AREA

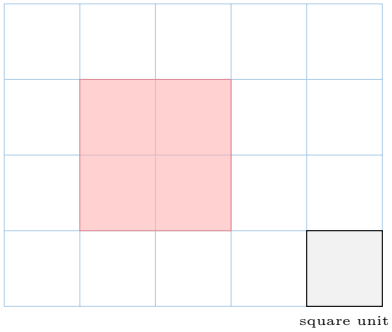
A DEFINITION

Definition Area

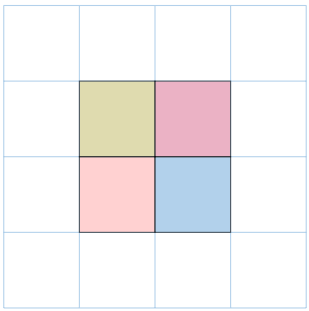
The **area** of a shape is how much space it covers on a flat surface.
We measure area by counting how many **square units** fit inside the shape.

To find the area of a shape, we can place it on a grid and count the total number of squares it covers. You can think of it like tiling a floor — the area is the total number of tiles you use.

Ex: Find the area of the green shape. Each small square in the grid is **1 square unit**.



Answer: To find the area, we count each square unit inside the shape.



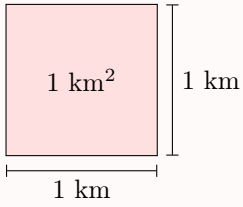
There are **4** small squares inside the shape.
So, the area is **4 square units**.

B UNITS OF AREA

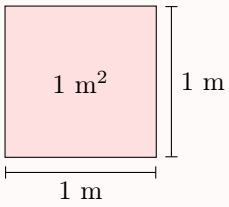
Discover: When we measure area, it is important to use **standard units** so that everyone gets the same measurement. Non-standard units, such as books or tiles of different sizes, can give different answers because they are not all the same size. For area, we use standard units like the **square centimeter** and the **square meter**.

Definition **Units of Area**

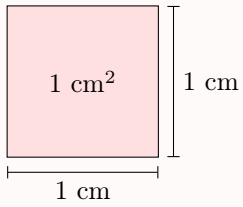
- Square Kilometer $\left(\text{km}^2\right)$: A very large unit of area, about the size of a small town.



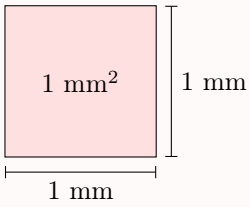
- Square Meter $\left(\text{m}^2\right)$: A larger unit of area, about the space it takes for you to lie down with your arms by your sides.



- Square Centimeter $\left(\text{cm}^2\right)$: A small unit of area, about the size of a big toe nail for a 6-year-old boy.

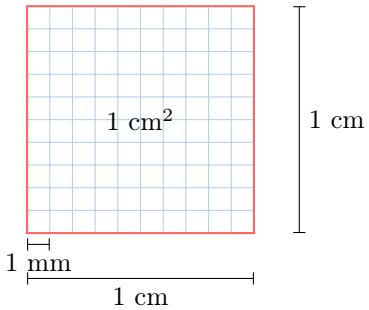


- Square Millimeter $\left(\text{mm}^2\right)$: A very small unit of area, about the size of a tiny dot made by a pencil.



C CONVERSION OF AREA UNITS

Discover: Let's see how area units are related. Consider a square with an area of **1 cm²**. Since 1 cm = 10 mm, each side of this square is 10 mm long.



Each small square is 1 mm². To find the area in mm², we multiply its length in mm by its width in mm:

$$\begin{aligned} 1\,\text{cm}^2 &= 1\,\text{cm} \times 1\,\text{cm} \\ &= 10\,\text{mm} \times 10\,\text{mm} \\ &= 100\,\text{mm}^2 \end{aligned}$$

So, **1 cm² is equal to 100 mm²**. The conversion factor is squared!

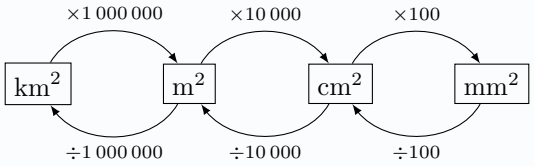
Proposition **Conversion of Area Units**

Because we multiply two lengths to get an area, the conversion factors are squared.

- $1\,\text{cm}^2 = (10 \times 10)\,\text{mm}^2 = \mathbf{100}\,\text{mm}^2$
- $1\,\text{m}^2 = (100 \times 100)\,\text{cm}^2 = \mathbf{10,000}\,\text{cm}^2$
- $1\,\text{km}^2 = (1000 \times 1000)\,\text{m}^2 = \mathbf{1,000,000}\,\text{m}^2$

Method **Converting Using Multiplication or Division**

- Use **multiplication** to go from a larger unit to a smaller one (like square meters to square centimeters).
- Use **division** to go from a smaller unit to a larger one (like square centimeters to square meters).



Method **Converting Using a Place Value Table**

For area, each unit in the place value table is split into **two columns**. Let's convert 10.5 m^2 to cm^2.

1. **Draw the area conversion table.** Each unit has two columns.

km²		ha				m²				cm²		mm²	

2. **Place the number in the table.** The rule is: the digit in the **ones place** goes into the **right-hand column** of the starting unit. For 10.5 m^2, the ones digit is **0**, so it goes in the right-hand column of $\mathbf{m^2}$. Then place the other digits in the neighbouring columns, keeping their order (tens to the left, decimal digits to the right).

km²		ha				m²				cm²		mm²	
						1	**0**	**5**					

3. **Move the decimal point** to the right side of your target unit's columns. Our target is $\mathbf{cm^2}$. Fill any empty columns with zeros.

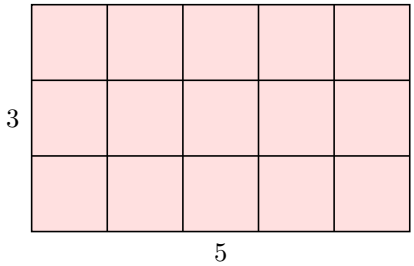
km²		ha				m²				cm²		mm²	
						1	0	5	**0**	**0**	**0.**		

4. **Read the final number.** The decimal point is now at the far right.

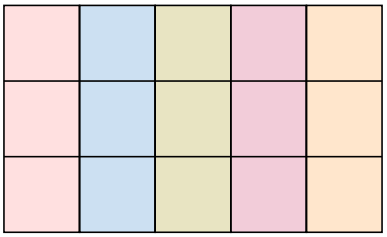
So, 10.5 $\mathbf{m^2}$ = 105 000 $\mathbf{cm^2}$.

D AREA OF A RECTANGLE OR A SQUARE

Discover: Counting every single square to find the area can take a long time. Let's see if there is a shortcut. Consider a rectangle that is 5 units long and 3 units wide.



We can find its area by adding up the squares in each column.

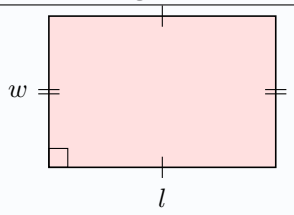
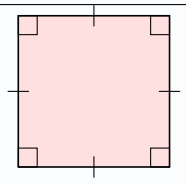


The area is $\underbrace{3 + 3 + 3 + 3 + 3}_{5 \text{ times}} = 5 \times 3 = 15$ square units.

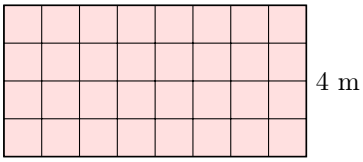
This shows that we can find the area of a rectangle by simply multiplying its **length** by its **width**.

Proposition **Area Formulas**

To find the area of a rectangle, multiply its **length** by its **width**.To find the area of a square, multiply the **side length** by itself.

Shape	Diagram	Area Formula
Rectangle		$A = l \times w$
Square		$A = s \times s$

Ex: Find the area of the rectangle:



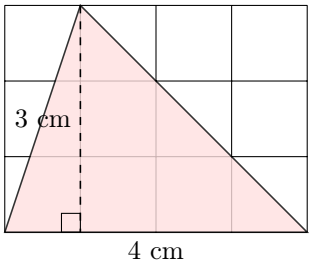
Answer: This is a rectangle with length $l = 8$ m and width $w = 4$ m. Using the formula for the area of a rectangle:

$$\begin{aligned} A &= l \times w \\ &= 8 \times 4 \\ &= 32\,\text{m}^2 \end{aligned}$$

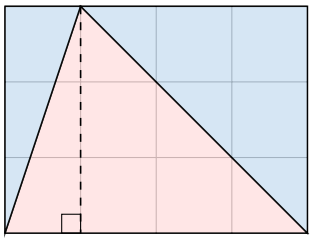
The area is 32 square meters (we read $32\,\text{m}^2$ as "32 square meters").

E AREA OF A TRIANGLE

Discover: To find the area of a triangle, we can cut it along its height to form two smaller triangles, then rearrange them to make a rectangle. Let's see how this works step by step with the triangle below:



1. Cut the triangle along the height CH to form two smaller triangles. Rotate and rearrange these two triangles to form a rectangle:



2. The area of the rectangle is the length multiplied by the height: $4 \times 3 = 12\,\text{cm}^2$. Since the area of the rectangle is equal to twice the area of the original triangle, the area of the triangle is half the area of the rectangle:

$$\begin{aligned} A_\triangle &= \frac{\text{base} \times \text{height}}{2} \\ &= \frac{4 \times 3}{2} \\ &= 6\,\text{cm}^2. \end{aligned}$$

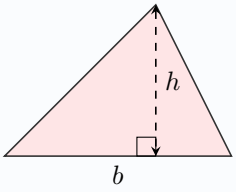
Proposition Area of a Triangle

The area of a triangle is found by multiplying the base by the height and dividing by 2:

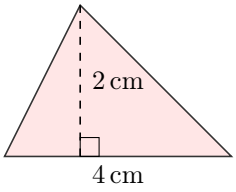
$$\text{Area of a triangle} = \frac{\text{base} \times \text{height}}{2}$$

$$A = \frac{b \times h}{2}$$

where b is the length of the base and h is the corresponding height.



Ex: Find the area of the triangle:



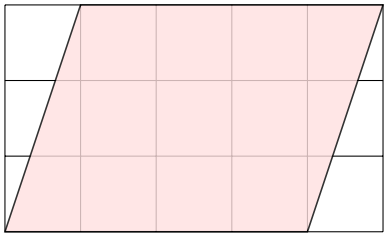
Answer:

$$\begin{aligned} A &= \frac{b \times h}{2} \\ &= \frac{4 \times 2}{2} \\ &= 4\,\text{cm}^2 \end{aligned}$$

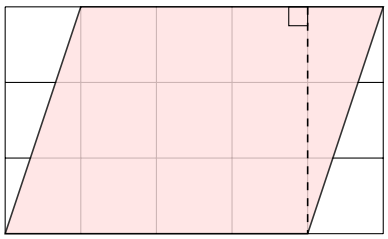
So, the area of the triangle is $4\,\text{cm}^2$.

F AREA OF A PARALLELOGRAM

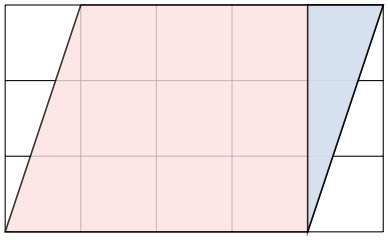
Discover: We can discover the formula for the area of a parallelogram by rearranging it into a rectangle.



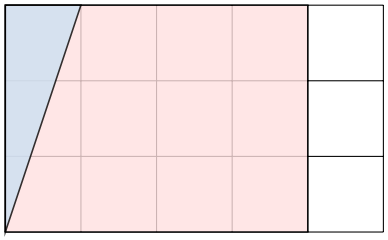
1. Draw the height, which is a line from the top side to the bottom side that is perpendicular to the base:



2. Cut the triangle on the right side:



3. Move the triangle to the left side to form a rectangle:



4. Now we have a rectangle with a length (base) of 4 cm and a height of 3 cm. The area of the parallelogram is the same as the area of this rectangle, which is the base times the height: $4 \times 3 = 12\,\text{cm}^2$.

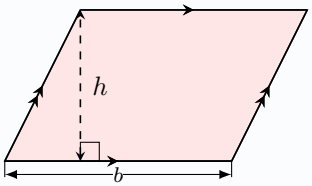
Proposition **Area of a Parallelogram**

The area of a parallelogram is found by multiplying the base by the height:

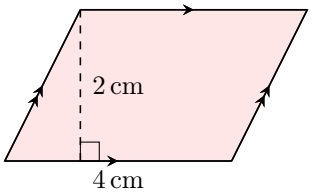
$$\text{Area of a parallelogram} = \text{base} \times \text{height}$$

$$A = b \times h,$$

where b is the base and h is the height.



Ex: Find the area of the parallelogram:



Answer:

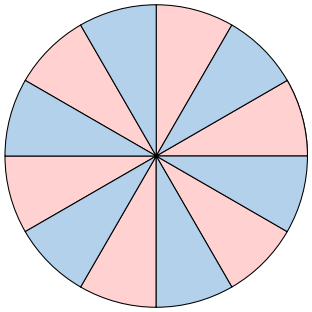
$$\begin{aligned} A &= b \times h \\ &= 4 \times 2 \\ &= 8\,\text{cm}^2 \end{aligned}$$

So, the area of the parallelogram is $8\,\text{cm}^2$.

G AREA OF A CIRCLE

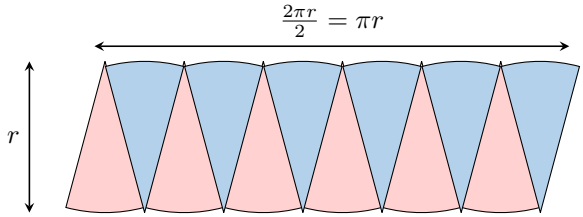
Discover: To find the area of a circle, we can divide it into smaller parts and rearrange them to approximate a parallelogram. Let's see how this works step by step:

1. Divide the circle into 12 equal parts, like slices of a pie:



2. Imagine cutting these 12 parts from the circle.

3. Rearrange the parts by alternating them to form a shape that looks like a parallelogram:



4. The base of the parallelogram is approximately half the circumference of the circle (πr), and its height is approximately the radius (r). So, the area of the circle is the area of the parallelogram:

$$\begin{aligned} A_{\text{circle}} &= (\pi r) \times r \\ &= \pi \times r \times r \\ &= \pi r^2. \end{aligned}$$

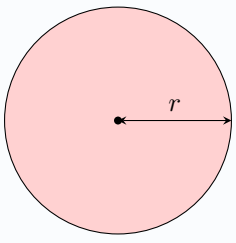
We read this as "pi r squared".

Proposition Area of a Circle

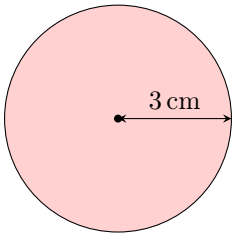
The area of a circle is found by multiplying pi by the radius squared:

$$\text{Area of a circle} = \pi \times \text{radius} \times \text{radius}$$

$$A = \pi r \times r = \pi r^2$$



Ex: Find the area of the circle:



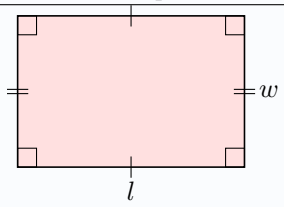
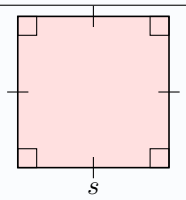
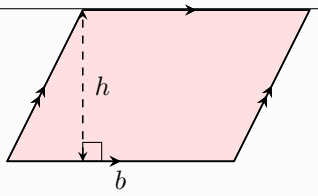
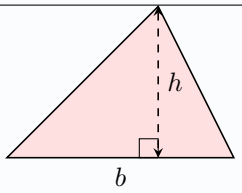
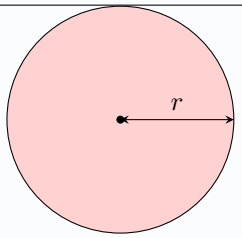
Answer:

$$\begin{aligned} A &= \pi r^2 \\ &= \pi 3^2 \\ &\approx 28.3\,\text{cm}^2 \end{aligned}$$

H AREA FORMULAS

Proposition Area Formulas

Here are the area formulas for some common shapes.

Name	Shape	Area
Rectangle		$A = l \times w$
Square		$A = s \times s$ $= s^2$
Parallelogram		$A = b \times h$
Triangle		$A = \dfrac{b \times h}{2}$
Circle		$A = \pi \times r \times r$ $= \pi r^2$

I AREA OF COMPOSITE FIGURES

Definition Composite Figure

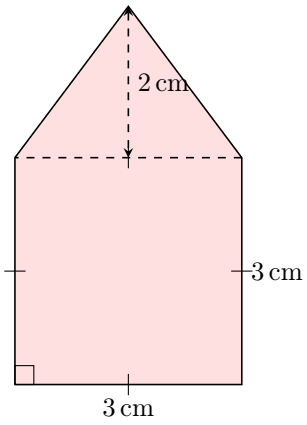
A **composite figure** is made up of two or more simple geometric shapes, like rectangles, triangles, or circles, combined together.

Method **Finding the Area of a Composite Figure**

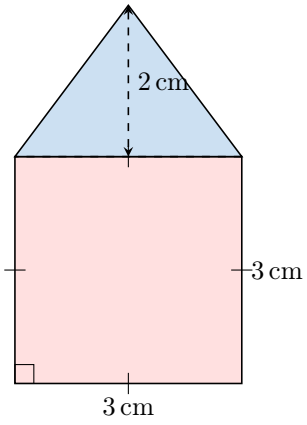
To find the area of a composite figure, follow these steps:

1. Divide the figure into simple, non-overlapping shapes, such as rectangles, triangles, or circles.
2. Find the area of each simpler shape using the appropriate formula.
3. Add the areas together to find the total area of the composite figure.

Ex: Find the area of the composite figure below, which is made up of a square and a triangle:



Answer:



$$
\begin{aligned}
A &= \text{Area of square} + \text{Area of triangle} \\
&= s \times s + \frac{b \times h}{2} \\
&= 3 \times 3 + \frac{3 \times 2}{2} \\
&= 9 + 3 \\
&= 12\,\text{cm}^2
\end{aligned}
$$