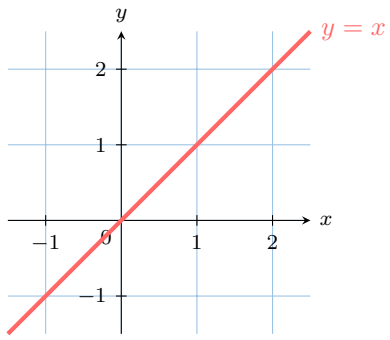
APPLICATIONS OF INTEGRATION IN GEOMETRY

A CALCULATING GEOMETRIC AREA

A.1 EVALUATING THE TOTAL GEOMETRIC AREAS USING GEOMETRIC FORMULAS

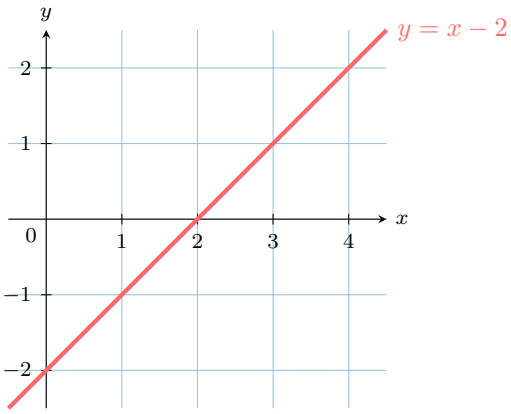
Ex 1:



Find:

$$\int_{-1}^{2} |x| \, dx = \square$$

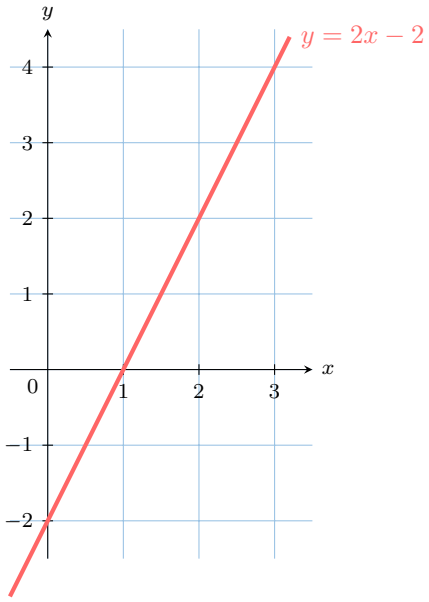
Ex 2:



Find:

$$\int_{0}^{4} |x - 2| \, dx = \square$$

Ex 3:

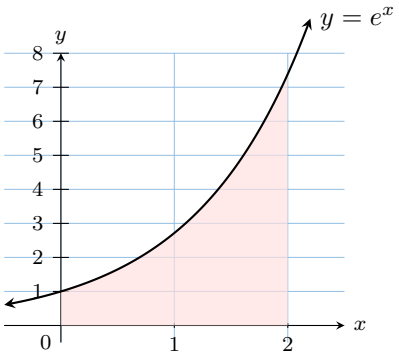


Find:

$$\int_{0}^{3} |2x - 2| \, dx = \square$$

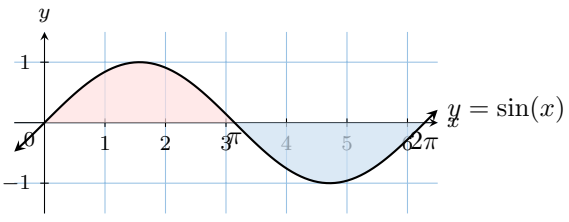
A.2 CALCULATING GEOMETRIC AREA

Ex 4:



Find the area of the region enclosed by the x-axis, the curve $y = e^x$, and the lines $x = 0$ and $x = 2$.

Ex 5:

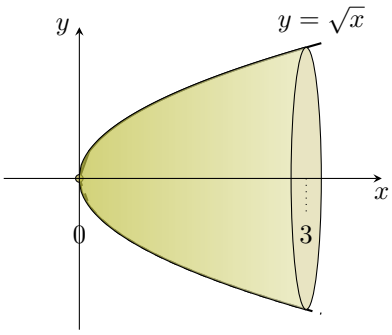


Find the total area of the region enclosed by the x-axis and the curve $y = \sin(x)$ from $x = 0$ to $x = 2\pi$.

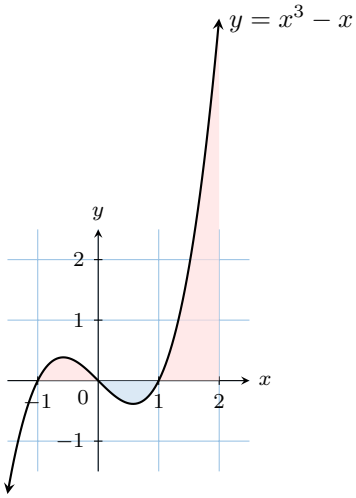
B VOLUMES OF REVOLUTION

B.1 CALCULATING VOLUMES OF REVOLUTION ABOUT THE X-AXIS

Ex 7: Find the volume of the solid generated by revolving the region under the curve $y = \sqrt{x}$ from $x = 0$ to $x = 3$ around the x-axis.

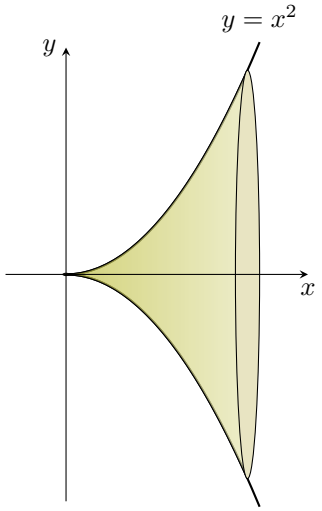


Ex 6:

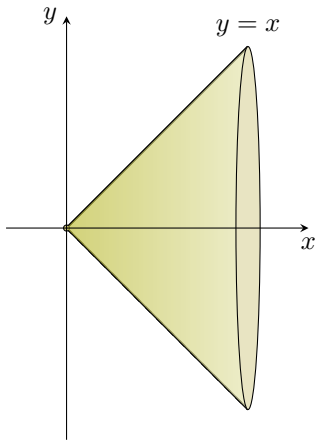


Find the total area of the region enclosed by the x-axis and the curve $y = x^3 - x$ from $x = -1$ to $x = 2$.

Ex 8: Find the volume of the solid generated by revolving the region under the curve $y = x^2$ from $x = 0$ to $x = 3$ around the x-axis.

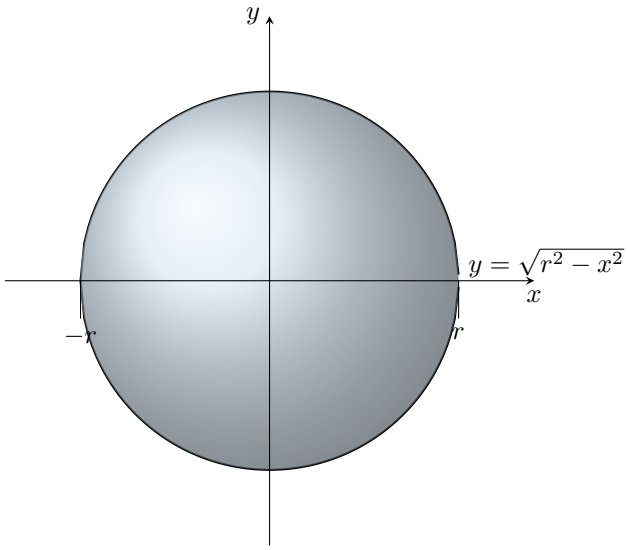


Ex 9: The area bounded by the line $y = x$ and the x-axis is revolved around the x-axis to form a cone.



1. Find the volume of the cone generated if the region is from $x = 0$ to $x = 2$.
2. Find a general formula for the volume of a cone with height h and radius r by revolving the line $y = \frac{r}{h}x$ from $x = 0$ to $x = h$.

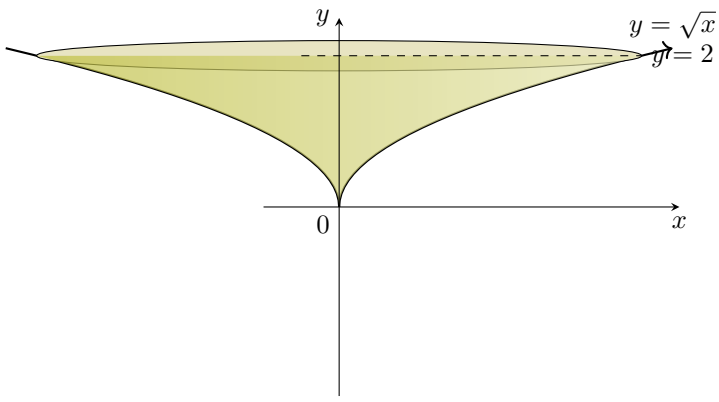
Ex 10: A sphere of radius r can be generated by revolving the semi-circle $y = \sqrt{r^2 - x^2}$ from $x = -r$ to $x = r$ around the x-axis.



Use the method of volumes of revolution to prove the formula for the volume of a sphere, $V = \frac{4}{3}\pi r^3$.

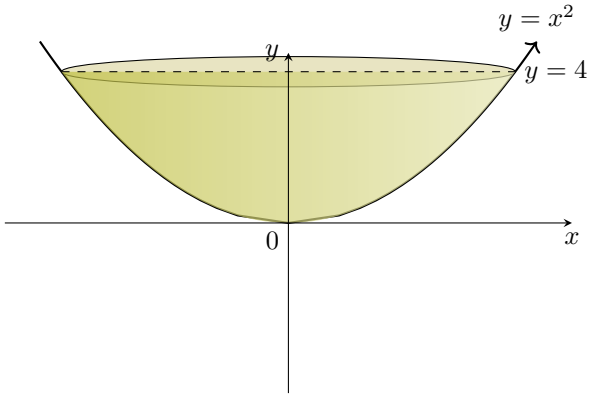
B.2 CALCULATING VOLUMES OF REVOLUTION ABOUT THE Y-AXIS

Ex 11: Consider the region bounded by the curve $y = \sqrt{x}$, the y-axis, and the line $y = 2$. This region is revolved around the y-axis to generate a solid.



1. Express the boundary curve in the form $x = g(y)$.
2. Find the volume of the solid generated.

Ex 12: Consider the region bounded by the curve $y = x^2$, the y-axis, and the line $y = 4$. This region is revolved around the y-axis to generate a solid.



1. Express the boundary curve in the form $x = g(y)$.
2. Find the volume of the solid generated.