COUNTING

A BASIC COUNTING PRINCIPLES

A.1 APPLYING PRODUCT RULE

Ex 1: Imagine you're getting ready for school and opening your cupboard. You have 3 t-shirts and 2 pairs of jeans to choose from. How many different outfits can you create by picking one t-shirt and one pair of jeans?

☐ outfits

Ex 2: You're getting dressed for a day out and check your closet. You find 4 hats and 3 pairs of shoes to choose from. How many different outfits can you create by picking one hat and one pair of shoes?

☐ outfits

Ex 3: You're at an ice cream shop with 5 flavors of ice cream and 2 types of toppings. How many different dessert combinations can you create by picking one ice cream flavor and one topping?

☐ combinations

Ex 4: Su is creating a 4-digit PIN for his debit card. Each digit can be any number from 0 to 9, and he can repeat digits. His PIN starts with 94.

9	4		

How many different PIN codes are possible for the remaining two digits?

☐ codes

Ex 5: At a school, there are 20 students in Year 4 and 18 students in Year 5. The headteacher wants to select one student from Year 4 and one student from Year 5 for an interview. How many different ways can the headteacher choose these two students?

☐ ways

Ex 6: You're packing for a trip and have 2 pairs of shoes, 3 pairs of pants, and 5 t-shirts to choose from. How many different outfits can you create by picking one pair of shoes, one pair of pants, and one t-shirt?

☐ outfits

A.2 ADDITION RULE

Ex 7: A student is shopping for a new computer and can choose between 3 desktop models and 4 laptop models. What is the total number of computer options they can consider?

☐ computer options

Ex 8: A person is picking a movie to watch and can choose from 6 action movies or 4 comedy movies. What is the total number of movie options they can consider?

☐ movie options

Ex 9: At a café, a customer can choose between 4 types of coffee or 5 types of tea. What is the total number of beverage options they can consider?

☐ beverage options

Ex 10: A customer is shopping for a new phone and can choose between 5 smartphone models or 3 flip phone models. What is the total number of phone options they can consider?

☐ phone options

B FACTORIALS

B.1 EVALUATING WITHOUT USING A CALCULATOR

Ex 11: Evaluate without using a calculator

$3! =$ ☐.

Ex 12: Evaluate without using a calculator

$4! =$ ☐

Ex 13: Evaluate without using a calculator

$5! =$ ☐

Ex 14: Evaluate without using a calculator

$6! =$ ☐

B.2 EVALUATING WITH A CALCULATOR

Ex 15: Use a calculator to evaluate:

$7! =$ ☐

Ex 16: Use a calculator to evaluate:

$\frac{8!}{3!} =$ ☐

Ex 17: Use a calculator to evaluate:

$\frac{9!}{3!6!} =$ ☐

Ex 18: Use a calculator to evaluate:

$\binom{20}{17} =$ ☐

Ex 19: Use a calculator to evaluate:

$\binom{15}{10} =$ ☐

B.3 EXPRESSING IN FACTORIAL FORM

Ex 20: Express in factorial form:

$$\frac{4 \times 3 \times 2 \times 1}{2 \times 1} = \square$$

Ex 21: Express in factorial form:

$$4 \times 3 = \square$$

Ex 22: Express in factorial form:

$$\frac{5 \times 4 \times 3 \times 2 \times 1}{3 \times 2 \times 1} = \square$$

Ex 23: Express in factorial form:

$$5 \times 4 = \square$$

Ex 24: Express in factorial form:

$$\frac{7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{4 \times 3 \times 2 \times 1} = \square$$

Ex 25: Express in factorial form:

$$7 \times 6 \times 5 = \square$$

B.4 BINOMIAL COEFFICIENT

Ex 26: Evaluate without using a calculator

$$\binom{5}{3} = \square$$

Ex 27: Evaluate without using a calculator

$$\binom{6}{4} = \square$$

Ex 28: Evaluate without using a calculator

$$\binom{7}{2} = \square$$

Ex 29: Evaluate without using a calculator

$$\binom{7}{4} = \square$$

C ORDERED DRAWS WITH REPLACEMENT

C.1 SOLVING REAL-WORLD PROBLEMS

Ex 30: You're packing for a trip and need to set a combination lock on your suitcase. The lock uses a 3-digit code, with each digit ranging from 0 to 9. How many different combinations can you create for the lock?

$\square$ combinations

Ex 31: You're setting up a security code for your mobile phone, which requires a 4-digit PIN, with each digit ranging from 0 to 9. How many different codes can you create?

$\square$ codes

Ex 32: You're taking a 5-question True/False test in class. Each question has two possible answers: True or False. How many different ways can you answer the test?

$\square$ ways

Ex 33: You're taking a multiple-choice test with 20 questions, and each question has 3 answer choices (A, B, or C). How many distinctly different answer keys are possible? (Use a calculator to compute this.)

$\square$ answer keys

D ORDERED DRAWS WITHOUT REPLACEMENT

D.1 SOLVING REAL-WORLD PROBLEMS

Ex 34: You're watching a race with 20 competitors, and the top three finishers will stand on the podium. How many different possible podiums (first, second, and third place) can there be? (Use a calculator to compute this.)

$\square$ podiums

Ex 35: You're organizing a group photo and need to arrange 5 people in a single row. How many different ways can you line them up? (Use a calculator to compute this.)

$\square$ ways

Ex 36: King Art is hosting a feast at his round table, sitting at the head position (of course). His seven knights need to sit around him in a circle. How many different ways can the knights be arranged? (Use a calculator to compute this.)

$\square$ ways

Ex 37: You're watching a race with 20 competitors, including Emile. How many different possible podiums (first, second, and third place) can there be if Emile must finish in first place? (Use a calculator to compute this.)

$\square$ podiums

Ex 38: You're watching a race with 20 competitors, including Emile. How many different possible podiums (first, second, and third place) can there be if Emile must be one of the three podium finishers? (Use a calculator to compute this.)

☐ podiums

MCQ 39: Mr. T has 5 algebra books, 3 geometry books, and 4 analysis books. In how many ways can he arrange them on a shelf in his library?

☐ 12^{12}

☐ $3! \times 5! \times 3! \times 4!$

☐ $5! \times 3! \times 4!$

☐ $12!$

MCQ 40: Books Mr. T has 5 algebra books, 3 geometry books, and 4 analysis books. In how many ways can he arrange them on a shelf in his library by grouping them by subject?

☐ 12^{12}

☐ $3! \times 5! \times 3! \times 4!$

☐ $5! \times 3! \times 4!$

☐ $12!$

MCQ 41: In a town, there are four bakeries that close one day a week. Determine the number of ways to assign a weekly closing day to each bakery.

☐ $4!$

☐ 7^4

☐ $7 \times 6 \times 5 \times 4$

MCQ 42: In a town, there are four bakeries that close one day a week. Determine the number of ways to assign a weekly closing day to each bakery if no two bakeries can close on the same day.

☐ $4!$

☐ 7^4

☐ $7 \times 6 \times 5 \times 4$

E UNORDERED DRAWS WITHOUT REPLACEMENT

E.1 SOLVING REAL-WORLD PROBLEMS

Ex 43: You're part of a sports squad with 5 players named P, Q, R, S, and T. How many different teams of 2 players can you form, where the order of players doesn't matter and no player is repeated?

☐ teams

Ex 44: You're part of a sports squad with 6 players named A, B, C, D, E, and F. How many different teams of 4 players can you form, where the order of players doesn't matter and no player is repeated?

☐ teams

Ex 45: Your company needs to form a committee of 4 members from a group of 9 employees. How many different teams of 4 can you choose, where the order of members doesn't matter and no employee is repeated? (Use a calculator to compute this.)

☐ teams

Ex 46: You're playing a word game and need to find the number of different anagrams of the word "TOTO" (an anagram is created by rearranging the letters of "TOTO," for example, "TTOO" is an anagram of "TOTO"). Use the analogy of drawing balls, where the number of the ball corresponds to the position of the letter T.

☐ anagrams

Ex 47: You're playing a number game and need to find the number of different anagrams of the sequence "00111" (an anagram is created by rearranging the digits "00111," for example, "01011" is an anagram of "00111"). Use the analogy of drawing balls, where the number of the ball corresponds to the position of the digit 0.

☐ anagrams

Ex 48: In your class of 15 students, there are 9 boys and 6 girls. The teacher wants to choose a group of 4 students, with exactly 2 boys and 2 girls. How many different ways can you form such a group? (Use a calculator to compute this.)

☐ ways

Ex 49: You have a box with 10 balls, of which 6 are red and 4 are blue. How many different ways can you select 3 balls from the box such that exactly 2 of them are red? (Use a calculator to compute this.)

☐ ways

Ex 50: Your company has 12 employees and wants to form a project team with 1 team leader, 1 deputy leader, and 4 team members. How many different ways can you form this project team, where the positions are distinct and no employee is repeated? (Use a calculator to compute this.)

☐ ways

Ex 51: Your club has 10 members, and it wants to form a committee with 1 president, 1 secretary, and 3 members. How many different ways can you form this committee, where the positions are distinct and no member is repeated? (Use a calculator to compute this.)

☐ ways

E.2 COUNTING POKER HANDS

Ex 52: You're playing poker, and a hand consists of 5 cards drawn from a standard deck of 52 playing cards (13 ranks and 4 suits). How many different 5-card poker hands are possible? (Use a calculator to compute this.)

[ ] hands

Ex 53: In poker, a four-of-a-kind (also known as a square) is a hand where four cards have the same rank, and the fifth card is of a different rank. How many different four-of-a-kind hands are possible in a standard 52-card deck? (Use a calculator to compute this.)

[ ] hands

Ex 54: In poker, a full house is a hand that contains three cards of one rank and two cards of another rank. How many different full house hands are possible in a standard 52-card deck? (Use a calculator to compute this.)

[ ] hands

Ex 55: In poker, a three of a kind is a hand that contains three cards of one rank and two other cards of different ranks, which are not the same as each other or the same as the rank of the three cards. How many different three-of-a-kind hands are possible in a standard 52-card deck? (Use a calculator to compute this.)

[ ] hands

Ex 56: In poker, a two pair is a hand that contains two cards of one rank, two cards of another rank, and one card of a different rank from the other four cards. How many different two-pair hands are possible in a standard 52-card deck? (Use a calculator to compute this.)

[ ] hands

Ex 57: In poker, a one pair is a hand that contains two cards of one rank and three other cards of different ranks, which are not the same as each other or the same as the rank of the pair. How many different one-pair hands are possible in a standard 52-card deck? (Use a calculator to compute this.)

[ ] hands