FRACTIONS

A DEFINING AND REPRESENTING FRACTIONS

A.1 WRITING FRACTIONS FROM WORDS

Ex 1: Write as a fraction:

$$x \text{ over } 2 = \boxed{\frac{x}{2}}$$

Answer:

$$x ext{ over } 2 = \frac{x}{2}$$

Ex 2: Write as a fraction:

$$x$$
 squared over $6 = \boxed{\frac{x^2}{6}}$

Answer:

$$x$$
 squared over $6 = \frac{x^2}{6}$

Ex 3: Write as a fraction:

3 over
$$x = \boxed{\frac{3}{x}}$$

Answer:

3 over
$$x = \frac{3}{r}$$

Ex 4: Write as a fraction:

$$x+1 \text{ over } 2 = \boxed{\frac{x+1}{2}}$$

Answer:

$$x + 1 \text{ over } 2 = \frac{x+1}{2}$$

B EQUIVALENT FRACTIONS

B.1 SIMPLIFYING ALGEBRAIC FRACTIONS

Ex 5:

$$\frac{6x}{9} = \frac{2x}{3}$$

Answer:

$$\frac{6x}{9} = \frac{\cancel{3} \times 2x}{\cancel{3} \times 3}$$
$$= \frac{2x}{3}$$

Ex 6:

$$\frac{10x^2}{4} = \frac{5x^2}{\boxed{2}}$$

Answer:

$$\frac{10x^2}{4} = \frac{\cancel{2} \times 5x^2}{\cancel{2} \times 2}$$
$$= \frac{5x^2}{2}$$

Ex 7:

$$\frac{9x^3}{30x} = \frac{3x^2}{10}$$

Answer:

$$\frac{9x^3}{30x} = \frac{\cancel{3} \times 3x^2 \times \cancel{x}}{\cancel{3} \times 10 \times \cancel{x}}$$
$$= \frac{3x^2}{10}$$

Ex 8:

$$\frac{3x^2}{2x} = \frac{3x}{\boxed{2}}$$

Answer:

$$\frac{3x^2}{2x} = \frac{3 \times x \times \cancel{x}}{2 \times \cancel{x}}$$
$$= \frac{3x}{2}$$

Ex 9:

$$\frac{2}{4x} = \frac{1}{2x}$$

Answer:

$$\frac{2}{4x} = \frac{\cancel{2}}{\cancel{2} \times 2x}$$
$$= \frac{1}{2x}$$

C SIMPLIFICATION

C.1 SIMPLIFYING ALGEBRAIC FRACTIONS

Ex 10: Simplify:

$$\frac{4x}{6} = \boxed{\frac{2x}{3}}$$

Answer:

$$\frac{4x}{6} = \frac{\cancel{2} \times 2x}{\cancel{2} \times 3}$$
$$= \frac{2x}{3}$$

Ex 11: Simplify:

$$\frac{x^2}{3x} = \boxed{\frac{x}{3}}$$

Answer:

$$\frac{x^2}{3x} = \frac{\cancel{x} \times x}{\cancel{x} \times 3}$$
$$= \frac{x}{3}$$

Ex 12: Simplify:

$$\frac{6x^3}{9x} = \boxed{\frac{2x^2}{3}}$$

$$\frac{6x^3}{9x} = \frac{\cancel{3} \times 2x^2 \times \cancel{x}}{\cancel{3} \times 3 \times \cancel{x}}$$
$$= \frac{2x^2}{3}$$

Ex 13: Simplify:

$$\frac{8x^4}{12x} = \boxed{\frac{2x^3}{3}}$$

Answer:

$$\begin{aligned} \frac{8x^4}{12x} &= \underbrace{\frac{\cancel{4} \times 2x^3 \times \cancel{x}}{\cancel{4} \times 3 \times \cancel{x}}}_{=} \\ &= \frac{2x^3}{3} \end{aligned}$$

Ex 14: Simplify:

$$\frac{15x^2}{25x} = \boxed{\frac{3x}{5}}$$

Answer:

$$\frac{15x^2}{25x} = \frac{\cancel{5} \times 3x \times \cancel{x}}{\cancel{5} \times 5 \times \cancel{x}}$$
$$= \frac{3x}{5}$$

Ex 15: Simplify:

$$\frac{14x^5}{21x^2} = \boxed{\frac{2x^3}{3}}$$

Answer:

$$\begin{aligned} \frac{14x^5}{21x^2} &= \frac{\cancel{7} \times 2x^3 \times \cancel{x}^{\cancel{2}}}{\cancel{7} \times 3 \times \cancel{x}^{\cancel{2}}} \\ &= \frac{2x^3}{3} \end{aligned}$$

Ex 16: Simplify:

$$\frac{4x^3}{16x} = \boxed{\frac{x^2}{4}}$$

Answer:

$$\frac{4x^3}{16x} = \frac{\cancel{4} \times x^2 \times \cancel{x}}{\cancel{4} \times 4 \times \cancel{x}}$$
$$= \frac{x^2}{\cancel{4}}$$

D CROSS MULTIPLICATION

D.1 SOLVING PROPORTIONS USING CROSS-MULTIPLICATION

Ex 17: Solve
$$x$$
 for $\frac{12}{4} = \frac{x}{6}$: $x = \boxed{18}$

Answer:

$$\frac{12}{4} \times x$$

$$4 \times x = 12 \times 6 \qquad \text{(cross multiplication)}$$

$$x = 12 \times 6 \div 4 \qquad \text{(dividing both sides by 4)}$$

$$x = 18$$

Ex 18: Solve
$$x$$
 for $\frac{11}{10} = \frac{x}{5}$: $x = 5.5$

Answer:

$$\frac{11}{10} \times \frac{x}{5}$$

$$10 \times x = 11 \times 5 \qquad \text{(cross multiplication)}$$

$$x = 11 \times 5 \div 10 \quad \text{(dividing both sides by 10)}$$

$$x = 5.5$$

Ex 19: Solve
$$x$$
 for $\frac{12}{10} = \frac{18}{x}$: $x = \boxed{15}$

Answer:

$$\frac{12}{10} = \frac{18}{x}$$

$$12 \times x = 18 \times 10 \qquad \text{(cross multiplication)}$$

$$x = 18 \times 10 \div 12 \quad \text{(dividing both sides by 12)}$$

$$x = 15$$

Ex 20: Solve
$$x$$
 for $\frac{27}{x} = \frac{30}{10}$: $x = 9$

Answer:

D.2 SOLVING PROPORTIONS USING CROSS-MULTIPLICATION

Ex 21: Solve for x in the equation $\frac{x}{3} = \frac{x+1}{2}$. $x = \boxed{-3}$

Answer:

$$rac{x}{3} = rac{x+1}{2}$$
 $2x = 3(x+1)$ (cross multiplication) $2x = 3x+3$ (distributing) $2x-3x=3$ (subtracting $3x$ from both sides) $-x=3$ $x=-3$

Ex 22: Solve for x in the equation $\frac{x}{2} = \frac{x-2}{3}$.

$$x = \boxed{-4}$$

$$rac{x}{2}=rac{x-2}{3}$$
 $3x=2(x-2)$ (cross multiplication) $3x=2x-4$ (distributing) $3x-2x=-4$ (subtracting $2x$ from both sides) $x=-4$

Ex 23: Solve for x in the equation $\frac{2}{x+1} = \frac{1}{x}$.

$$x = \boxed{1}$$

Answer:

$$\frac{2}{x+1} = \frac{1}{x}$$

$$2x = 1(x+1) \quad \text{(cross multiplication)}$$

$$2x = x+1 \quad \text{(distributing)}$$

$$2x-x=1 \quad \text{(subtracting x from both sides)}$$

$$x=1$$

Ex 24: Solve for x in the equation $\frac{2x+1}{4} = \frac{x+2}{3}$.

$$x = \boxed{\frac{5}{2}}$$

Answer:

$$\frac{2x+1}{4} = \frac{x+2}{3}$$

$$3(2x+1) = 4(x+2)$$
 (cross multiplication)
$$6x+3 = 4x+8$$
 (distributing)
$$6x-4x = 8-3$$
 (collecting)
$$2x = 5$$
 (simplify)
$$x = \frac{5}{2}$$

E ADDITION AND SUBTRACTION

E.1 ADDING AND SUBTRACTING ALGEBRAIC FRACTIONS

Ex 25: Calculate and simplify:

$$\frac{x}{6} + \frac{3x}{6} = \boxed{\frac{2x}{3}}$$

Answer

$$\frac{x}{6} + \frac{3x}{6} = \frac{x+3x}{6}$$
 (collecting like terms)
$$= \frac{4x}{6}$$

$$= \frac{\cancel{2} \times 2 \times x}{\cancel{2} \times 3}$$
 (simplifying by canceling common factors)
$$= \frac{2x}{2}$$

Ex 26: Calculate and simplify:

$$\frac{x}{2} + \frac{3x}{4} = \boxed{\frac{5x}{4}}$$

Answer:

$$\frac{x}{2} + \frac{3x}{4} = \frac{x \times 2}{2 \times 2} + \frac{3x}{4}$$
 (finding a common denominator, which is 4)
$$= \frac{2x}{4} + \frac{3x}{4}$$

$$= \frac{2x + 3x}{4}$$
 (combining fractions with common denominator)
$$= \frac{5x}{4}$$

Ex 27: Calculate and simplify:

$$\frac{3x^2}{2} + \frac{5x^2}{3} = \boxed{\frac{19x^2}{6}}$$

Answer:

$$\begin{split} \frac{3x^2}{2} + \frac{5x^2}{3} &= \frac{3x^2 \times 3}{2 \times 3} + \frac{5x^2 \times 2}{3 \times 2} & \text{ (finding a common denominator, which is 6)} \\ &= \frac{9x^2}{6} + \frac{10x^2}{6} & \\ &= \frac{9x^2 + 10x^2}{6} & \text{ (combining fractions with common denominator)} \\ &= \frac{19x^2}{6} & \end{split}$$

Ex 28: Calculate and simplify:

$$\frac{5x}{3} - \frac{x}{6} = \boxed{\frac{3x}{2}}$$

Answer:

$$\frac{5x}{3} - \frac{x}{6} = \frac{5x \times 2}{3 \times 2} - \frac{x \times 1}{6 \times 1}$$
 (finding a common denominator, which is 6)
$$= \frac{10x}{6} - \frac{x}{6}$$

$$= \frac{10x - x}{6}$$
 (combining fractions with common denominator)
$$= \frac{9x}{6}$$

$$= \frac{\cancel{3} \times \cancel{3} \times \cancel{x}}{\cancel{3} \times \cancel{2}}$$
 (simplifying by canceling common factors)
$$= \frac{3x}{2}$$

Ex 29: Calculate and simplify:

$$\frac{2x^2}{5} + \frac{3x^2}{10} = \boxed{\frac{7x^2}{10}}$$

Answer.

$$\frac{2x^2}{5} + \frac{3x^2}{10} = \frac{2x^2 \times 2}{5 \times 2} + \frac{3x^2 \times 1}{10 \times 1}$$
 (finding a common denominator, which is 10)
$$= \frac{4x^2}{10} + \frac{3x^2}{10}$$

$$= \frac{4x^2 + 3x^2}{10}$$
 (combining fractions with common denominator)
$$= \frac{7x^2}{10}$$

Ex 30: Calculate and simplify:

$$\frac{7x^3}{4} - \frac{2x^3}{3} = \boxed{\frac{13x^3}{12}}$$

 $4x^2 \times \frac{x}{8} = \left| \frac{x^3}{2} \right|$

$$\frac{7x^3}{4} - \frac{2x^3}{3} = \frac{7x^3 \times 3}{4 \times 3} - \frac{2x^3 \times 4}{3 \times 4} \qquad \text{(finding a common denominator, which is 12)}$$

$$= \frac{21x^3}{12} - \frac{8x^3}{12}$$

$$= \frac{21x^3 - 8x^3}{12} \qquad \text{(combining fractions with common denominator)}$$

$$13x^3$$

$$4x^2 imes rac{x}{8} = rac{4x^2 imes x}{8}$$
 (multiply the number by the numerator)
$$= rac{4x^3}{8}$$

$$= rac{\cancel{A} imes x^3}{\cancel{A} imes 2}$$
 (simplify by canceling common factors)
$$= rac{x^3}{8}$$

MULTIPLYING FRACTION BY A **NUMBER**

F.1 MULTIPLYING OF ALGEBRAIC FRACTIONS BY **NUMBERS**

Ex 31: Calculate and simplify:

$$x \times \frac{x}{2} = \boxed{\frac{x^2}{2}}$$

Answer:

$$x imes rac{x}{2} = rac{x imes x}{2}$$
 (multiply the number by the numerator)
$$= rac{x^2}{2}$$

Ex 32: Calculate and simplify:

$$3x \times \frac{2x}{9} = \boxed{\frac{2x^2}{3}}$$

Answer:

$$3x imes rac{2x}{9} = rac{3x imes 2x}{9}$$
 (multiply the number by the numerator)
$$= rac{6x^2}{9}$$

$$= rac{3 imes 2 imes x^2}{3 imes 3}$$
 (simplify by canceling common factors)
$$= rac{2x^2}{3}$$

Ex 33: Calculate and simplify:

$$2x \times \frac{x^3}{6} = \boxed{\frac{x^4}{3}}$$

Answer:

$$2x imes rac{x^3}{6} = rac{2x imes x^3}{6}$$
 (multiply the number by the numerator)
$$= rac{2x^4}{6}$$

$$= rac{2 imes x^4}{2 imes 3}$$
 (simplify by canceling common factors)
$$= rac{x^4}{2}$$

Ex 34: Calculate and simplify:

Ex 35: Calculate and simplify:

$$\frac{5}{x} \times x^2 = \boxed{5x}$$

Answer:

Answer.

$$\frac{5}{x} \times x^2 = \frac{5 \times x^2}{x}$$
 (multiply the numerator and denominator)
$$= \frac{5x^2}{x}$$

$$= 5x$$
 (simplify by canceling x in numerator and denominator)

Ex 36: Calculate and simplify:

$$\frac{x^4}{3} \times 6 = \boxed{2x^4}$$

$$\frac{x^4}{3} \times 6 = \frac{x^4 \times 6}{3}$$
 (multiply the numerator and denominator)
$$= \frac{6x^4}{3}$$

$$= \frac{\cancel{3} \times 2 \times x^4}{\cancel{3}}$$
 (simplify by canceling common factors)
$$= 2x^4$$

G MULTIPLICATION OF FRACTIONS

G.1 MULTIPLYING OF ALGEBRAIC FRACTIONS

Ex 37: Calculate and simplify:

$$\frac{2}{3} \times \frac{x}{2} = \boxed{\frac{x}{3}}$$

Answer.

$$egin{align*} rac{2}{3} imes rac{x}{2} &= rac{2 imes x}{3 imes 2} & ext{(multiply the numerators and denominators)} \ &= rac{2x}{6} \ &= rac{2 imes x}{2 imes 3} & ext{(simplify by canceling common factors)} \ &= rac{x}{3} \ \end{aligned}$$

Ex 38: Calculate and simplify:

$$\frac{5}{2} \times \frac{x}{5} = \boxed{\frac{x}{2}}$$

$$\begin{split} \frac{5}{2} \times \frac{x}{5} &= \frac{5 \times x}{2 \times 5} & \text{(multiply the numerators and denominators)} \\ &= \frac{5x}{10} \\ &= \frac{\cancel{5} \times x}{\cancel{5} \times 2} & \text{(simplify by canceling common factors)} \\ &= \frac{x}{2} \end{split}$$

Ex 39: Calculate and simplify:

$$\frac{x}{3} \times \frac{2}{x} = \boxed{\frac{2}{3}}$$

Answer:

$$\begin{split} \frac{x}{3} \times \frac{2}{x} &= \frac{x \times 2}{3 \times x} & \text{(multiply the numerators and denominators)} \\ &= \frac{2x}{3x} \\ &= \frac{\cancel{x} \times 2}{\cancel{x} \times 3} & \text{(simplify by canceling common factors)} \\ &= \frac{2}{3} \end{split}$$

Ex 40: Calculate and simplify:

$$\frac{4}{x} \times \frac{1}{2} = \boxed{\frac{2}{x}}$$

Answer:

$$\begin{split} \frac{4}{x} \times \frac{1}{2} &= \frac{4 \times 1}{x \times 2} & \text{(multiply the numerators and denominators)} \\ &= \frac{4}{2x} \\ &= \frac{\cancel{2} \times 2}{\cancel{2} \times x} & \text{(simplify by canceling common factors)} \\ &= \frac{2}{2} \end{split}$$

G.2 MULTIPLYING OF ALGEBRAIC FRACTIONS

Ex 41: Calculate and simplify:

$$\frac{x}{2} \times \frac{2x}{3} = \boxed{\frac{x^2}{3}}$$

Answer:

$$\frac{x}{2} \times \frac{2x}{3} = \frac{x \times 2x}{2 \times 3}$$
 (multiply the numerators and denominators)
$$= \frac{2x^2}{6}$$

$$= \frac{2 \times x^2}{2 \times 3}$$
 (simplify by canceling common factors)
$$= \frac{x^2}{3}$$

Ex 42: Calculate and simplify:

$$\frac{x}{2} \times \frac{x^2}{3} = \boxed{\frac{x^3}{6}}$$

Answer

$$\frac{x}{2} imes \frac{x^2}{3} = \frac{x imes x^2}{2 imes 3}$$
 (multiply the numerators and denominators)
$$= \frac{x^3}{6}$$

Ex 43: Calculate and simplify:

$$\frac{2}{x} \times \frac{x^2}{3} = \boxed{\frac{2x}{3}}$$

Answer:

$$\frac{2}{x} \times \frac{x^2}{3} = \frac{2 \times x^2}{x \times 3}$$
 (multiply the numerators and denominators)
$$= \frac{2x^2}{3x}$$

$$= \frac{\cancel{x} \times 2 \times \cancel{x}}{\cancel{x} \times 3}$$
 (simplify by canceling common factors)
$$= \frac{2x}{3}$$

Ex 44: Calculate and simplify:

$$\left(\frac{x}{2}\right)^2 = \boxed{\frac{x^2}{4}}$$

Answer:

$$\left(\frac{x}{2}\right)^2 = \frac{x \times x}{2 \times 2}$$
 (square the numerator and the denominator) $= \frac{x^2}{4}$

H FRACTIONS AS THE RESULT OF DIVISION

H.1 DIVIDING ALGEBRAIC FRACTIONS

Ex 45: Calculate and simplify:

$$\frac{2}{3} \div \frac{2}{x} = \boxed{\frac{x}{3}}$$

Answer.

$$\frac{2}{3} \div \frac{2}{x} = \frac{2}{3} \times \frac{x}{2} \quad \text{(dividing by a fraction is multiplying by its reciprocal)}$$

$$= \frac{\cancel{2} \times x}{3 \times \cancel{2}} \quad \text{(simplify by canceling common factors)}$$

$$= \frac{x}{3}$$

Ex 46: Calculate and simplify:

$$\frac{2x}{3} \div 2 = \boxed{\frac{x}{3}}$$

Answer:

$$\frac{2x}{3} \div 2 = \frac{2x}{3} \times \frac{1}{2}$$
 (dividing by a number is multiplying by its reciprocal)
$$= \frac{\cancel{2} \times x \times 1}{3 \times \cancel{2}}$$
 (simplify by canceling common factors)
$$= \frac{x}{2}$$

Ex 47: Calculate and simplify:

$$\frac{3}{x} \div \frac{6}{x} = \boxed{\frac{1}{2}}$$

$$\frac{3}{x} \div \frac{6}{x} = \frac{3}{x} \times \frac{x}{6}$$
 (dividing by a fraction is multiplying by its reciprocal)
$$= \frac{\cancel{3} \times \cancel{x}}{\cancel{x} \times \cancel{3} \times 2}$$
 simplify by canceling common factor)
$$= \frac{1}{2}$$

Ex 48: Calculate and simplify:

$$\frac{4x}{5} \div x = \boxed{\frac{4}{5}}$$

Answer:

$$\frac{4x}{5} \div x = \frac{4x}{5} \times \frac{1}{x}$$
 (dividing by a number is multiplying by its reciproca)
$$= \frac{4 \times \cancel{x} \times 1}{5 \times \cancel{x}}$$
 (simplify by canceling common factors)
$$= \frac{4}{5}$$

H.2 DIVIDING ALGEBRAIC FRACTIONS

Ex 49: Calculate and simplify:

$$\frac{\frac{2}{3}}{\frac{2}{x}} = \boxed{\frac{x}{3}}$$

Answer:

$$\frac{\frac{2}{3}}{\frac{2}{x}} = \frac{2}{3} \times \frac{x}{2} \quad \text{(dividing by a fraction is multiplying by its reciprocal)}$$

$$= \frac{\cancel{2} \times x}{3 \times \cancel{2}} \quad \text{(simplify by canceling common factors)}$$

$$= \frac{x}{3}$$

Ex 50: Calculate and simplify:

$$\frac{\frac{2x}{3}}{2} = \boxed{\frac{x}{3}}$$

Answer:

$$\begin{aligned} \frac{\frac{2x}{3}}{2} &= \frac{2x}{3} \times \frac{1}{2} & \text{(dividing by a number is multiplying by its reciprocal)} \\ &= \frac{\cancel{2} \times x \times 1}{3 \times \cancel{2}} & \text{(simplify by canceling common factors)} \\ &= \frac{x}{3} \end{aligned}$$

Ex 51: Calculate and simplify:

$$\frac{\frac{3}{x}}{\frac{6}{x}} = \boxed{\frac{1}{2}}$$

Answer:

$$\frac{\frac{3}{x}}{\frac{6}{x}} = \frac{3}{x} \times \frac{x}{6}$$
 (dividing by a fraction is multiplying by its reciprocal)
$$= \frac{\cancel{3} \times \cancel{x}}{\cancel{x} \times \cancel{3} \times 2}$$
 (simplify by canceling common factors)
$$= \frac{1}{2}$$

Ex 52: Calculate and simplify:

$$\frac{\frac{4x^2}{5}}{2} = \boxed{\frac{2x^2}{5}}$$

Answer:

$$\frac{\frac{4x^2}{5}}{2} = \frac{4x^2}{5} \div \frac{2}{1} \qquad \text{(dividing by a number is multiplying by its reciproca}$$

$$= \frac{4x^2}{5} \times \frac{1}{2} \qquad \text{(multiply the numerators and denominators)}$$

$$= \frac{4x^2 \times 1}{5 \times 2}$$

$$= \frac{4x^2}{10}$$

$$= \frac{2 \times 2 \times x^2}{2 \times 5} \qquad \text{(simplify by canceling common factors)}$$

$$= \frac{2x^2}{5}$$

Ex 53: Calculate and simplify:

$$\frac{\frac{4x}{5}}{x} = \boxed{\frac{4}{5}}$$

Answer:

$$\frac{\frac{4x}{5}}{x} = \frac{4x}{5} \times \frac{1}{x}$$
 (dividing by a fraction is multiplying by its reciprocal)
$$= \frac{4 \times \cancel{x} \times 1}{5 \times \cancel{x}}$$
 (simplify by canceling common factors)
$$= \frac{4}{5}$$

Ex 54: Calculate and simplify:

$$\frac{\frac{x^2}{2}}{\frac{x}{4}} = \boxed{2x}$$

Answer:

$$\frac{\frac{x^2}{2}}{\frac{x}{4}} = \frac{x^2}{2} \times \frac{4}{x}$$
 (dividing by a fraction is multiplying by its reciprocal)
$$= \frac{\cancel{2} \times 2 \times x \times \cancel{x}}{\cancel{2} \times \cancel{x}}$$
 (simplify by canceling common factors)
$$= 2x$$

I SIGN CONVENTIONS FOR FRACTIONS

I.1 SIMPLIFYING ALGEBRAIC FRACTIONS WITH RELATIVE NUMBERS

Ex 55: Simplify:

$$\frac{-15x}{-30} = \boxed{\frac{x}{2}}$$

Answer:

$$\frac{-15x}{-30} = \underbrace{\frac{-15 \times x}{-15 \times 2}}_{= \frac{x}{2}}$$
 (simplify by canceling common factors)

Ex 56: Simplify:

$$\frac{-6x}{12} = \boxed{-\frac{x}{2}}$$

Answer:

$$\frac{-6x}{12} = \frac{-\cancel{6} \times x}{\cancel{6} \times 2}$$
 (simplify by canceling common factors)
$$= \frac{-x}{2}$$

$$= -\frac{x}{2}$$

Ex 57: Simplify:

$$\frac{-12x^4}{-2x^2} = \boxed{6x^2}$$

Answer:

$$\frac{-12x^4}{-2x^2} = \frac{\cancel{2} \times 6 \times x^2 \times \cancel{x}}{\cancel{2} \times 1 \times \cancel{x}}$$
 (simplify by canceling common factors)
$$= \frac{6x^2}{1}$$

$$= 6x^2$$

Ex 58: Simplify:

$$\frac{3x^3}{-9x} = \boxed{-\frac{x^2}{3}}$$

Answer:

$$\frac{3x^3}{-9x} = \frac{\cancel{3} \times x^2 \times \cancel{x}}{\cancel{3} \times -3 \times \cancel{x}}$$
 (simplify by canceling common factors)
$$= \frac{x^2}{-3}$$

$$= -\frac{x^2}{3}$$

Ex 59: Simplify:

$$\frac{-21x^3}{-7x} = \boxed{3x^2}$$

Answer:

$$\frac{-21x^3}{-7x} = \frac{\cancel{7} \times 3 \times x^2 \times \cancel{x}}{\cancel{7} \times 1 \times \cancel{x}} \quad \text{(simplify by canceling common factors)}$$

$$= \frac{3x^2}{1}$$

$$= 3x^2$$

Ex 60: Simplify:

$$\frac{-4x^4}{8x^2} = \boxed{-\frac{x^2}{2}}$$

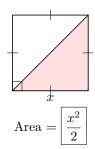
Answer:

$$\begin{split} \frac{-4x^4}{8x^2} &= \frac{-\cancel{A} \times x^2 \times \cancel{x^2}}{\cancel{A} \times 2 \times \cancel{x^2}} & \text{ (simplify by canceling common factors)} \\ &= \frac{-x^2}{2} \end{split}$$

J FRACTION AS QUOTIENT

J.1 FORMULATING ALGEBRAIC EXPRESSIONS

Ex 61: Express the colored area in the following diagram:



Answer: Let's figure out the area of the colored region step by step:

• The diagram shows a square with side length x. The area of this square is:

Area of square
$$= x \times x = x^2$$

• The colored region is a right triangle that occupies half the square. Thus:

Area of colored region
$$=\frac{x^2}{2}$$

Ex 62: You have x marbles and want to share them equally among four friends. Express the number of marbles each friend receives.

Marbles per friend =
$$\sqrt{\frac{x}{4}}$$

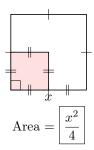
Answer

- You have a total of x marbles.
- These marbles are shared equally among 4 friends. The number of marbles each friend receives is:

$$x \div 4 = \frac{x}{4}$$



Ex 63: Express the colored area in the following diagram:



Answer:

• The area of the square is:

Area of square
$$= x \times x = x^2$$

• The colored region is one of the four equal smaller squares (quadrants), so its area is:

Area of colored region
$$=\frac{x^2}{4}$$

Ex 64: A baker bakes 1000 cookies. He wants to put these cookies into x boxes, with the same number of cookies in each box. Express the number of cookies in each box.

Cookies per box =
$$\left[\frac{1000}{x}\right]$$

Answer:

- The total number of cookies is 1000.
- ullet These cookies are distributed into x boxes. The number of cookies per box is:

$$1000 \div x = \frac{1000}{x}$$

K FRACTION AS A RATIO AND OPERATOR

K.1 APPLYING RATIOS TO ALGEBRAIC EXPRESSIONS

Ex 65: An item is priced at x dollars. If it is discounted by a ratio of $\frac{1}{4}$ of its price, write a simplified expression for the new price.

New Price
$$= \boxed{\frac{3x}{4}}$$

Un article a un prix de x dollars. S'il est réduit d'un rapport de $\frac{1}{4}$ de son prix, écrire une expression simplifiée pour le nouveau prix.

Nouveau Prix =
$$\boxed{\frac{3x}{4}}$$

Answer: The problem requires subtracting a fraction of the original price from the original price.

New Price =
$$x - \left(\frac{1}{4} \text{ of } x\right)$$

= $x - \frac{1}{4}x$ (Apply the 'fraction of' rule)
= $\frac{4}{4}x - \frac{1}{4}x$ (Express x with a common denominating $\frac{3}{4}x$ or $\frac{3x}{4}$

Le problème demande de soustraire une fraction du prix d'origine du prix initial.

Nouveau Prix =
$$x - \left(\frac{1}{4} \text{ de } x\right)$$

= $x - \frac{1}{4}x$ (Appliquer la règle « fraction de »
= $\frac{4}{4}x - \frac{1}{4}x$ (Exprimer x avec un dénominate $\frac{3}{4}x$ ou $\frac{3x}{4}$

Ex 66: Calculate and simplify: $\frac{2}{3}$ of (x+9).

$$\boxed{\frac{2x+18}{3}}$$

Calculer et simplifier : $\frac{2}{3}$ de (x+9).

$$\boxed{\frac{2x+18}{3}}$$

Answer: Applying the rule "fraction of a quantity":

$$\frac{2}{3} \text{ of } (x+9) = \frac{2}{3} \times (x+9)$$

$$= \frac{2(x+9)}{3} \qquad \text{(Multiply the fraction by the expression }$$

$$= \frac{2x+18}{3}$$

This can also be expressed as $\frac{2}{3}x+6. \mathrm{En}$ appliquant la règle « fraction d'une quantité » :

$$\frac{2}{3} \text{ de } (x+9) = \frac{2}{3} \times (x+9)$$

$$= \frac{2(x+9)}{3} \qquad \text{(Multiplier la fraction par l'expression)}$$

$$= \frac{2x+18}{3}$$

Cela peut aussi être exprimé par $\frac{2}{3}x + 6$.

Ex 67: Calculate and simplify: one-half of x plus one-third of x.

$$\frac{5x}{6}$$

Calculer et simplifier : la moitié de x plus un tiers de x.

$$\frac{5x}{6}$$

Answer: First, express the problem as an algebraic expression.

$$\frac{1}{2}x + \frac{1}{3}x$$

To combine the like terms, find a common denominator for the coefficients. The least common multiple of 2 and 3 is 6.

$$\frac{1}{2}x + \frac{1}{3}x = \left(\frac{1\times3}{2\times3}\right)x + \left(\frac{1\times2}{3\times2}\right)x$$
$$= \frac{3}{6}x + \frac{2}{6}x$$
$$= \left(\frac{3+2}{6}\right)x$$
$$= \frac{5}{6}x \text{ or } \frac{5x}{6}$$

D'abord, exprimer le problème sous forme d'expression algébrique.

$$\frac{1}{2}x + \frac{1}{3}x$$

Pour combiner les termes semblables, trouver un dénominateur commun pour les coefficients. Le plus petit commun multiple de 2 et 3 est 6.

$$\frac{1}{2}x + \frac{1}{3}x = \left(\frac{1\times3}{2\times3}\right)x + \left(\frac{1\times2}{3\times2}\right)x$$
$$= \frac{3}{6}x + \frac{2}{6}x$$
$$= \left(\frac{3+2}{6}\right)x$$
$$= \frac{5}{6}x \text{ ou } \frac{5x}{6}$$

Ex 68: A quantity x is increased by $\frac{2}{5}$ of its value. Write a simplified expression for the new quantity

New Quantity =
$$\boxed{\frac{7x}{5}}$$

Une quantité x est augmentée de $\frac{2}{5}$ de sa valeur. Écrire une Calculer et simplifier : expression simplifiée pour la nouvelle quantité.

Nouvelle Quantité =
$$\boxed{\frac{7x}{5}}$$

Answer: The problem is represented by the expression:

$$x + \frac{2}{5}x$$

To simplify, combine the like terms by finding a common denominator.

$$1x + \frac{2}{5}x = \frac{5}{5}x + \frac{2}{5}x$$
$$= \left(\frac{5+2}{5}\right)x$$
$$= \frac{7}{5}x \text{ or } \frac{7x}{5}$$

Le problème est représenté par l'expression :

$$x + \frac{2}{5}x$$

Pour simplifier, combiner les termes semblables en trouvant un dénominateur commun.

$$1x + \frac{2}{5}x = \frac{5}{5}x + \frac{2}{5}x$$
$$= \left(\frac{5+2}{5}\right)x$$
$$= \frac{7}{5}x \text{ ou } \frac{7x}{5}$$

L FRACTIONS AS DECIMAL NUMBERS

L.1 SIMPLIFYING EXPRESSIONS WITH FRACTIONAL AND DECIMAL COEFFICIENTS

Ex 69: Calculate and simplify:

$$0.5x - \frac{x}{2} = \boxed{0}$$

Calculer et simplifier :

$$0,5x-\frac{x}{2}=\boxed{0}$$

Answer: To simplify the expression, we first convert the decimal coefficient to its equivalent fractional form.

$$0.5x - \frac{x}{2} = \frac{1}{2}x - \frac{x}{2} \qquad \text{(Convert 0.5 to a fraction)}$$
$$= \frac{1}{2}x - \frac{1}{2}x \qquad \text{(Rewrite } \frac{x}{2} \text{ as } \frac{1}{2}x\text{)}$$
$$= 0$$

Pour simplifier l'expression, nous convertissons d'abord le coefficient décimal en sa forme fractionnaire équivalente.

$$0,5x - \frac{x}{2} = \frac{1}{2}x - \frac{x}{2} \qquad \text{(Convertir 0,5 en fraction)}$$

$$= \frac{1}{2}x - \frac{1}{2}x \qquad \text{(Réécrire } \frac{x}{2} \text{ en } \frac{1}{2}x\text{)}$$

$$= 0$$

Ex 70: Calculate and simplify:

$$0.25y + \frac{3y}{4} = y$$

$$0,25y + \frac{3y}{4} = y$$

Answer: To simplify the expression, convert the decimal to a fraction and find a common denominator.

$$0.25y + \frac{3y}{4} = \frac{1}{4}y + \frac{3}{4}y \qquad \text{(Convert 0.25 to a fraction)}$$
$$= \left(\frac{1}{4} + \frac{3}{4}\right)y \quad \text{(Factor out y)}$$
$$= \frac{4}{4}y$$
$$= y$$

Pour simplifier l'expression, convertir le nombre décimal en fraction et trouver un dénominateur commun.

$$0,25y + \frac{3y}{4} = \frac{1}{4}y + \frac{3}{4}y$$
 (Convertir 0,25 en fraction)

$$= \left(\frac{1}{4} + \frac{3}{4}\right)y$$
 (Factoriser y)

$$= \frac{4}{4}y$$

$$= y$$

Ex 71: Calculate and simplify:

$$a - 0.2a - \frac{2a}{5} = \boxed{0.4a}$$

Calculer et simplifier :

$$a - 0, 2a - \frac{2a}{5} = \boxed{0.4a}$$

Answer: To simplify, convert all terms to a consistent format (either decimal or fraction). Here, we will convert the fraction to a decimal.

$$a - 0.2a - \frac{2a}{5} = 1a - 0.2a - (2 \div 5)a$$
 (Convert fraction to decimal)
= $1a - 0.2a - 0.4a$
= $(1 - 0.2 - 0.4)a$
= $0.4a$

Alternatively, converting all terms to fractions with a common denominator gives $\frac{2}{5}a$. Pour simplifier, convertir tous les termes dans un format cohérent (soit décimal, soit fractionnaire). Ici, nous convertirons la fraction en décimal.

$$a-0,2a-\frac{2a}{5}=1a-0,2a-(2\div 5)a \quad \text{(Convertir la fraction en décimal)}$$

$$=1a-0,2a-0,4a$$

$$=(1-0,2-0,4)a$$

$$=0.4a$$

Alternativement, la conversion de tous les termes en fractions avec un dénominateur commun donne $\frac{2}{5}a$.

Ex 72: Calculate and simplify:

$$0.75 imes rac{z}{3} = \boxed{z/4}$$

Calculer et simplifier :

$$0,75 \times \frac{z}{3} = \boxed{z/4}$$

Answer: To simplify, convert the decimal to a fraction and then multiply.

$$0.75 \times \frac{z}{3} = \frac{3}{4} \times \frac{z}{3}$$
 (Convert 0.75 to a fraction)

$$= \frac{3 \times z}{4 \times 3}$$

$$= \frac{3z}{12}$$
 (Simplify by dividing by the common factor $z = \frac{z}{1}$

Pour simplifier, convertir le nombre décimal en fraction, puis multiplier.

$$0,75 \times \frac{z}{3} = \frac{3}{4} \times \frac{z}{3}$$
 (Convertir 0,75 en fraction)

$$= \frac{3 \times z}{4 \times 3}$$

$$= \frac{3z}{12}$$
 (Simplifier en divisant par le facteur con

Ex 73: Calculate and simplify:

$$0.25x + \frac{3x}{4} = \boxed{x}$$

Calculer et simplifier :

$$0,25x + \frac{3x}{4} = \boxed{x}$$

Answer: To simplify the expression, convert the decimal to a fraction to achieve a common format.

$$0.25x + \frac{3x}{4} = \frac{1}{4}x + \frac{3}{4}x \qquad \text{(Convert 0.25 to a fraction)}$$
$$= \left(\frac{1}{4} + \frac{3}{4}\right)x \quad \text{(Factor out x)}$$
$$= \frac{4}{4}x$$
$$= x$$

Pour simplifier l'expression, convertir le nombre décimal en fraction pour obtenir un format commun.

$$0,25x + \frac{3x}{4} = \frac{1}{4}x + \frac{3}{4}x \qquad \text{(Convertir 0,25 en fraction)}$$

$$= \left(\frac{1}{4} + \frac{3}{4}\right)x \quad \text{(Factoriser x)}$$

$$= \frac{4}{4}x$$

$$= x$$

Ex 74: Calculate and simplify:

$$x - 0.2x - \frac{2x}{5} = 0.4x$$

Calculer et simplifier :

$$x - 0, 2x - \frac{2x}{5} = \boxed{0.4x}$$

 ${\it Answer:}$ To simplify, convert all terms to a consistent format. In this case, converting the fraction to a decimal is efficient.

$$x - 0.2x - \frac{2x}{5} = 1x - 0.2x - (2 \div 5)x$$
 (Convert fraction to decimal $x - 0.2x - 0.4x$)
= $(1 - 0.2 - 0.4)x$
= $0.4x$

Alternatively, converting all terms to fractions with a common (Simplify by dividing by the common factor 3) denominator gives the equivalent answer $\frac{2}{5}x$. Pour simplifier, convertir tous les termes dans un format cohérent. Dans ce cas, il est efficace de convertir la fraction en nombre décimal.

$$x - 0, 2x - \frac{2x}{5} = 1x - 0, 2x - (2 \div 5)x$$
 (Convertir la fraction en d
$$= 1x - 0, 2x - 0, 4x$$
$$= (1 - 0, 2 - 0, 4)x$$
$$= 0, 4x$$

Alternativement, la conversion de tous les termes en fractions (Simplifier en divisant par le facteur commun \mathfrak{B}) et un dénominateur commun donne la réponse équivalente $\frac{2}{5}x$.

Ex 75: Calculate and simplify:

$$0.75 \times \frac{x}{3} = \sqrt{x/4}$$

Calculer et simplifier :

$$0,75 \times \frac{x}{3} = x/4$$

Answer: To simplify, convert the decimal to a fraction and then perform the multiplication.

$$0.75 \times \frac{x}{3} = \frac{3}{4} \times \frac{x}{3}$$
 (Convert 0.75 to a fraction)
$$= \frac{3 \times x}{4 \times 3}$$
 (Simplify by canceling the common factor of 3))
$$= \frac{x}{4}$$
 (Simplify by canceling the common factor of 3))
$$= \frac{x}{4}$$
 (Simplify by canceling the common factor of 3)
$$x \times \frac{3x + x}{4 + 2} = x \times \frac{4x}{6}$$
 (simplifier le numérateur et le dénominateur)

Pour simplifier, convertir le nombre décimal en fraction, puis effectuer la multiplication.

$$0,75 \times \frac{x}{3} = \frac{3}{4} \times \frac{x}{3}$$
 (Convertir 0,75 en fraction)
= $\frac{\cancel{3} \times x}{4 \times \cancel{3}}$ (Simplifier en annulant le facteur commun de 3)
= $\frac{x}{4}$

ORDER OF OPERATIONS

M.1 CALCULATING ALGEBRAIC EXPRESSIONS

Ex 76: Calculate and simplify:

$$\frac{x+7x}{3\times 4} = \boxed{\frac{2x}{3}}$$

Calcule et simplifie :

$$\frac{x+7x}{3\times 4} = \boxed{\frac{2x}{3}}$$

Answer:

$$\frac{x+7x}{3\times 4} = \frac{8x}{12}$$
 (simplify numerator and denominator)
$$= \frac{\cancel{4}\times 2\times x}{\cancel{4}\times 3}$$
 (simplify by canceling common factors)
$$= \frac{2x}{3}$$

$$\frac{x+7x}{3\times 4} = \frac{8x}{12}$$
 (simplifier le numérateur et le dénominateur):
$$= \frac{\cancel{4}\times 2\times x}{\cancel{4}\times 3}$$
 (simplifier en annulant les facteurs communs)
$$= \frac{2x}{\cancel{2}}$$

Ex 77: Calculate and simplify:

$$x \times \frac{3x + x}{4 + 2} = \boxed{\frac{2x^2}{3}}$$

Calcule et simplifie :

$$x \times \frac{3x + x}{4 + 2} = \boxed{\frac{2x^2}{3}}$$

$$x imes rac{3x+x}{4+2} = x imes rac{4x}{6}$$
 (simplify the numerator and the denominato
$$= rac{x imes 4x}{6}$$
 (multiply the numerators and denominators)
$$= rac{4x^2}{6}$$

$$= rac{2 imes 2 imes x^2}{2 imes 3}$$
 (simplify by canceling common factors)
$$= rac{2x^2}{3}$$
)
$$x imes rac{3x+x}{4+2} = x imes rac{4x}{6}$$
 (simplifier le numérateur et le dénominateur)

$$=\frac{x\times 4x}{6}$$
 (multiplier les numérateurs et les dénominateurs et les des dénominateurs et les dénominateurs et les des des des des d

Ex 78: Calculate and simplify:

$$\frac{2x^3}{2x-x} = 2x^2$$

Calcule et simplifie :

$$\frac{2x^3}{2x-x} = \boxed{2x^2}$$

Answer.

$$\frac{2x^3}{2x - x} = \frac{2x^3}{x}$$
 (combine like terms in the denominator
$$= \frac{2 \times x^2 \times \cancel{t}}{\cancel{t}}$$
 (simplify by canceling common factors)
$$= 2x^2$$

$$\frac{2x^3}{2x-x} = \frac{2x^3}{x} \qquad \text{(additionner les termes semblables au dénominateur)}$$

$$= \frac{2\times x^2 \times \cancel{x}}{\cancel{x}} \qquad \text{(simplifier en annulant les facteurs communs)}$$

$$= 2x^2$$

Ex 79: Calculate and simplify:

$$4x \times \frac{6x-2x}{2\times 8} = \boxed{x^2}$$

Calcule et simplifie :

$$4x \times \frac{6x - 2x}{2 \times 8} = \boxed{x^2}$$

$$4x \times \frac{6x - 2x}{2 \times 8} = 4x \times \frac{4x}{16} \quad \text{(simplify the numerator and the denominator)}$$

$$= \frac{4x \times 4x}{16} \quad \text{(multiply the numerators and denominators)}$$

$$= \frac{16x^2}{16}$$

$$= \frac{\cancel{16} \times x^2}{\cancel{16} \times 1} \quad \text{(simplify by canceling common factors)}$$

$$= x^2$$

$$4x \times \frac{6x - 2x}{2 \times 8} = 4x \times \frac{4x}{16} \qquad \text{(simplifier le numérateur et le dénominateur)}$$

$$= \frac{4x \times 4x}{16} \qquad \text{(multiplier les numérateurs et les dénominateurs)}$$

$$= \frac{16x^2}{16}$$

$$= \frac{\cancel{16} \times x^2}{\cancel{16} \times 1} \qquad \text{(simplifier en annulant les facteurs communs)}$$

$$= x^2$$

M.2 CALCULATING ALGEBRAIC EXPRESSIONS

Ex 80: Write as a single fraction:

$$2 - \frac{x+1}{3} = \boxed{\frac{5-x}{3}}$$

Answer:

$$\begin{aligned} 2 - \frac{x+1}{3} &= \frac{2 \times 3}{3} - \frac{x+1}{3} & \text{(common denominator = 3)} \\ &= \frac{6}{3} - \frac{x+1}{3} \\ &= \frac{6 - (x+1)}{3} & \text{(combine with common denominator)} \\ &= \frac{6 - x - 1}{3} & \text{(distribute the negative sign)} \\ &= \frac{5 - x}{3} \end{aligned}$$

Ex 81: Write as a single fraction:

$$3x - \frac{2-x}{4} = \boxed{\frac{13x-2}{4}}$$

Answer:

$$3x - \frac{2-x}{4} = \frac{3x \times 4}{4} - \frac{2-x}{4} \qquad \text{(common denominator = 4)}$$

$$= \frac{12x}{4} - \frac{2-x}{4}$$

$$= \frac{12x - (2-x)}{4} \qquad \text{(combine with common denominator)}$$

$$= \frac{12x - 2 + x}{4} \qquad \text{(distribute the negative sign)}$$

$$= \frac{13x - 2}{4}$$

Ex 82: Write as a single fraction:

$$\frac{x}{2} - \frac{x+1}{3} = \boxed{\frac{x-2}{6}}$$

Answer:

$$\frac{x}{2} - \frac{x+1}{3} = \frac{x \times 3}{2 \times 3} - \frac{(x+1) \times 2}{3 \times 2} \qquad \text{(common denominator = 6)}$$

$$= \frac{3x}{6} - \frac{2(x+1)}{6}$$

$$= \frac{3x - 2(x+1)}{6} \qquad \text{(combine with common denominator }$$

$$= \frac{3x - 2x - 2}{6} \qquad \text{(distribute the negative sign)}$$

$$= \frac{x-2}{6} \qquad \text{(combine like terms)}$$

Ex 83: Write as a single fraction:

$$\frac{x+1}{3} - \frac{x+4}{2} = \boxed{\frac{-x-10}{6}}$$

Answer:

$$\begin{array}{l} \frac{x+1}{3} - \frac{x+4}{2} = \frac{(x+1)\times 2}{3\times 2} - \frac{(x+4)\times 3}{2\times 3} & \text{(common denominator = 6)} \\ &= \frac{2(x+1)}{6} - \frac{3(x+4)}{6} \\ &= \frac{2(x+1) - 3(x+4)}{6} \\ &= \frac{2x+2 - 3x - 12}{6} & \text{(distribute the negative sign} \\ &= \frac{-x - 10}{6} & \text{(combine like terms)} \end{array}$$