COUNTING

A BASIC COUNTING PRINCIPLES

A.1 APPLYING THE PRODUCT RULE

Ex 1: You have 3 t-shirts and 2 pairs of jeans. How many different outfits can you create by picking one t-shirt and one pair of jeans?

$\square$ outfits

Ex 2: You are at an ice cream shop with 5 flavors of ice cream and 2 types of toppings. How many different desserts can you create by picking one ice cream flavor and one topping?

$\square$ desserts

Ex 3: Su is creating a 4-digit PIN for his debit card. Each digit can be any number from 0 to 9, and digits can be repeated. His PIN starts with 94.

9	4		

How many different PINs are possible for the remaining two digits?

$\square$ PINs

Ex 4: A headteacher wants to select one student from Year 4 and one student from Year 5 for an interview. There are 20 students in Year 4 and 18 students in Year 5. How many different pairs of students can be chosen?

$\square$ pairs

Ex 5: You are packing for a trip with 2 pairs of shoes, 3 pairs of pants, and 5 t-shirts. How many different outfits can you create by picking one of each item?

$\square$ outfits

A.2 APPLYING THE ADDITION RULE

Ex 6: A student can choose a new computer from either 3 desktop models or 4 laptop models. What is the total number of computer options?

$\square$ options

Ex 7: To travel from Paris to Lyon, a traveler can choose from 6 different high-speed trains or 2 flights. How many different travel options does the traveler have for their journey?

$\square$ options

Ex 8: A film enthusiast wants to watch a movie. They subscribe to two streaming services. Service A has 8 exclusive action movies, and Service B has 5 exclusive comedy movies. How many different movies can they choose to watch?

$\square$ movies

Ex 9: A community center offers weekend workshops. On Saturday, there are 3 painting workshops and 2 pottery workshops. On Sunday, there are 4 creative writing workshops. If a person can only sign up for one workshop for the entire weekend, how many choices do they have?

$\square$ choices

B FACTORIALS

B.1 EVALUATING WITHOUT A CALCULATOR

Ex 10: Evaluate:

$$3! = \square.$$

Ex 11: Evaluate:

$$4! = \square$$

Ex 12: Evaluate:

$$5! = \square$$

Ex 13: Evaluate:

$$6! = \square$$

B.2 EVALUATING WITH A CALCULATOR

Ex 14: Evaluate:

$$7! = \square$$

Ex 15: Evaluate:

$$\frac{8!}{3!} = \square$$

Ex 16: Evaluate:

$$\frac{9!}{3!6!} = \square$$

Ex 17: Evaluate:

$$\binom{20}{17} = \square$$

Ex 18: Evaluate:

$$\binom{15}{10} = \square$$

B.3 EXPRESSING PRODUCTS IN FACTORIAL FORM

Ex 19: Express in factorial form:

$$\frac{4 \times 3 \times 2 \times 1}{2 \times 1} = \square$$

Ex 20: Express in factorial form:

$$4 \times 3 = \square$$

Ex 21: Express in factorial form:

$$\frac{5\times4\times3\times2\times1}{3\times2\times1}=\square$$

Ex 22: Express in factorial form:

$$5\times4=\square$$

Ex 23: Express in factorial form:

$$\frac{7\times6\times5\times4\times3\times2\times1}{4\times3\times2\times1}=\square$$

Ex 24: Express in factorial form:

$$7\times6\times5=\square$$

B.4 EVALUATING BY SIMPLIFICATION

Ex 25: Evaluate

$$\binom{5}{3}=\square$$

Ex 26: Evaluate

$$\binom{6}{4}=\square$$

Ex 27: Evaluate

$$\binom{7}{2}=\square$$

Ex 28: Evaluate

$$\binom{7}{4}=\square$$

C ORDERED DRAWS WITH REPLACEMENT (P-LISTS)

C.1 SOLVING REAL-WORLD PROBLEMS

Ex 29: A suitcase lock requires a 3-digit code, with each digit ranging from 0 to 9. How many different codes are possible?

$\square$ codes

Ex 30: A mobile phone requires a 4-digit PIN, with each digit ranging from 0 to 9. How many different PINs can be created?

$\square$ PINs

Ex 31: You are taking a 5-question True/False test. How many different ways can you answer the entire test?

$\square$ ways

Ex 32: A multiple-choice test has 20 questions, and each question has 3 answer choices (A, B, or C). How many different answer keys are possible?

$\square$ answer keys

D ORDERED DRAWS WITHOUT REPLACEMENT (ARRANGEMENTS)

D.1 SOLVING REAL-WORLD PROBLEMS

Ex 33: You're watching a race with 20 competitors, and the top three finishers will stand on the podium. How many different podiums (first, second, and third place) are possible?

$\square$ podiums

Ex 34: You're organizing a group photo and need to arrange 5 people in a single row. How many different ways can you line them up?

$\square$ ways

Ex 35: You're watching a race with 20 competitors, including Emile. How many different possible podiums (first, second, and third place) are there if Emile must finish in first place?

$\square$ podiums

Ex 36: You're watching a race with 20 competitors, including Emile. How many different podiums are possible if Emile must be one of the three podium finishers?

$\square$ podiums

MCQ 37: Mr. T has 5 algebra books, 3 geometry books, and 4 analysis books. In how many ways can he arrange them on a shelf if he groups them by subject?

- ☐ 12^{12}
- ☐ $3!\times5!\times3!\times4!$
- ☐ $5!\times3!\times4!$
- ☐ $12!$

E UNORDERED DRAWS WITHOUT REPLACEMENT (COMBINATIONS)

E.1 SOLVING REAL-WORLD PROBLEMS

Ex 38: You are on a sports team with 5 players: P, Q, R, S, and T. How many different teams of 2 players can be formed?

$\square$ teams

Ex 39: You are part of a squad with 6 players. How many different teams of 4 players can be formed?

teams

Ex 40: How many different anagrams can be formed by rearranging the letters of the word "TOTO"?

anagrams

Ex 41: In a class of 15 students, there are 9 boys and 6 girls. The teacher wants to form a group of 4 students, consisting of exactly 2 boys and 2 girls. How many different groups can be formed?

groups

Ex 42: Your company has 12 employees. A project team is to be formed with 1 team leader, 1 deputy leader, and 4 team members. How many different ways can this team be formed?

ways

F SUMMARY OF COUNTING FORMULAS

F.1 COUNTING POKER HANDS

Ex 43: A standard deck of 52 playing cards has 13 ranks and 4 suits. A poker hand consists of 5 cards drawn from this deck. How many different 5-card poker hands are possible?

hands

Ex 44: A **four-of-a-kind** is a hand with four cards of one rank and one card of another rank. How many different four-of-a-kind hands are possible?

hands

Ex 45: A **full house** is a hand with three cards of one rank and two cards of another rank. How many different full house hands are possible?

hands

Ex 46: A **three-of-a-kind** is a hand with three cards of one rank, and two other cards of two different ranks. How many are possible?

hands

Ex 47: A **two pair** hand has two cards of one rank, two cards of another rank, and one card of a third rank. How many are possible?

hands

Ex 48: A **one pair** hand has two cards of one rank, and three other cards of three different ranks. How many are possible?

hands

G APPLICATIONS TO PROBABILITY

G.1 CALCULATING PROBABILITIES USING COUNTING PRINCIPLES

Ex 49: A student council has 10 members: 6 seniors and 4 juniors. A 3-person committee is to be selected at random. What is the probability that the committee will consist entirely of seniors?

Ex 50: In a race with 20 horses, you bet on 3 horses to finish first, second, and third in exact order (a "triple forecast"). What's the probability of winning your bet?

Ex 51: In a race with 20 horses, you bet on 3 horses to finish in the top 3 positions in any order (a "trio" bet). What's the probability of winning your bet?

Ex 52: In Lotto, you pick 6 numbers from a grid of 49. What is the probability of winning the jackpot by matching all 6 numbers?

Ex 53: In Lotto, you pick 6 numbers from a grid of 49. What is the probability of matching exactly 5 of the 6 winning numbers?

Ex 54: In a class of 30 students, what is the probability that at least two students share the same birthday? Assume a year has 365 days and birthdays are equally likely.