COMPLEX NUMBERS: ALGEBRAIC APPROACH

A THE NUMBER i AND THE SET OF COMPLEX NUMBERS

A.1 IDENTIFYING REAL AND IMAGINARY PARTS

Ex 1: Find the real part of the following complex number:

$$\mathrm{Re}(2+3i) = \square$$

Ex 2: Find the imaginary part of the following complex number:

$$\mathrm{Im}(2-3i) = \square$$

Ex 3: Find the real part of the following complex number:

$$\mathrm{Re}(3i) = \square$$

Ex 4: Find the imaginary part of the following complex number:

$$\mathrm{Im}(2) = \square$$

Ex 5: Find the real part of the following complex number:

$$\mathrm{Re}(1+\sqrt{2}+3i) = \square$$

A.2 CLASSIFYING COMPLEX NUMBERS

MCQ 6: $1+2i$ is a purely imaginary number.

☐ True

☐ False

MCQ 7: $\sqrt{2}i$ is a purely imaginary number.

☐ True

☐ False

MCQ 8: $1+2i$ is a real number.

☐ True

☐ False

MCQ 9: $\sqrt{2}$ is a real number.

☐ True

☐ False

B OPERATIONS WITH COMPLEX NUMBERS

B.1 CALCULATING WITH COMPLEX NUMBERS

Ex 10: For $z = 2+3i$ and $w = 4-5i$, write in standard form:

$$z+w = \square$$

Ex 11: For $z = 2-4i$ and $w = -2+3i$, write in standard form:

$$z-w = \square$$

Ex 12: For $z = i$ and $w = 2+i$, write in standard form:

$$zw = \square$$

Ex 13: For $z = 2-i$ and $w = 1+3i$, write in standard form:

$$zw = \square$$

B.2 DIVIDING COMPLEX NUMBERS

Ex 14: For $z = 2$ and $w = 1+i$, write in standard form:

$$\frac{z}{w} = \square$$

Ex 15: For $z = 2i$ and $w = i-1$, write in standard form:

$$\frac{z}{w} = \square$$

Ex 16: For $z = 2-i$ and $w = i-1$, write in standard form:

$$\frac{z}{w} = \square$$

Ex 17: For $z = i$ and $w = 2-i$, write in standard form:

$$\frac{z}{w} = \square$$

B.3 SIMPLIFYING EXPRESSIONS TO FIND REAL AND IMAGINARY PARTS

Ex 18: Find the real part of the following complex number:

$$\mathrm{Re}(i(2+i)) = \square$$

Ex 19: Find the imaginary part of the following complex number:

$$\mathrm{Im}((1+i)(3-2i)) = \square$$

Ex 20: Find the real part of the following complex number:

$$\mathrm{Re}((2+i)^2) = \square$$

Ex 21: Find the imaginary part of the following complex number:

$$\mathrm{Im}\left(\frac{3+i}{1-i}\right) = \square$$

B.4 CALCULATING POWERS OF THE IMAGINARY UNIT

Ex 22: Write in terms of i:

- $i^0 = \square$
- $i^1 = \square$
- $i^2 = \square$
- $i^3 = \square$

Ex 23: Prove that $i^{4n} = 1$ for n a natural number.

Ex 24: Prove that $i^{4n+1} = i$ for n a natural number.

Ex 25: Write in terms of i:

- $i^{10} =$
- $i^{21} =$
- $i^{400} =$

B.5 EVALUATING POLYNOMIAL EXPRESSIONS OF A COMPLEX NUMBER

Ex 26: For $z = 1 + 2i$, write in standard form:

$$z^2 =$$

Ex 27: For $z = 1 + i$, write in standard form:

$$z - z^2 =$$

Ex 28: For $z = 1 + i$, write in standard form:

$$z^2 - z + 1 =$$

Ex 29: For $z = \frac{1}{2} + \frac{\sqrt{3}}{2}i$, write in standard form:

$$z^2 - z + 1 =$$

C EQUALITY OF COMPLEX NUMBERS

C.1 SOLVING LINEAR EQUATIONS

Ex 30: Solve the equation $\frac{z+1}{z-1} = 2$ in $\mathbb{C}$.

Ex 31: Solve the equation $z(1 + i) = i$ in $\mathbb{C}$.

Ex 32: Solve the equation $\frac{z+1}{z-1} = i$ in $\mathbb{C}$.

C.2 SOLVING EQUATIONS BY EQUATING REAL AND IMAGINARY PARTS

Ex 33: For x, y real numbers, solve the equation $x(1+i) = 2y + 1$.

Ex 34: For x, y real numbers, solve the equation $(x+i)(2+i) = 1 + yi$.

Ex 35: For x, y real numbers, solve the equation $(x+2i)(1-i) = 2 + yi$.

D COMPLEX CONJUGATE

D.1 FINDING THE CONJUGATE OF A COMPLEX NUMBER

Ex 36: Find the conjugate of the following complex number:

$$\overline{1+i} = \square$$

Ex 37: Find the conjugate of the following complex number:

$$\overline{-i+1} = \square$$

Ex 38: Find the conjugate of the following complex number:

$$\overline{\frac{2-3i}{2}} = \square$$

Ex 39: Find the conjugate of the following complex number:

$$\overline{2(1+i)} = \square$$

D.2 PROVING PROPERTIES OF THE COMPLEX CONJUGATE

Ex 40: Given a complex number z, prove that $\overline{\overline{z}} = z$.

Ex 41: Given two complex numbers z and w, prove that $\overline{z+w} = \overline{z} + \overline{w}$.

Ex 42: Given two complex numbers z and w, prove that $\overline{z-w} = \overline{z} - \overline{w}$.

Ex 43: Given two complex numbers z and w, prove that $\overline{zw} = \overline{z} \cdot \overline{w}$.

Ex 44: Given a complex number z, prove that $z + \overline{z}$ is a real number.

Ex 45: Given a complex number z, prove that $z \cdot \overline{z}$ is a non-negative real number.

D.3 SOLVING COMPLEX EQUATIONS INVOLVING THE CONJUGATE

Ex 46: Solve the equation $z = 2\overline{z} + 1$ in $\mathbb{C}$. (Hint: Let $z = a + bi$ where a and b are real numbers.)

Ex 47: Solve the equation $2z + 3\overline{z} = 1 + i$ in $\mathbb{C}$. (Hint: Let $z = a + bi$ where a and b are real numbers.)

Ex 48: Solve the equation $(1 + i)z + \overline{z} = 2 - i$ in $\mathbb{C}$. (Hint: Let $z = a + bi$ where a and b are real numbers.)