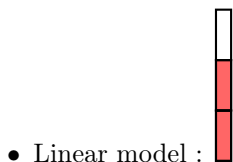# ALGEBRAIC FRACTIONS

A DEFINITIONS

Definition Fraction

A **fraction** consists of two numbers: the **numerator**, a, and the **denominator**, $b \neq 0$, separated by a horizontal bar:

$$\frac{a}{b}$$

$a \longleftarrow$ **numerator**: number of equal parts considered

$b \longleftarrow$ **denominator**: number of equal parts the unit is divided

A fraction can be represented as:

- Symbol : $\dfrac{2}{3}$

- Words : two thirds or two over three

- Linear model :

B FRACTION AS QUOTIENT

Proposition Fraction as Quotient

A fraction is a quotient that represents the result of **division**. It tells us how much of something we have when we divide it into equal parts.

- **The top number (numerator)** is the whole.

- **The bottom number (denominator)** is the number of equal parts the whole is divided into.

The fraction $\dfrac{a}{b}$ is the same as saying **"a divided by b"**.

$$\frac{a}{b} = a \div b$$

The fraction $\dfrac{a}{b}$ is the number which, when multiplied by b, gives a:

$$\frac{a}{b} \times b = a$$

Ex:

$$2 \div 3 = \qquad = \frac{2}{3}$$

C EQUIVALENT FRACTIONS

Definition Equivalent Fractions

- When you multiply the numerator and the denominator by the same number, the fractions are equals.

$$\frac{a}{b} \; \overset{\times k}{\underset{\times k}{=}} \; \frac{k \times a}{k \times b}$$

- When you divide the numerator and the denominator by the same number, the fractions are equals.

$$\frac{k \times a}{k \times b} \; \overset{\div k}{\underset{\div k}{=}} \; \frac{a}{b}$$

Ex:

$$= \; \frac{1}{3} \; \overset{\times 2}{\underset{\times 2}{=}} \; \frac{2 \times 1}{2 \times 3} = \; \frac{2}{6} \; =$$

Ex:

$$= \; \frac{3}{6} \; \overset{\div 3}{\underset{\div 3}{=}} \; \frac{3 \times 1}{3 \times 2} = \; \frac{1}{2} \; =$$

D CROSS MULTIPLICATION

Proposition Cross Multiplication Property

$$\frac{a}{b} \; \diagup\!\!\!\!\diagup \; \frac{c}{d} \quad \text{if and only if} \quad a \times d = b \times c$$

Ex: Solve x for $\dfrac{10}{5} = \dfrac{x}{8}$.

Answer:

$$\frac{10}{5} \; \diagup\!\!\!\!\diagup \; \frac{x}{8}$$

$$
\begin{aligned}
5 \times x &= \; 10 \times 8 && \text{(cross mutiplication)} \\
x &= 10 \times 8 \div 5 && \text{(dividing both sides by 5)} \\
x &= \; 16
\end{aligned}
$$

E SIMPLIFICATION

Definition Simplest form

A fraction is in **simplest form** if it is written with the smallest possible whole number numerator and denominator, that is, if its numerator and denominator have no common factors other than 1.

Ex:

- $\frac{2}{3}$ is in simplest form.

- $\frac{4}{6}$ is **not** in simplest form because we can write $\frac{4}{6} = \frac{2}{3}$.

Method **Simplifying a fraction**

To simplify a fraction (or to write a fraction in its simplest form), we cancel the greatest common factor of the numerator and the denominator .

Ex: Simplify $\frac{4}{6}$.

Answer:

$$\begin{aligned} \frac{4}{6} &= \frac{2 \times \cancel{2}}{3 \times \cancel{2}} \\ &= \frac{2}{3} \end{aligned}$$

F ADDITION AND SUBTRACTION

Definition **Addition and Subtraction of Fractions with Common Denominators**

- When we **add** fractions with common denominators, we keep the denominator the same and add the numerators:

$$\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b}$$

- When we **subtract** fractions with common denominators, we keep the denominator the same and subtract the numerators:
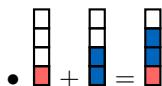
$$\frac{a}{b} - \frac{c}{b} = \frac{a-c}{b}$$

Ex: Calculate $\frac{1}{4} + \frac{2}{4}$.

Answer:

-

$$\begin{aligned} \frac{1}{4} + \frac{2}{4} &= \frac{1+2}{4} \\ &= \frac{3}{4} \end{aligned}$$

-

Method **Addition or Subtraction of Fractions with Different Denominators**

To add or subtract fractions with different denominators:

- **Find a common denominator**: Choose a common multiple of the denominators.

- **Convert each fraction**: Rewrite each fraction so it has the common denominator.

- **Add or subtract the numerators**: Add or subtract the numerators and keep the denominator the same.
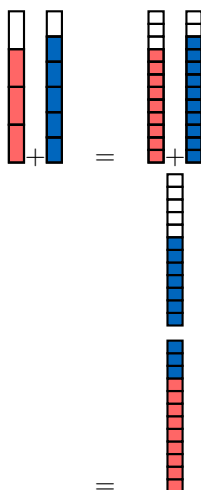

Ex: Calculate $\frac{3}{4} + \frac{5}{6}$.

Answer:

- **Find a common denominator:** To add fractions, they must have the same denominator.

  - Multiples of 4: 4, 8, **12**, 16, 20, ...
  - Multiples of 6: 6, **12**, 18, 24, ...
  - The smallest common denominator is **12**.

- $\begin{aligned} \frac{3}{4} + \frac{5}{6} &= \frac{3 \times 3}{4 \times 3} + \frac{5 \times 2}{6 \times 2} \\ &= \frac{9}{12} + \frac{10}{12} \qquad \text{(common denominator} = 12) \\ &= \frac{9+10}{12} \qquad \text{(adding numerators)} \\ &= \frac{19}{12} \end{aligned}$

- **Visual representation**:

G MULTIPLICATION OF A FRACTION BY A NUMBER

Definition **Multiplication of a Fraction by a Number**

To multiply a fraction by a whole number:

1. Multiply the numerator by the number.

2. Keep the denominator the same.

$$a \times \frac{b}{c} = \frac{a \times b}{c}$$

Ex: Calculate $3 \times \frac{2}{5}$.

Answer:

- Mathematical calculation:

$$3 \times \frac{2}{5} = \frac{3 \times 2}{5}$$
$$= \frac{6}{5}$$

- Visual representation:

H MULTIPLICATION OF FRACTIONS

Definition **Multiplication of Fractions**

To multiply fractions, **tu multiplies** the numerators and **tu multiplies** the denominators:

$$\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$$

Ex: Calculate $\frac{5}{2} \times \frac{3}{4}$.

Answer:

$$\frac{5}{2} \times \frac{3}{4} = \frac{5 \times 3}{2 \times 4}$$
$$= \frac{15}{8}$$

Ex: Calculate $\dfrac{31}{7} \times \dfrac{12}{31}$.

Answer:

$$\frac{31}{7} \times \frac{12}{31} = \frac{\cancel{31} \times 12}{7 \times \cancel{31}} \quad \text{(cancel the common factor 31)}$$
$$= \frac{12}{7}$$

I DIVISION OF FRACTIONS

Definition **Reciprocal**

The **reciprocal** of a number is a number that, when multiplied by the original number, gives 1.

Proposition **Reciprocal of a fraction**

The reciprocal of the fraction $\dfrac{a}{b}$ is $\dfrac{b}{a}$.

Ex: State the reciprocal of $\dfrac{5}{7}$.

Answer: The reciprocal of $\dfrac{5}{7}$ is $\dfrac{7}{5}$.

Definition **Division of fractions**

To divide by a fraction, you multiply by its reciprocal:

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c},$$

or equivalently,

$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \times \frac{d}{c}.$$

Ex: Calculate $\dfrac{2}{3} \div \dfrac{5}{7}$.

Answer:

$$\frac{2}{3} \div \frac{5}{7} = \frac{2}{3} \times \frac{7}{5} \quad \text{(multiply by the reciprocal)}$$
$$= \frac{2 \times 7}{3 \times 5} \quad \text{(multiply numerators and denominators)}$$
$$= \frac{14}{15}.$$

J SIGN RULES

Proposition **Sign rules**

$$\frac{-a}{b} = \frac{a}{-b} = -\frac{a}{b},$$

and

$$\frac{-a}{-b} = \frac{a}{b}.$$

Ex: Simplify $\dfrac{-4}{-6}$.

Answer:

$$\frac{-4}{-6} = \frac{4}{6} \quad \text{(a negative divided by a negative is positive)}$$
$$= \frac{2 \times 2}{3 \times 2} \quad \text{(cancel the common factor 2)}$$
$$= \frac{2}{3}.$$

K ORDER OF OPERATIONS

> **Definition Order of Operations**
>
> The division line in a fraction acts as a grouping symbol (like parentheses). This means that, according to the order of operations (PEMDAS), you must first evaluate the numerator and the denominator before performing the division.

Ex: Simplify $\frac{1+7}{3\times4}$.

Answer:

$$\frac{1+7}{3\times4} = \frac{8}{12} \quad \text{(evaluate numerator and denominator)}$$
$$= \frac{2\times\cancel{4}}{3\times\cancel{4}} \quad \text{(cancel common factor)}$$
$$= \frac{2}{3}$$