BINOMIAL EXPANSION

A BINOMIAL EXPANSION FOR n=2 AND n=3

A.1 EXPANDING PERFECT SQUARE BINOMIALS (ADDITION)

Ex 1: Expand and simplify

$$(x+2)^2 =$$

Ex 2: Expand and simplify

$$(3+x)^2 =$$

Ex 3: Expand and simplify

$$(2x+1)^2 =$$

Ex 4: Expand and simplify

$$(2+3x)^2 =$$

A.2 EXPANDING PERFECT SQUARE BINOMIALS (SUBTRACTION)

Ex 5: Expand and simplify

$$(x-2)^2 = \boxed{}$$

Ex 6: Expand and simplify

$$(3-x)^2 = \boxed{}$$

Ex 7: Expand and simplify

$$(2x-1)^2 =$$

Ex 8: Expand and simplify

$$(2-3x)^2 = \boxed{}$$

A.3 EXPANDING PERFECT CUBE BINOMIALS (ADDITION)

Ex 9: Expand and simplify

$$(x+1)^3 = \boxed{}$$

Ex 10: Expand and simplify

$$(x+3)^3 =$$

Ex 11: Expand and simplify

$$(2x+2)^3 = \boxed{}$$

Ex 12: Expand and simplify

A.4 EXPANDING PERFECT CUBE BINOMIALS (SUBTRACTION)

Ex 13: Expand and simplify

$$(x-1)^3 = \boxed{}$$

Ex 14: Expand and simplify

$$(x-2)^3 = \boxed{}$$

Ex 15: Expand and simplify

$$(3-x)^3 = \boxed{}$$

Ex 16: Expand and simplify

$$(2x-1)^3 = \boxed{}$$

B PASCAL'S TRIANGLE

B.1 BUILDING PASCAL'S TRIANGLE

Ex 17: Write down the first five rows of Pascal's triangle.

Ex 18: Write down the first six rows of Pascal's triangle.

B.2 USING THE FOURTH ROW OF PASCAL'S TRIANGLE

Ex 19: Use Pascal's triangle to find the coefficients of the expansion of $(a + b)^4$, then expand and simplify.

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Ex 27: Evaluate:

4! =

Ex 28: Evaluate:

5! =

Ex 29: Evaluate:

6! =

C.2 EVALUATING FACTORIALS WITH A CALCULATOR

Ex 30: Evaluate:

7! =

 $\frac{8!}{3!} = \boxed{}$

Ex 32: Evaluate:

 $\frac{9!}{3!6!} = \boxed{}$

Ex 33: Evaluate:

 $\binom{20}{17} = \boxed{}$

Ex 34: $\mathbb{E}^{\mathbb{E}}$ Evaluate:

 $\binom{15}{10} = \boxed{}$

C.3 EXPRESSING PRODUCTS IN FACTORIAL FORM

Ex 35: Express in factorial form:

 $\frac{4 \times 3 \times 2 \times 1}{2 \times 1} = \boxed{}$

Ex 36: Express in factorial form:

 $4 \times 3 =$

Ex 37: Express in factorial form:

$$\frac{5 \times 4 \times 3 \times 2 \times 1}{3 \times 2 \times 1} =$$

Ex 38: Express in factorial form:

 $5 \times 4 =$

Ex 39: Express in factorial form:

$$\frac{7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{4 \times 3 \times 2 \times 1} =$$

Ex 40: Express in factorial form:

$$7 \times 6 \times 5 =$$

Ex 20: Use Pascal's triangle to expand and simplify:

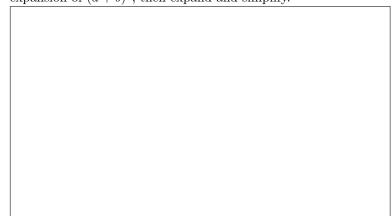
$$(x-1)^4 =$$

Ex 21: Use Pascal's triangle to expand and simplify

$$(2x-1)^4 = \boxed{}$$

B.3 USING THE FIFTH ROW OF PASCAL'S TRIANGLE

Ex 22: Use Pascal's triangle to find the coefficients of the expansion of $(a + b)^5$, then expand and simplify.



Ex 23: Use Pascal's triangle to expand and simplify:

$$(x+1)^5 =$$

Ex 24: Use Pascal's triangle to expand and simplify:

$$(x-1)^5 =$$

Ex 25: Use Pascal's triangle to expand and simplify:

$$(2-x)^5 = \boxed{$$

C THE BINOMIAL THEOREM

C.1 EVALUATING FACTORIALS WITHOUT A CALCULATOR

Ex 26: Evaluate:

C.4 EVALUATING FACTORIALS BY SIMPLIFICATION

Ex 41: Evaluate

$$\binom{5}{3} = \boxed{}$$

Ex 42: Evaluate

$$\binom{6}{4} = \square$$

Ex 43: Evaluate

$$\binom{7}{2} = \square$$

Ex 44: Evaluate

$$\binom{7}{4} = \square$$

C.5 FINDING A SPECIFIC TERM IN AN EXPANSION

Ex 45: Find the coefficient of x^3 in the expansion of $(x-1)^6$.



Ex 46: Find the coefficient of x^5 in the expansion of $(2x + 3)^7$.



Ex 47: Find the constant term (term independent of x) in the expansion of $\left(x+\frac{1}{x}\right)^6$.



Ex 48: Find the coefficient of x^4 in the expansion of $(3x-2)^7$.



Ex 49: Find the constant term (term independent of x) in the expansion of $\left(2x - \frac{1}{x}\right)^6$.

C.6 DERIVING IDENTITIES FROM THE BINOMIAL THEOREM

Ex 50:

1. Show that

$$(1+x)^n = \binom{n}{0} + \binom{n}{1}x + \binom{n}{2}x^2 + \ldots + \binom{n}{n-1}x^{n-1} + \binom{n}{n}x^n.$$

2. Hence deduce that:

(a)
$$\binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \ldots + \binom{n}{n-1} + \binom{n}{n} = 2^n$$

(b)
$$\binom{n}{0} - \binom{n}{1} + \binom{n}{2} - \dots + (-1)^n \binom{n}{n} = 0$$

Ex 51:

1. By differentiating the binomial expansion of $(1+x)^n$, show that

$$n(1+x)^{n-1} = \sum_{k=1}^{n} k \binom{n}{k} x^{k-1}.$$

2. Hence, deduce that:

$$\binom{n}{1} + 2\binom{n}{2} + 3\binom{n}{3} + \dots + n\binom{n}{n} = n2^{n-1}$$

