BINOMIAL EXPANSION

A BINOMIAL EXPANSION FOR $n = 2$ AND $n = 3$

A.1 EXPANDING PERFECT SQUARE BINOMIALS (ADDITION)

Ex 1: Expand and simplify

$$(x+2)^2 = \boxed{}$$

Ex 2: Expand and simplify

$$(3+x)^2 = \boxed{}$$

Ex 3: Expand and simplify

$$(2x+1)^2 = \boxed{}$$

Ex 4: Expand and simplify

$$(2+3x)^2 = \boxed{}$$

A.2 EXPANDING PERFECT SQUARE BINOMIALS (SUBTRACTION)

Ex 5: Expand and simplify

$$(x-2)^2 = \boxed{}$$

Ex 6: Expand and simplify

$$(3-x)^2 = \boxed{}$$

Ex 7: Expand and simplify

$$(2x-1)^2 = \boxed{}$$

Ex 8: Expand and simplify

$$(2-3x)^2 = \boxed{}$$

A.3 EXPANDING PERFECT CUBE BINOMIALS (ADDITION)

Ex 9: Expand and simplify

$$(x+1)^3 = \boxed{}$$

Ex 10: Expand and simplify

$$(x+3)^3 = \boxed{}$$

Ex 11: Expand and simplify

$$(2x+2)^3 = \boxed{}$$

Ex 12: Expand and simplify

$$(2x+3)^3 = \boxed{}$$

A.4 EXPANDING PERFECT CUBE BINOMIALS (SUBTRACTION)

Ex 13: Expand and simplify

$$(x-1)^3 = \boxed{}$$

Ex 14: Expand and simplify

$$(x-2)^3 = \boxed{}$$

Ex 15: Expand and simplify

$$(3-x)^3 = \boxed{}$$

Ex 16: Expand and simplify

$$(2x-1)^3 = \boxed{}$$

B PASCAL'S TRIANGLE

B.1 BUILDING PASCAL'S TRIANGLE

Ex 17: Write down the first five rows of Pascal's triangle.

Ex 18: Write down the first six rows of Pascal's triangle.

B.2 USING THE FOURTH ROW OF PASCAL'S TRIANGLE

Ex 19: Use Pascal's triangle to find the coefficients of the expansion of $(a+b)^4$, then expand and simplify.

Ex 20: Use Pascal's triangle to expand and simplify:

$$(x-1)^4 = \square$$

Ex 21: Use Pascal's triangle to expand and simplify

$$(2x-1)^4 = \square$$

B.3 USING THE FIFTH ROW OF PASCAL'S TRIANGLE

Ex 22: Use Pascal's triangle to find the coefficients of the expansion of $(a+b)^5$, then expand and simplify.

Ex 23: Use Pascal's triangle to expand and simplify:

$$(x+1)^5 = \square$$

Ex 24: Use Pascal's triangle to expand and simplify:

$$(x-1)^5 = \square$$

Ex 25: Use Pascal's triangle to expand and simplify:

$$(2-x)^5 = \square$$

C THE BINOMIAL THEOREM

C.1 EVALUATING FACTORIALS WITHOUT A CALCULATOR

Ex 26: Evaluate:

$$3! = \square.$$

Ex 27: Evaluate:

$$4! = \square$$

Ex 28: Evaluate:

$$5! = \square$$

Ex 29: Evaluate:

$$6! = \square$$

C.2 EVALUATING FACTORIALS WITH A CALCULATOR

Ex 30: Evaluate:

$$7! = \square$$

Ex 31: Evaluate:

$$\frac{8!}{3!} = \square$$

Ex 32: Evaluate:

$$\frac{9!}{3!6!} = \square$$

Ex 33: Evaluate:

$$\binom{20}{17} = \square$$

Ex 34: Evaluate:

$$\binom{15}{10} = \square$$

C.3 EXPRESSING PRODUCTS IN FACTORIAL FORM

Ex 35: Express in factorial form:

$$\frac{4\times 3\times 2\times 1}{2\times 1} = \square$$

Ex 36: Express in factorial form:

$$4\times 3 = \square$$

Ex 37: Express in factorial form:

$$\frac{5\times 4\times 3\times 2\times 1}{3\times 2\times 1} = \square$$

Ex 38: Express in factorial form:

$$5\times 4 = \square$$

Ex 39: Express in factorial form:

$$\frac{7\times 6\times 5\times 4\times 3\times 2\times 1}{4\times 3\times 2\times 1} = \square$$

Ex 40: Express in factorial form:

$$7\times 6\times 5 = \square$$

C.4 EVALUATING FACTORIALS BY SIMPLIFICATION

Ex 41: Evaluate

$$\binom{5}{3} = \square$$

Ex 42: Evaluate

$$\binom{6}{4} = \square$$

Ex 43: Evaluate

$$\binom{7}{2} = \square$$

Ex 44: Evaluate

$$\binom{7}{4} = \square$$

C.5 FINDING A SPECIFIC TERM IN AN EXPANSION

Ex 45: Find the coefficient of x^3 in the expansion of $(x-1)^6$.

Ex 46: Find the coefficient of x^5 in the expansion of $(2x+3)^7$.

Ex 47: Find the constant term (term independent of x) in the expansion of $\left(x+\frac{1}{x}\right)^6$.

Ex 48: Find the coefficient of x^4 in the expansion of $(3x-2)^7$.

Ex 49: Find the constant term (term independent of x) in the expansion of $\left(2x-\frac{1}{x}\right)^6$.

C.6 DERIVING IDENTITIES FROM THE BINOMIAL THEOREM

Ex 50:

1. Show that
$$(1+x)^n = \binom{n}{0}+\binom{n}{1}x+\binom{n}{2}x^2+\ldots+\binom{n}{n-1}x^{n-1}+\binom{n}{n}x^n.$$
2. Hence deduce that:
   (a) $\binom{n}{0}+\binom{n}{1}+\binom{n}{2}+\ldots+\binom{n}{n-1}+\binom{n}{n}=2^n$
   (b) $\binom{n}{0}-\binom{n}{1}+\binom{n}{2}-\ldots+(-1)^n\binom{n}{n}=0$

Ex 51:

1. By differentiating the binomial expansion of $(1+x)^n$, show that
$$n(1+x)^{n-1}=\sum_{k=1}^{n} k\binom{n}{k}x^{k-1}.$$
2. Hence, deduce that:
$$\binom{n}{1}+2\binom{n}{2}+3\binom{n}{3}+\ldots+n\binom{n}{n}=n2^{n-1}$$