f'(x) = \dfrac{(x+3)(x-3)}{x^2}$.
2. Draw the sign diagram for $f'(x)$.
3. Hence, find the intervals where $y = f(x)$ is increasing or decreasing.

Answer:

1. **Show that $f'(x) = \dfrac{(x+3)(x-3)}{x^2}$:**
   First, rewrite the function with a negative exponent: $f(x) = x + 9x^{-1}$. Now, differentiate term by term using the power rule:
   $$f'(x) = 1 + 9(-1x^{-2}) = 1 - 9x^{-2}$$
   Combine into a single fraction:
   $$f'(x) = 1 - \frac{9}{x^2} = \frac{x^2}{x^2} - \frac{9}{x^2} = \frac{x^2 - 9}{x^2}$$
   Factor the numerator as a difference of squares:
   $$f'(x) = \frac{(x+3)(x-3)}{x^2}$$
2. **Sign diagram for $f'(x)$:**
   The sign of $f'(x)$ is determined by its numerator, $(x+3)(x-3)$, as the denominator x^2 is always positive for $x \neq 0$. The derivative is undefined at $x = 0$. The roots of the numerator are $x = -3$ and $x = 3$.

x	$-\infty$		-3		0		3		$+\infty$
Sign of $f'(x)$		$+$	0	$-$	‖	$-$	0	$+$	

3. **Intervals of increase and decrease:**
   Based on the sign diagram:

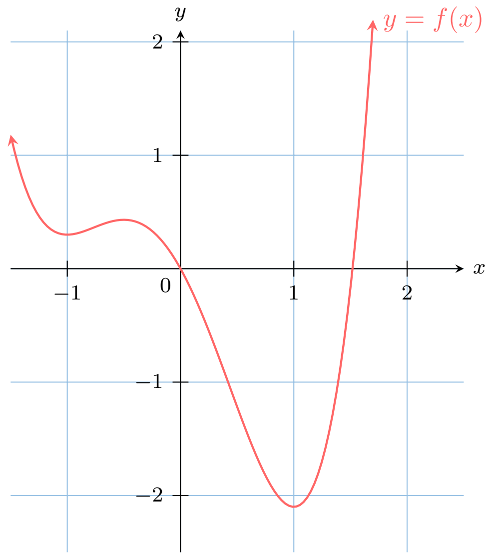
   - $f(x)$ is **increasing** when $f'(x) \geq 0$, which is on the intervals $(-\infty, -3]$ and $[3, \infty)$.
   - $f(x)$ is **decreasing** when $f'(x) \leq 0$, which is on the intervals $[-3, 0)$ and $(0, 3]$.

C EXTREMA OF FUNCTIONS

C.1 DEFINITIONS

C.1.1 IDENTIFYING EXTREMA FROM A GRAPH

MCQ 24: Consider the function f whose graph is shown below. Which of the following statements is true?



☐ The function has a global minimum at $x = -1$ and a local minimum at $x = 1$.

☐ The function has global minima at $x = -1$ and $x = 1$.

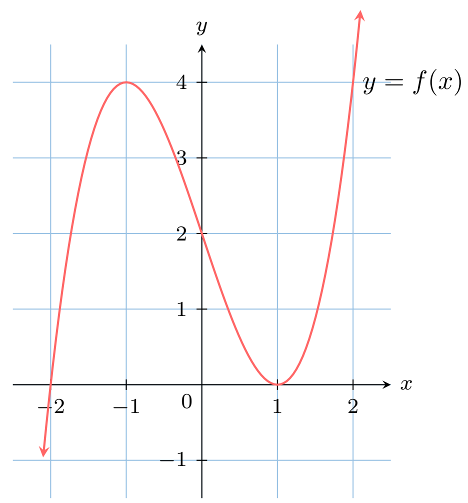
☒ The function has a local minimum at $x = -1$ and a global minimum at $x = 1$.

Answer:

- A **local minimum** is a point that is lower than all of its immediate neighbors. From the graph, we can see that the function has low points at both $x = -1$ and $x = 1$. Therefore, both are local minima.
- A **global minimum** is the lowest point on the entire graph. By comparing the two local minima, we can see that the point at $x = 1$ (approximately $(1, -2.1)$) is lower than the point at $x = -1$ (approximately $(-1, 0.3)$).

Therefore, the function has a local minimum at $x = -1$ and a global minimum at $x = 1$.

MCQ 25: Consider the function f whose graph is shown below. Which of the following statements is true?



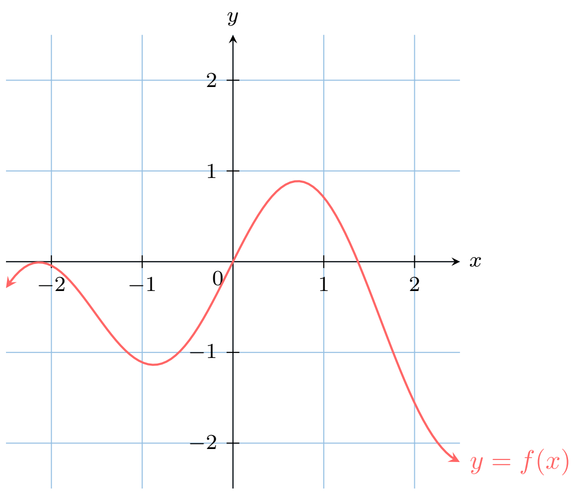
☐ The function has a global maximum at $x = -1$ and a local minimum at $x = 1$.

☒ The function has a local maximum at $x = -1$ and a local minimum at $x = 1$.

☐ The function has a global maximum at $x = -1$ and a global minimum at $x = 1$.

Answer: By observing the graph, we can identify a peak (a local maximum) at $x = -1$ and a valley (a local minimum) at $x = 1$. Since the function's value tends to $+\infty$ as x increases and to $-\infty$ as x decreases, these extrema cannot be global. Therefore, the function has a **local maximum** at $x = -1$ and a **local minimum** at $x = 1$.

MCQ 26: Consider the function f whose graph is shown below. Which of the following statements is true?



☒ The function has a global maximum at $x \approx 0.7$ and a local maximum at $x \approx -2.5$.

☐ The function has a local maximum at $x \approx 0.7$ and no global maximum.

☐ The function has a local maximum at $x \approx 0.7$ and a global maximum at $x \approx -2.5$.

Answer:

- A **local maximum** is a point that is higher than all of its immediate neighbors. From the graph, we can see that the function has high points (peaks) at approximately $x \approx -2.5$ and $x \approx 0.7$. Therefore, both are local maxima.
- A **global maximum** is the highest point on the entire graph shown. By comparing the two local maxima, we can see that the point at $x \approx 0.7$ (approximately $(0.7, 0.8)$) is higher than the point at $x \approx -2.5$ (approximately $(-2.5, -0.4)$).

Therefore, the function has a global maximum at $x \approx 0.7$ and a local maximum at $x \approx -2.5$.

C.2 FIRST DERIVATIVE TEST FOR LOCAL EXTREMA

C.2.1 FINDING AND CLASSIFYING EXTREMA: LEVEL 1

Ex 27: Let $f(x) = x^2 - 4x + 3$.

1. Find the derivative, $f'(x)$.
2. Find the x-coordinate of the stationary point of the function.
3. Hence, classify the stationary point as a local maximum or a local minimum.

Answer:

1. **Find the derivative:**
$$f'(x) = 2x - 4$$
2. **Find the stationary point by solving $f'(x) = 0$:**
$$\begin{aligned} 2x - 4 &= 0 \\ 2x &= 4 \\ x &= 2 \end{aligned}$$
The stationary point is at $x = 2$.
3. **Classify the stationary point:**
We use a sign diagram for the linear function $f'(x) = 2x - 4$.

x	$-\infty$		2		$+\infty$
Sign of $f'(x)$		$-$	0	$+$	

Since the sign of $f'(x)$ changes from negative to positive at $x = 2$, this indicates a **local minimum**.

Ex 28: Let $f(x) = -x^2 - 2x + 8$.

1. Find the derivative, $f'(x)$.
2. Find the x-coordinate of the stationary point of the function.
3. Hence, classify the stationary point as a local maximum or a local minimum.

Answer:

1. **Find the derivative:**
$$f'(x) = -2x - 2$$
2. **Find the stationary point by solving $f'(x) = 0$:**
$$\begin{aligned} -2x - 2 &= 0 \\ -2x &= 2 \\ x &= -1 \end{aligned}$$
The stationary point is at $x = -1$.
3. **Classify the stationary point:**
We use a sign diagram for the linear function $f'(x) = -2x - 2$.

x	$-\infty$		-1		$+\infty$
Sign of $f'(x)$		$+$	0	$-$	

Since the sign of $f'(x)$ changes from positive to negative at $x = -1$, this indicates a **local maximum**.

Ex 29: Let $f(x) = 2x^3 - 3x^2 - 12x + 5$.

1. Find the derivative, $f'(x)$.

2. Find the x-coordinates of the stationary points of the function.

3. Hence, classify each stationary point as a local maximum or a local minimum.

Answer:

1. **Find the derivative:**

$$f'(x) = 6x^2 - 6x - 12$$

2. **Find stationary points by solving $f'(x) = 0$:**

$$6x^2 - 6x - 12 = 0$$
$$6(x^2 - x - 2) = 0$$
$$6(x-2)(x+1) = 0$$

The stationary points are at $x = -1$ and $x = 2$.

3. **Classify the stationary points:**
We use a sign diagram for $f'(x) = 6(x-2)(x+1)$. This is a quadratic function with a positive leading coefficient ($a = 6 > 0$), so its graph is an upward-opening parabola. It is positive outside its roots and negative between them.

x	$-\infty$		-1		2		$+\infty$
Sign of $f'(x)$		+	0	−	0	+	

- At $x = -1$, the sign of $f'(x)$ changes from positive to negative, which indicates a **local maximum**.
- At $x = 2$, the sign of $f'(x)$ changes from negative to positive, which indicates a **local minimum**.

C.2.2 FINDING AND CLASSIFYING EXTREMA: LEVEL 2

Ex 30: Let $f(x) = x\sqrt{4-x}$ for $x \leq 4$.

1. Show that the derivative is $f'(x) = \dfrac{8-3x}{2\sqrt{4-x}}$.

2. Find the coordinates of the stationary point on the graph of $y = f(x)$.

3. Using the first derivative test, determine the nature of this stationary point.

4. Find the global maximum and global minimum values of the function on the interval $[-5, 4]$.

Answer:

1. Using the product rule with $u = x$ and $v = (4-x)^{1/2}$:

$$u' = 1, \quad v' = \frac{1}{2}(4-x)^{-1/2}(-1) = -\frac{1}{2\sqrt{4-x}}$$

$$\begin{aligned} f'(x) &= (1)\sqrt{4-x} + x\left(-\frac{1}{2\sqrt{4-x}}\right) \\ &= \sqrt{4-x} - \frac{x}{2\sqrt{4-x}} \\ &= \frac{\sqrt{4-x}\cdot 2\sqrt{4-x} - x}{2\sqrt{4-x}} \\ &= \frac{2(4-x) - x}{2\sqrt{4-x}} \\ &= \frac{8 - 2x - x}{2\sqrt{4-x}} \\ &= \frac{8-3x}{2\sqrt{4-x}} \end{aligned}$$

2. Set the derivative $f'(x) = 0$. This occurs when the numerator is zero.

$$8 - 3x = 0 \implies x = \frac{8}{3}$$

The y-coordinate is $f(8/3) = \frac{8}{3}\sqrt{4 - \frac{8}{3}} = \frac{8}{3}\sqrt{\frac{4}{3}} = \frac{8}{3}\cdot\frac{2}{\sqrt{3}} = \frac{16}{3\sqrt{3}}$. The stationary point is $(\frac{8}{3}, \frac{16}{3\sqrt{3}})$.

3. We create a sign diagram for $f'(x)$ around $x = 8/3$. The denominator is always positive. So the sign of $f'(x)$ is given by the numerator $8 - 3x$

x	$-\infty$		$8/3$		4
Sign of $f'(x)$		+	0	−	‖

Since the sign of $f'(x)$ changes from positive to negative, the point at $x = 8/3$ is a **local maximum**.

4. To find the global extrema on $[-5, 4]$, we test the stationary point and the endpoints.

- Endpoint: $f(-5) = -5\sqrt{4-(-5)} = -5\sqrt{9} = -15$.
- Stationary Point: $f(8/3) = \frac{16}{3\sqrt{3}} \approx 3.08$.
- Endpoint: $f(4) = 4\sqrt{4-4} = 0$.

Comparing these values, the **global maximum is** $\frac{16}{3\sqrt{3}}$ and the **global minimum is -15**.

Ex 31: Let $f(x) = \frac{\ln x}{x}$ for $x > 0$.

1. Show that the derivative is $f'(x) = \dfrac{1 - \ln x}{x^2}$.

2. Find the exact coordinates of the stationary point on the graph of $y = f(x)$.

3. Using the first derivative test, determine the nature of this stationary point.

4. Find the global maximum and global minimum values of the function on the interval $[1, 4]$.

Answer:

1. Using the quotient rule with $u = \ln x$ and $v = x$:

$$u' = \frac{1}{x}, \quad v' = 1$$

$$\begin{aligned} f'(x) &= \frac{u'v - uv'}{v^2} \\ &= \frac{(\frac{1}{x})(x) - (\ln x)(1)}{x^2} \\ &= \frac{1 - \ln x}{x^2} \end{aligned}$$

2. Set the derivative $f'(x) = 0$. This occurs when the numerator is zero.

$$1 - \ln x = 0 \implies \ln x = 1 \implies x = e$$

The y-coordinate is $f(e) = \frac{\ln e}{e} = \frac{1}{e}$. The stationary point is $(e, \frac{1}{e})$.

3. We create a sign diagram for $f'(x)$ around $x = e$. The denominator x^2 is always positive. The sign of $f'(x)$ is determined by the numerator $1 - \ln x$.

x	0		e		$+\infty$
Sign of $f'(x)$	‖	$+$	0	$-$	

Since the sign of $f'(x)$ changes from positive to negative, the point at $x = e$ is a **local maximum**.

4. To find the global extrema on $[1, 4]$, we test the stationary point and the endpoints.

   - Endpoint: $f(1) = \frac{\ln 1}{1} = 0$.
   - Stationary Point: $f(e) = \frac{1}{e} \approx 0.368$.
   - Endpoint: $f(4) = \frac{\ln 4}{4} \approx \frac{1.386}{4} \approx 0.347$.

   Comparing these values, the **global maximum is** $\frac{1}{e}$ and the **global minimum is 0**.

Ex 32: Let $f(x) = xe^{-x}$.

1. Show that the derivative is $f'(x) = \dfrac{1-x}{e^x}$.
2. Find the coordinates of the stationary point on the graph of $y = f(x)$.
3. Using the first derivative test, determine the nature of this stationary point.
4. Find the global maximum and global minimum values of the function on the interval $[-1, 3]$.

Answer:

1. Using the product rule with $u = x$ and $v = e^{-x}$:

$$u' = 1, \quad v' = e^{-x} \cdot (-1) = -e^{-x}$$

$$\begin{aligned} f'(x) &= u'v + uv' \\ &= (1)(e^{-x}) + (x)(-e^{-x}) \\ &= e^{-x} - xe^{-x} \\ &= e^{-x}(1 - x) \\ &= \frac{1-x}{e^x} \end{aligned}$$

2. Set the derivative $f'(x) = 0$. This occurs when the numerator is zero.

$$1 - x = 0 \implies x = 1$$

The y-coordinate is $f(1) = 1 \cdot e^{-1} = \frac{1}{e}$. The stationary point is $(1, \frac{1}{e})$.

3. We create a sign diagram for $f'(x)$. The denominator e^x is always positive. The sign of $f'(x)$ is determined by the numerator $1 - x$.

x	$-\infty$		1		$+\infty$
Sign of $f'(x)$		$+$	0	$-$	

Since the sign of $f'(x)$ changes from positive to negative, the point at $x = 1$ is a **local maximum**.

4. To find the global extrema on $[-1, 3]$, we test the stationary point and the endpoints.

   - Endpoint: $f(-1) = (-1)e^{-(-1)} = -e \approx -2.718$.
   - Stationary Point: $f(1) = \frac{1}{e} \approx 0.368$.
   - Endpoint: $f(3) = 3e^{-3} = \frac{3}{e^3} \approx 0.149$.

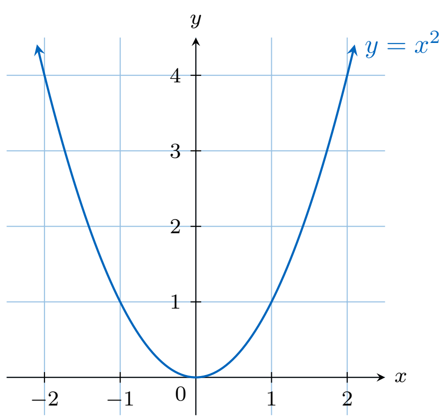
   Comparing these values, the **global maximum is** $\frac{1}{e}$ and the **global minimum is** $-e$.

D CONCAVITY

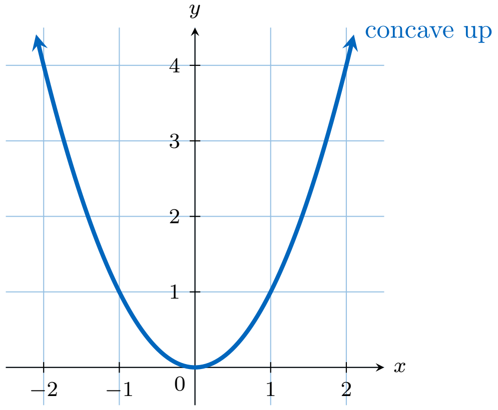
D.1 DEFINITION

D.1.1 DETERMINING CONCAVITY GRAPHICALLY

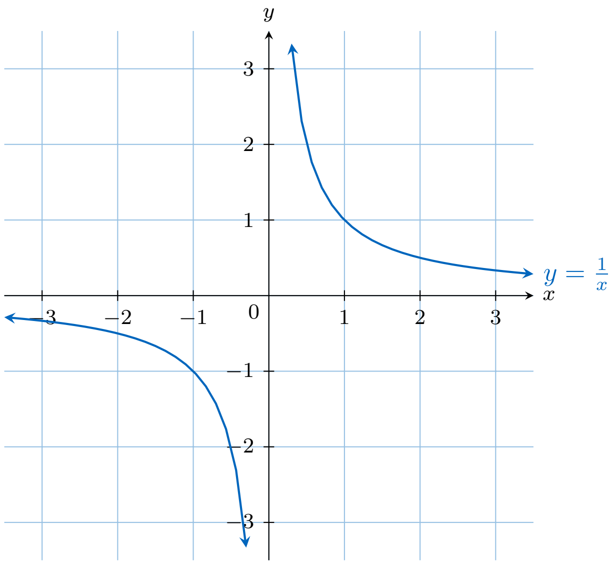
Ex 33: Graphically, determine the concavity of the function $f(x) = x^2$.



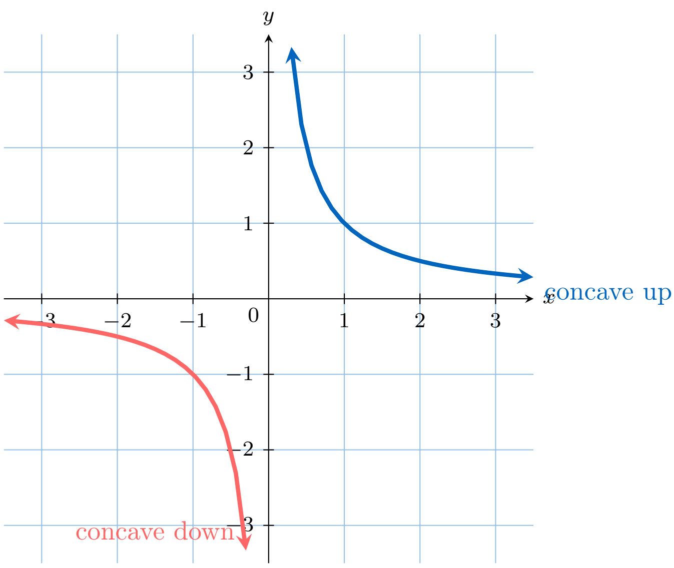
Answer: The function is **concave up** on its entire domain, $(-\infty, \infty)$.



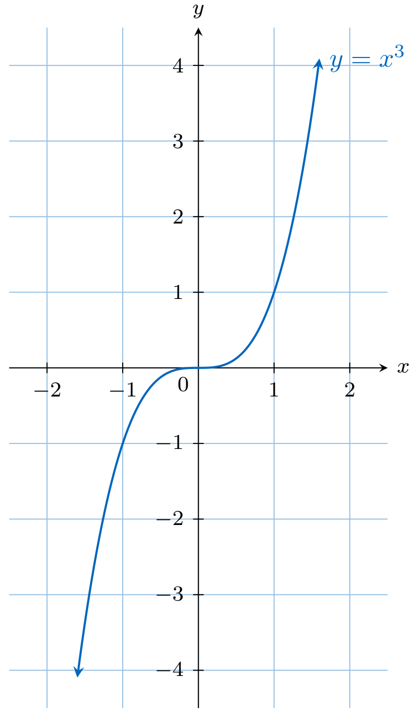
Ex 34: Graphically, determine the intervals of concavity for the function $f(x) = \frac{1}{x}$.



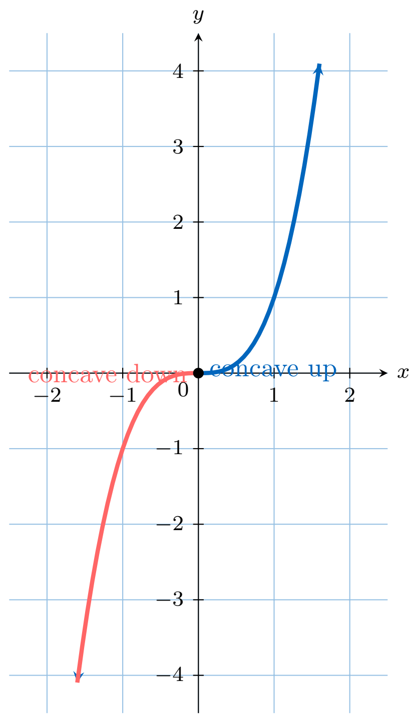
Answer: The function is **concave down** on the interval $(-\infty, 0)$ and **concave up** on the interval $(0, \infty)$.



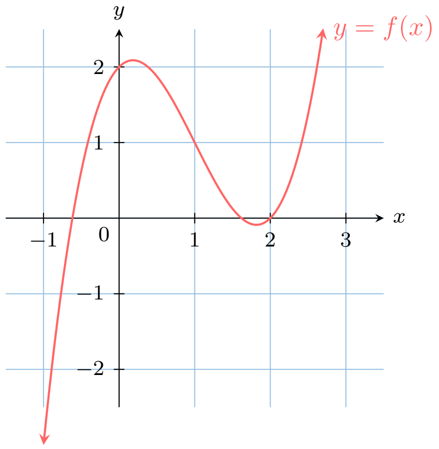
Ex 35: Graphically, determine the concavity of the function $f(x) = x^3$.



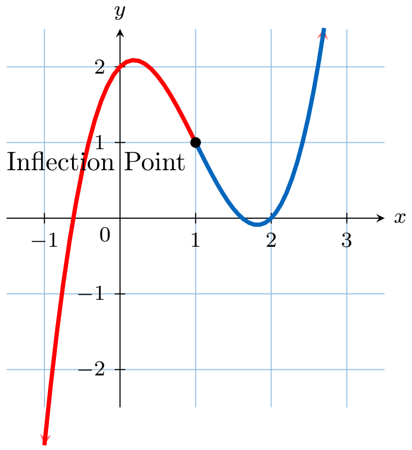
Answer: The function is **concave down** on the interval $(-\infty, 0)$ and **concave up** on the interval $(0, \infty)$.



Ex 36: Graphically, find the point of inflection and describe the concavity for the function $f(x)$ shown below.



Answer: The function is concave down on $(-\infty, 1)$ and **concave up** on $(1, \infty)$. The concavity changes at the point $(1, 1)$, which is a **point of inflection**.



D.2 SECOND DERIVATIVE TEST FOR CONCAVITY

D.2.1 DETERMINING CONCAVITY: LEVEL 1

Ex 37: Let $f(x) = x^3$.

1. Find the second derivative, $f''(x)$.
2. Create a sign diagram for $f''(x)$.
3. Hence, determine the intervals where the function is concave up and concave down.

Answer:

1. **Find the second derivative:**

$$f'(x) = 3x^2 \implies f''(x) = 6x$$

2. **Create a sign diagram for $f''(x)$:**
We find where the second derivative is zero.

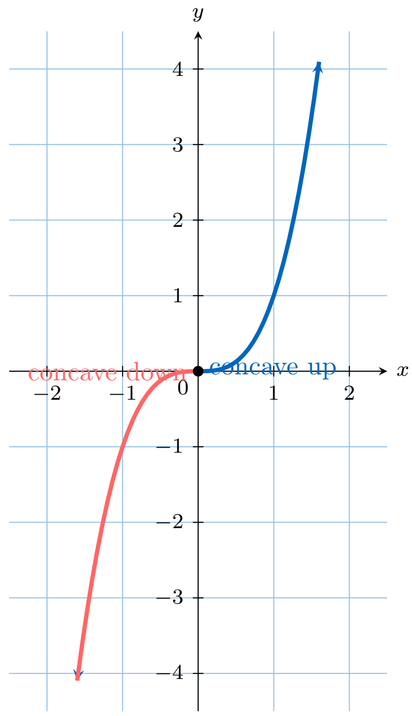
$$6x = 0 \implies x = 0$$

The sign of $f''(x) = 6x$ is negative for $x < 0$ and positive for $x > 0$.

x	$-\infty$	0	$+\infty$
Sign of $f''(x)$	$-$	0	$+$

3. **Determine concavity:**

- The function is **concave down** where $f''(x) \leq 0$, which is on the interval $(-\infty, 0]$.
- The function is **concave up** where $f''(x) \geq 0$, which is on the interval $[0, \infty)$.



Ex 38: Let $f(x) = \frac{1}{x}$.

1. Find the second derivative, $f''(x)$.
2. Create a sign diagram for $f''(x)$.
3. Hence, determine the intervals where the function is concave up and concave down.

Answer:

1. **Find the second derivative:**

$$f(x) = x^{-1}$$

$$f'(x) = -x^{-2} = -\frac{1}{x^2}$$

$$f''(x) = -(-2)x^{-3} = \frac{2}{x^3}$$

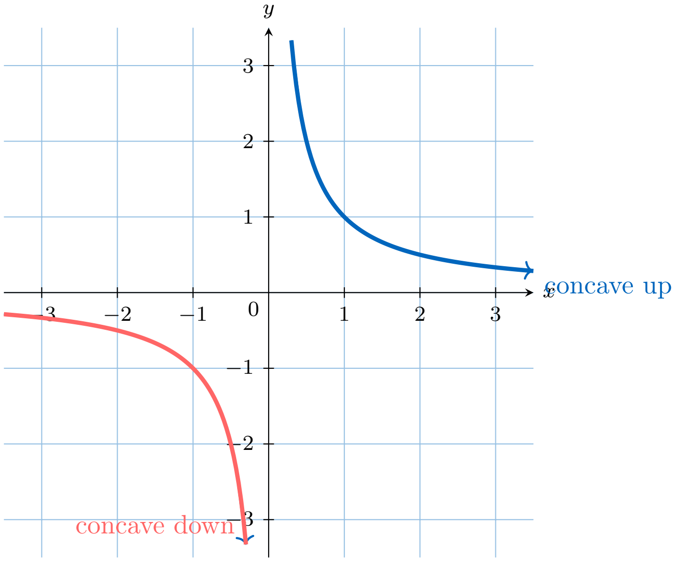
2. **Create a sign diagram for $f''(x)$:**
The second derivative $f''(x) = \frac{2}{x^3}$ is never zero. It is undefined at $x = 0$. The sign is determined by the sign of x^3.

x	$-\infty$	0	$+\infty$
Sign of $f''(x)$	$-$	‖	$+$

3. **Determine concavity:**

- The function is **concave down** where $f''(x) < 0$, which is on the interval $(-\infty, 0)$.

- The function is **concave up** where $f''(x) > 0$, which is on the interval $(0, \infty)$.



Ex 39: Let $f(x) = x^3 - 3x^2 + x$.

1. Find the second derivative, $f''(x)$.
2. Create a sign diagram for $f''(x)$.
3. Hence, determine the intervals where the function is concave up and concave down.

Answer:

1. **Find the second derivative:**

$$f'(x) = 3x^2 - 6x + 1$$

$$f''(x) = 6x - 6$$

2. **Create a sign diagram for $f''(x)$:**
We find where the second derivative is zero.

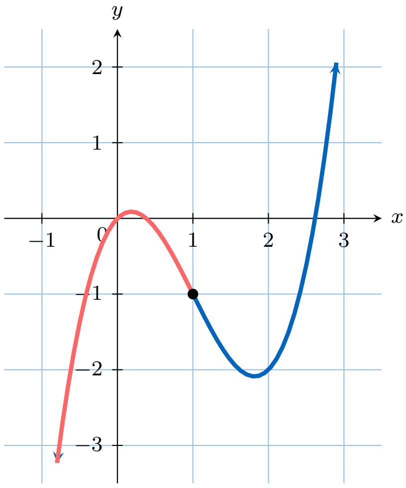
$$6x - 6 = 0 \implies x = 1$$

The sign of $f''(x) = 6(x-1)$ is negative for $x < 1$ and positive for $x > 1$.

x	$-\infty$		1		$+\infty$
Sign of $f''(x)$		$-$	0	$+$	

3. **Determine concavity:**

- The function is **concave down** where $f''(x) \leq 0$, which is on the interval $(-\infty, 1]$.
- The function is **concave up** where $f''(x) \geq 0$, which is on the interval $[1, \infty)$.



D.2.2 DETERMINING CONCAVITY: LEVEL 2

Ex 40: Let $f(x) = 2x^4 - 8x^3 + 12x^2 + 3$.

1. Show that $f''(x) = 24(x-1)^2$.
2. Hence, determine the concavity of the graph of $y = f(x)$.

Answer:

1.

$$\begin{aligned} f'(x) &= 8x^3 - 24x^2 + 24x \\ f''(x) &= 24x^2 - 48x + 24 \\ &= 24(x^2 - 2x + 1) \\ &= 24(x-1)^2 \end{aligned}$$

2. To determine the concavity, we analyze the sign of $f''(x) = 24(x-1)^2$. Since $(x-1)^2$ is a square, it is always greater than or equal to zero for all real values of x. It is only zero at $x = 1$. Therefore, $f''(x) \geq 0$ for all $x \in \mathbb{R}$. This means the function f is always **concave up**.

Ex 41: The function f is defined by $f(x) = e^x \cos(x)$ for $x \in [0, 2\pi]$.

1. Find an expression for $f'(x)$.
2. Show that $f''(x) = -2e^x \sin(x)$.
3. Hence, find the interval(s) where the graph of f is concave down.

Answer:

1. Using the product rule:

$$f'(x) = (e^x)(\cos x) + (e^x)(-\sin x) = e^x(\cos x - \sin x)$$

2. Differentiating $f'(x)$ using the product rule again:

$$\begin{aligned} f''(x) &= (e^x)(\cos x - \sin x) + (e^x)(-\sin x - \cos x) \\ &= e^x \cos x - e^x \sin x - e^x \sin x - e^x \cos x \\ &= -2e^x \sin(x) \end{aligned}$$

3. The graph of f is concave up when $f''(x) \geq 0$.

$$\begin{aligned} f''(x) &\geq 0 \\ -2e^x \sin(x) &\geq 0 \\ \sin(x) &\leq 0 \quad (\text{since } -2e^x < 0) \\ x &\in [\pi, 2\pi] \end{aligned}$$

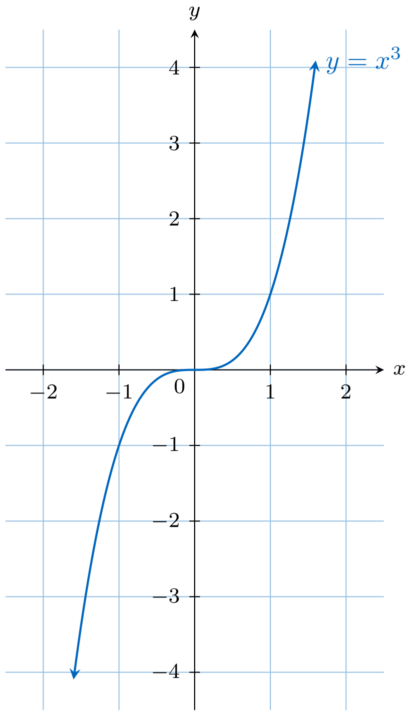
Therefore, the function is **concave up** on $[\pi, 2\pi]$ and **concave down** on $[0, \pi]$.

E POINTS OF INFLECTION

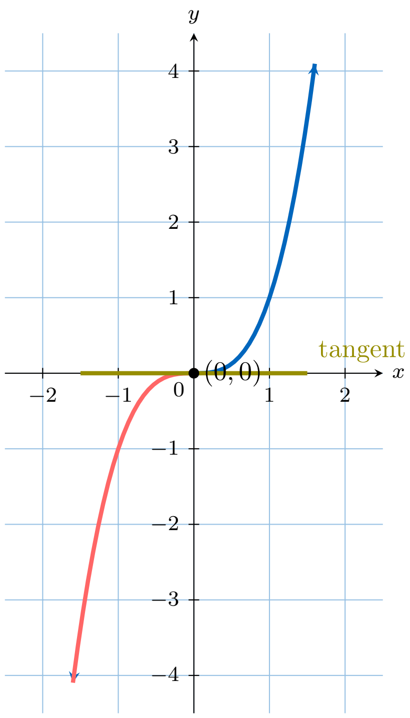
E.1 DEFINITION

E.1.1 IDENTIFYING POINTS OF INFLECTION FROM A GRAPH

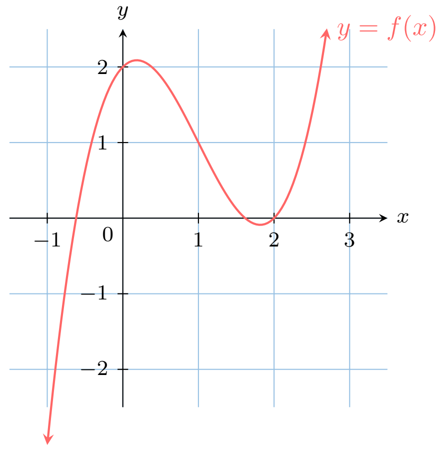
Ex 42: Graphically, find the point of inflection for the function $f(x) = x^3$.



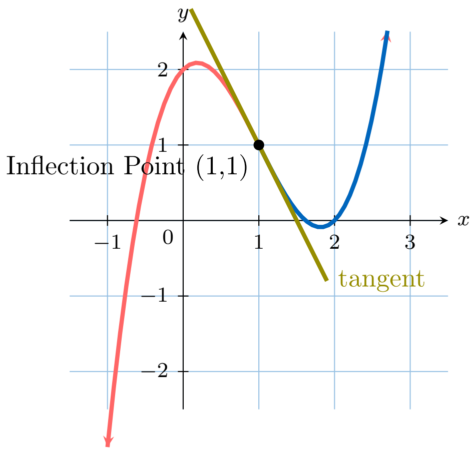
Answer: The concavity changes at the point $(0, 0)$, which is the **point of inflection**. The curve is concave down for $x < 0$ and concave up for $x > 0$. The tangent at this point is the horizontal line $y = 0$, which crosses the curve.



Ex 43: Graphically, find the point of inflection for the function $f(x)$ shown below.



Answer: The concavity changes at the point $(1, 1)$, which is the **point of inflection**. The curve is concave down for $x < 1$ and concave up for $x > 1$. The tangent at this point crosses the curve.



E.2 SECOND DERIVATIVE TEST FOR POINTS OF INFLECTION

E.2.1 DETERMINING POINTS OF INFLECTION: LEVEL 1

Ex 44: Let $f(x) = x^3$.

1. Find the second derivative, $f''(x)$.
2. Find the x-coordinate of the potential point of inflection by solving $f''(x) = 0$.
3. Use a sign diagram for $f''(x)$ to show that a point of inflection exists at this x-coordinate.
4. Find the coordinates of the point of inflection and classify it as stationary or non-stationary.

Answer:

1. **Find the second derivative:**
$$f'(x) = 3x^2 \implies f''(x) = 6x$$
2. **Find potential points of inflection:**
$$f''(x) = 0 \implies 6x = 0 \implies x = 0$$
3. **Use a sign diagram to confirm:**
The sign of $f''(x) = 6x$ changes at $x = 0$.

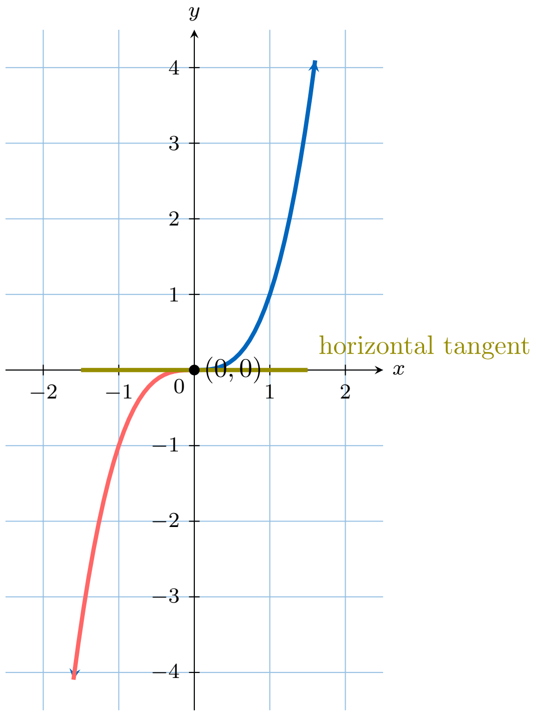
x	$-\infty$		0		$+\infty$
Sign of $f''(x)$		$-$	0	$+$	

Since the sign of $f''(x)$ changes, a point of inflection exists at $x = 0$.

4. **Classify the inflection point:**
We evaluate the first derivative at $x = 0$:

$$f'(0) = 3(0)^2 = 0$$

Since $f'(0) = 0$, the inflection is **stationary**. The coordinates are $(0, f(0)) = (0, 0^3) = (0, 0)$.



Ex 45: Let $f(x) = x^3 - 3x^2 + x + 2$.

1. Find the second derivative, $f''(x)$.
2. Find the x-coordinate of the potential point of inflection.
3. Use a sign diagram for $f''(x)$ to show that a point of inflection exists at this x-coordinate.
4. Find the coordinates of the point of inflection and classify it as stationary or non-stationary.

Answer:

1. **Find the second derivative:**

$$f'(x) = 3x^2 - 6x + 1$$

$$f''(x) = 6x - 6$$

2. **Find potential points of inflection:**
Set $f''(x) = 0$:

$$6x - 6 = 0 \implies x = 1$$

3. **Use a sign diagram to confirm:**
The sign of $f''(x) = 6(x - 1)$ changes at $x = 1$.

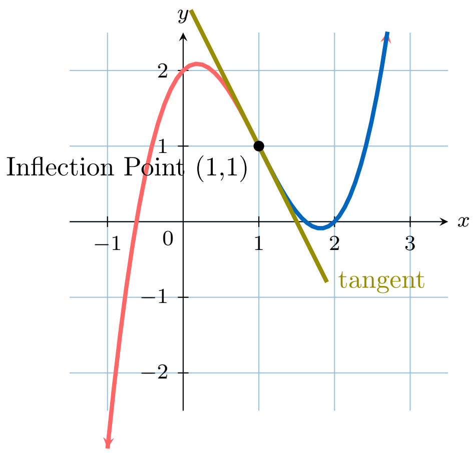
x	$-\infty$		1		$+\infty$
Sign of $f''(x)$		$-$	0	$+$	

Since the sign of $f''(x)$ changes, a point of inflection exists at $x = 1$.

4. **Classify the inflection point:**
We evaluate the first derivative at $x = 1$:

$$f'(1) = 3(1)^2 - 6(1) + 1 = 3 - 6 + 1 = -2$$

Since $f'(1) \neq 0$, the inflection is **non-stationary**. The y-coordinate is $f(1) = 1^3 - 3(1)^2 + 1 + 2 = 1 - 3 + 1 + 2 = 1$. The point of inflection is at $(1, 1)$.



Ex 46: Let $f(x) = \frac{1}{12}x^4 - \frac{1}{2}x^3 + x^2$.

1. Find the first and second derivatives of $f(x)$.
2. Find the x-coordinates of the potential points of inflection.
3. Use a sign diagram for $f''(x)$ to show that points of inflection exist at these x-coordinates.
4. Find the coordinates of the points of inflection and classify them as stationary or non-stationary.

Answer:

1. **Find the derivatives:**

$$f'(x) = \frac{4}{12}x^3 - \frac{3}{2}x^2 + 2x = \frac{1}{3}x^3 - \frac{3}{2}x^2 + 2x$$

$$f''(x) = x^2 - 3x + 2$$

2. **Find potential points of inflection:**
Set $f''(x) = 0$:

$$x^2 - 3x + 2 = 0 \implies (x - 1)(x - 2) = 0$$

The potential points of inflection are at $x = 1$ and $x = 2$.

3. **Use a sign diagram to confirm:**
The graph of $f''(x) = x^2 - 3x + 2$ is an upward-opening parabola. It changes sign at both of its roots.

x	$-\infty$		1		2		$+\infty$
Sign of $f''(x)$		$+$	0	$-$	0	$+$	

Since the sign of $f''(x)$ changes at both $x = 1$ and $x = 2$, they are both x-coordinates of points of inflection.

4. **Classify the inflection points:**
   We evaluate the first derivative $f'(x)$ at these points.
   - At $x = 1$: $f'(1) = \frac{1}{3}(1)^3 - \frac{3}{2}(1)^2 + 2(1) = \frac{1}{3} - \frac{3}{2} + 2 = \frac{2-9+12}{6} = \frac{5}{6}$. Since $f'(1) \neq 0$, it is a **non-stationary point of inflection**. The y-coordinate is $f(1) = \frac{1}{12} - \frac{1}{2} + 1 = \frac{7}{12}$. Point: $(1, \frac{7}{12})$.
   - At $x = 2$: $f'(2) = \frac{1}{3}(2)^3 - \frac{3}{2}(2)^2 + 2(2) = \frac{8}{3} - 6 + 4 = \frac{8}{3} - 2 = \frac{2}{3}$. Since $f'(2) \neq 0$, it is a **non-stationary point of inflection**. The y-coordinate is $f(2) = \frac{16}{12} - \frac{8}{2} + 4 = \frac{4}{3} - 4 + 4 = \frac{4}{3}$. Point: $(2, \frac{4}{3})$.

E.2.2 DETERMINING POINTS OF INFLECTION: LEVEL 2

Ex 47: Let $f(x) = x^3 - 6x^2 + 12x - 5$.

1. Find expressions for $f'(x)$ and $f''(x)$.
2. Find the coordinates of the stationary point of $f(x)$.
3. Find the coordinates of the point of inflection.
4. Show that the stationary point is also the point of inflection.

Answer:

1. **Find derivatives:**
$$f'(x) = 3x^2 - 12x + 12$$
$$f''(x) = 6x - 12$$
2. **Find the stationary point:**
   Set $f'(x) = 0$:
$$3x^2 - 12x + 12 = 0 \implies 3(x^2 - 4x + 4) = 0$$
$$3(x-2)^2 = 0 \implies x = 2$$
   The y-coordinate is $f(2) = (2)^3 - 6(2)^2 + 12(2) - 5 = 8 - 24 + 24 - 5 = 3$.
   The stationary point is $(2, 3)$.
3. **Find the point of inflection:**
   Set $f''(x) = 0$:
$$6x - 12 = 0 \implies x = 2$$
   We must check that the sign of $f''(x)$ changes at $x = 2$.
   - For $x < 2$, $f''(x) = 6(x-2) < 0$ (concave down).
   - For $x > 2$, $f''(x) = 6(x-2) > 0$ (concave up).

   Since the concavity changes, a point of inflection exists at $x = 2$. The coordinates are $(2, f(2)) = (2, 3)$.
4. **Show they are the same point:**
   From part (b), the stationary point occurs at $x = 2$. From part (c), the point of inflection occurs at $x = 2$. Since both occur at the same x-value, the point $(2, 3)$ is a stationary point of inflection.

Ex 48: Let $f(x) = xe^{-x}$.

1. Find expressions for $f'(x)$ and $f''(x)$.
2. Find the coordinates of the stationary point and determine its nature.
3. Find the coordinates of the point of inflection.
4. Find the interval(s) where the graph of f is concave down.

Answer:

1. **Find derivatives:**
   Using the product rule for $f'(x)$:
$$f'(x) = (1)(e^{-x}) + (x)(-e^{-x}) = e^{-x}(1-x) = \frac{1-x}{e^x}$$
   Using the product rule again for $f''(x)$:
$$f''(x) = (-e^{-x})(1-x) + (e^{-x})(-1) = -e^{-x} + xe^{-x} - e^{-x} = e^{-x}($$
2. **Find and classify the stationary point:**
   Set $f'(x) = 0 \implies 1 - x = 0 \implies x = 1$.
   The point is $(1, f(1)) = (1, e^{-1}) = (1, 1/e)$.
   The sign of $f'(x)$ is determined by $(1-x)$. It changes from $+$ to $-$ at $x = 1$, so this is a **local maximum**.
3. **Find the point of inflection:**
   Set $f''(x) = 0 \implies x - 2 = 0 \implies x = 2$.
   The sign of $f''(x)$ is determined by $(x-2)$.

x	$-\infty$		2		$+\infty$
$f''(x)$		$-$	0	$+$	

   Since the sign of $f''(x)$ changes at $x = 2$, it is a point of inflection.
   The coordinates are $(2, f(2)) = (2, 2e^{-2}) = (2, 2/e^2)$.
4. **Find the interval of concave down curvature:**
   The function is concave down when $f''(x) \leq 0$. From the sign diagram, this occurs on the interval $(-\infty, 2]$.