ALGEBRAIC FRACTIONS

A DEFINITIONS

A.1 WRITING FRACTIONS FROM WORDS

Ex 1: Write as fraction:

$$x \text{ over } 2 = \boxed{\frac{x}{2}}$$

Answer:

$$x \text{ sur } 2 = \frac{x}{2}$$

Ex 2: Write as a fraction:

$$x \text{ square over } 6 = \boxed{\frac{x^2}{6}}$$

Answer:

$$x \text{ carré sur } 6 = \frac{x^2}{6}$$

Ex 3: Write as a fraction:

$$3 \text{ over } x = \boxed{\frac{3}{x}}$$

Answer:

$$3 \text{ over } x = \frac{3}{x}$$

Ex 4: Write as a fraction:

$$x + 1 \text{ over } 2 = \boxed{\frac{x+1}{2}}$$

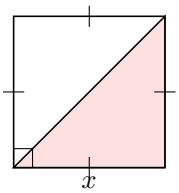
Answer:

$$x + 1 \text{ over } 2 = \frac{x+1}{2}$$

B FRACTION AS QUOTIENT

B.1 FORMULATING ALGEBRAIC EXPRESSIONS

Ex 5: Express the colored area in the following diagram:



$$\text{Area} = \boxed{\frac{x^2}{2}}$$

Answer: Let's figure out the area of the colored region step by step:

- The diagram shows a square with side length x. The area of this square is:

$$\begin{aligned}\text{Area of square} &= x \times x \\ &= x^2\end{aligned}$$

- The area of the colored region is half the area of the square:

$$\text{Area of colored region} = \frac{x^2}{2}$$

Ex 6: You have x marbles and want to share them equally among four friends. Express the number of marbles each friend receives.

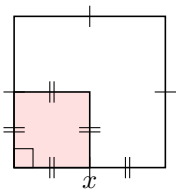
$$\text{Marbles per friend} = \boxed{\frac{x}{4}}$$

Answer:

- You have a total of x marbles.
- These marbles are shared equally among 4 friends. The number of marbles each friend receives is:

$$x \div 4 = \frac{x}{4}$$

Ex 7: Express the colored area in the following diagram:



$$\text{Area} = \boxed{\frac{x^2}{4}}$$

Answer:

- The area of the square is:

$$\begin{aligned}\text{Area of square} &= x \times x \\ &= x^2\end{aligned}$$

- Since the square is divided into four equal quadrants, the area of one quadrant is:

$$\text{Area of colored region} = \frac{x^2}{4}$$

Ex 8: Un boulanger cuit 1000 biscuits. Il veut mettre ces biscuits dans x boîtes, avec le même nombre de biscuits dans chaque boîte. Exprime le nombre de biscuits dans chaque boîte.

$$\text{Biscuits par boîte} = \boxed{\frac{1000}{x}}$$

Answer:

- The total number of cookies is 1000.
- These cookies are distributed into x boxes. The number of cookies per box is:

$$1000 \div x = \frac{1000}{x}$$

C EQUIVALENT FRACTIONS

C.1 SIMPLIFYING ALGEBRAIC FRACTIONS

Ex 9:

$$\frac{6x}{9} = \frac{2x}{\boxed{3}}$$

Answer:

Ex 10:

$$\frac{10x^2}{4} = \frac{5x^2}{\boxed{2}}$$

Answer:

$$\frac{10x^2}{4} = \frac{\cancel{2} \times 5 \times x^2}{\cancel{2} \times 2} = \frac{5x^2}{2}$$

Ex 11:

$$\frac{9x^3}{30x} = \frac{3x^2}{\boxed{10}}$$

Answer:

$$\frac{9x^3}{30x} = \frac{\cancel{3} \times 3 \times x^2 \times \cancel{x}}{\cancel{3} \times 10 \times \cancel{x}} = \frac{3x^2}{10}$$

Ex 12:

$$\frac{3x^2}{2x} = \frac{3x}{\boxed{2}}$$

Answer:

$$\frac{3x^2}{2x} = \frac{\cancel{3} \times x \times \cancel{x}}{\cancel{3} \times 2 \times \cancel{x}} = \frac{x}{2}$$

Ex 13:

$$\frac{2}{4x} = \frac{\boxed{1}}{2x}$$

Answer:

$$\frac{2}{4x} = \frac{\cancel{2}}{\cancel{2} \times 2 \times x} = \frac{1}{2x}$$

D SIMPLIFICATION

D.1 SIMPLIFYING ALGEBRAIC FRACTIONS

Ex 14: Simplify:

$$\frac{4x}{6} = \boxed{\frac{2x}{3}}$$

Answer:

$$\frac{4x}{6} = \frac{\cancel{2} \times 2 \times x}{\cancel{2} \times 3} = \frac{2x}{3}$$

Ex 15: Simplify:

$$\frac{x^2}{3x} = \boxed{\frac{x}{3}}$$

Answer:

$$\frac{x^2}{3x} = \frac{\cancel{x} \times x}{\cancel{x} \times 3} = \frac{x}{3}$$

Ex 16: Simplify:

$$\frac{6x^3}{9x} = \boxed{\frac{2x^2}{3}}$$

Answer:

$$\frac{6x^3}{9x} = \frac{\cancel{3} \times 2 \times x^2 \times \cancel{x}}{\cancel{3} \times 3 \times \cancel{x}} = \frac{2x^2}{3}$$

Ex 17: Simplify:

$$\frac{8x^4}{12x} = \boxed{\frac{2x^3}{3}}$$

Answer:

$$\frac{8x^4}{12x} = \frac{\cancel{4} \times 2 \times x^3 \times \cancel{x}}{\cancel{4} \times 3 \times \cancel{x}} = \frac{2x^3}{3}$$

Ex 18: Simplify:

$$\frac{15x^2}{25x} = \boxed{\frac{3x}{5}}$$

Answer:

$$\frac{15x^2}{25x} = \frac{\cancel{5} \times 3 \times x \times \cancel{x}}{\cancel{5} \times 5 \times \cancel{x}} = \frac{3x}{5}$$

Ex 19: Simplify:

$$\frac{14x^5}{21x^2} = \boxed{\frac{2x^3}{3}}$$

Answer:

$$\frac{14x^5}{21x^2} = \frac{\cancel{7} \times 2 \times x^3 \times \cancel{x^2}}{\cancel{7} \times 3 \times \cancel{x^2}} = \frac{2x^3}{3}$$

Ex 20: Simplify:

$$\frac{4x^3}{16x} = \boxed{\frac{x^2}{4}}$$

Answer:

$$\frac{4x^3}{16x} = \frac{\cancel{4} \times x^2 \times \cancel{x}}{\cancel{4} \times 4 \times \cancel{x}} = \frac{x^2}{4}$$

E CROSS MULTIPLICATION

E.1 SOLVING PROPORTIONS USING CROSS-MULTIPLICATION

Ex 21: Solve x for $\frac{12}{4} = \frac{x}{6}$ (you can use a calculator).

$$x = \boxed{18}$$

Answer:

$$\begin{aligned} 4 \times x &= 12 \times 6 && \text{(cross multiplication)} \\ x &= 12 \times 6 \div 4 && \text{(dividing both sides by 4)} \\ x &= 18 \end{aligned}$$

Ex 22: Solve x for $\frac{11}{10} = \frac{x}{5}$ (you can use a calculator).

$$x = \boxed{5.5}$$

Answer:

$$\begin{aligned} 10 \times x &= 11 \times 5 && \text{(cross multiplication)} \\ x &= 11 \times 5 \div 10 && \text{(dividing both sides by 10)} \\ x &= 5.5 \end{aligned}$$

Ex 23: Solve x for $\frac{12}{10} = \frac{18}{x}$ (you can use a calculator).

$$x = \boxed{15}$$

Answer:

$$\begin{aligned} 12 \times x &= 18 \times 10 && \text{(cross multiplication)} \\ x &= 18 \times 10 \div 12 && \text{(dividing both sides by 12)} \\ x &= 15 \end{aligned}$$

Ex 24: Solve x for $\frac{27}{x} = \frac{30}{10}$ (you can use a calculator).

$$x = \boxed{9}$$

Answer:

$$\begin{aligned} 30 \times x &= 27 \times 10 && \text{(cross multiplication)} \\ x &= 27 \times 10 \div 30 && \text{(dividing both sides by 30)} \\ x &= 9 \end{aligned}$$

E.2 SOLVING PROPORTIONS USING CROSS-MULTIPLICATION

Ex 25: Solve for x in the equation $\frac{x}{3} = \frac{x+1}{2}$.

$$x = \boxed{-3}$$

Answer:

$$\begin{aligned} \frac{x}{3} &= \frac{x+1}{2} \\ 2x &= 3(x+1) && \text{(cross multiplication)} \\ 2x &= 3x + 3 && \text{(distribute)} \\ 2x - 3x &= 3 && \text{(subtract } 3x \text{ from both sides)} \\ -x &= 3 \\ x &= -3 && \text{(multiply both sides by -1)} \end{aligned}$$

Ex 26: Solve for x in the equation $\frac{x}{2} = \frac{x-2}{3}$.

$$x = \boxed{-4}$$

Answer:

$$\begin{aligned} \frac{x}{2} &= \frac{x-2}{3} \\ 3x &= 2(x-2) && \text{(cross multiplication)} \\ 3x &= 2x - 4 && \text{(distribute)} \\ 3x - 2x &= -4 && \text{(subtract } 2x \text{ from both sides)} \\ x &= -4 \end{aligned}$$

Ex 27: Solve for x in the equation $\frac{2}{x+1} = \frac{1}{x}$.

$$x = \boxed{1}$$

Answer:

$$\begin{aligned} \frac{2}{x+1} &= \frac{1}{x} \\ 2x &= 1(x+1) && \text{(cross multiplication)} \\ 2x &= x + 1 && \text{(distribute)} \\ 2x - x &= 1 && \text{(subtract } x \text{ from both sides)} \\ x &= 1 \end{aligned}$$

Ex 28: Solve x for $\frac{2x+1}{4} = \frac{x+2}{3}$.

$$x = \boxed{\frac{5}{2}}$$

Answer:

Answer:

$$\begin{aligned}
\frac{2x+1}{4} &= \frac{x+2}{3} \\
3(2x+1) &= 4(x+2) && \text{(produit en croix)} \\
6x+3 &= 4x+8 && \text{(développer)} \\
6x-4x &= 8-3 && \text{(regrouper les termes)} \\
2x &= 5 && \text{(simplifier)} \\
x &= \frac{5}{2}
\end{aligned}$$

F ADDITION AND SUBTRACTION

F.1 ADDING AND SUBTRACTING ALGEBRAIC FRACTIONS

Ex 29: Calculate and simplify::

$$\frac{x}{6}+\frac{3x}{6} = \boxed{\frac{2x}{3}}$$

Answer:

$$\begin{aligned}
\frac{x}{6}+\frac{3x}{6} &= \frac{x+3x}{6} && \text{(combine fractions with common denominator)} \\
&= \frac{4x}{6} \\
&= \frac{\cancel{2}\times 2\times x}{\cancel{2}\times 3} && \text{(simplify by canceling common factors)} \\
&= \frac{2x}{3}
\end{aligned}$$

Ex 30: Calculate and simplify:

$$\frac{x}{2}+\frac{3x}{4} = \boxed{\frac{5x}{4}}$$

Answer:

$$\begin{aligned}
\frac{x}{2}+\frac{3x}{4} &= \frac{x\times 2}{2\times 2}+\frac{3x}{4} && \text{(find a common denominator, which is 4)} \\
&= \frac{2x}{4}+\frac{3x}{4} \\
&= \frac{2x+3x}{4} && \text{(combine fractions with common denominator)} \\
&= \frac{5x}{4}
\end{aligned}$$

Ex 31: Calculate and simplify:

$$\frac{3x^2}{2}+\frac{5x^2}{3} = \boxed{\frac{19x^2}{6}}$$

Answer:

$$\begin{aligned}
\frac{3x^2}{2}+\frac{5x^2}{3} &= \frac{3x^2\times 3}{2\times 3}+\frac{5x^2\times 2}{3\times 2} && \text{(find a common denominator, which is 6)} \\
&= \frac{9x^2}{6}+\frac{10x^2}{6} \\
&= \frac{9x^2+10x^2}{6} && \text{(combine fractions with common denominator)} \\
&= \frac{19x^2}{6}
\end{aligned}$$

Ex 32: Calculate and simplify:

$$\frac{5x}{3}-\frac{x}{6} = \boxed{\frac{3x}{2}}$$

Answer:

$$\begin{aligned}
\frac{5x}{3}-\frac{x}{6} &= \frac{5x\times 2}{3\times 2}-\frac{x\times 1}{6\times 1} && \text{(find a common denominator, which is 6)} \\
&= \frac{10x}{6}-\frac{x}{6} \\
&= \frac{10x-x}{6} && \text{(combine fractions with common denominator)} \\
&= \frac{9x}{6} \\
&= \frac{\cancel{3}\times 3\times x}{\cancel{3}\times 2} && \text{(simplify by canceling common factors)} \\
&= \frac{3x}{2}
\end{aligned}$$

Ex 33: Calculate and simplify:

$$\frac{2x^2}{5}+\frac{3x^2}{10} = \boxed{\frac{7x^2}{10}}$$

Answer:

$$\begin{aligned}
\frac{2x^2}{5}+\frac{3x^2}{10} &= \frac{2x^2\times 2}{5\times 2}+\frac{3x^2\times 1}{10\times 1} && \text{(find a common denominator, which is 10)} \\
&= \frac{4x^2}{10}+\frac{3x^2}{10} \\
&= \frac{4x^2+3x^2}{10} && \text{(combine fractions with common denominator)} \\
&= \frac{7x^2}{10}
\end{aligned}$$

Ex 34: Calculate and simplify:

$$\frac{7x^3}{4}-\frac{2x^3}{3} = \boxed{\frac{13x^3}{12}}$$

Answer:

$$\begin{aligned}
\frac{7x^3}{4}-\frac{2x^3}{3} &= \frac{7x^3\times 3}{4\times 3}-\frac{2x^3\times 4}{3\times 4} && \text{(find a common denominator, which is 12)} \\
&= \frac{21x^3}{12}-\frac{8x^3}{12} \\
&= \frac{21x^3-8x^3}{12} && \text{(combine fractions with common denominator)} \\
&= \frac{13x^3}{12}
\end{aligned}$$

G MULTIPLICATION OF A FRACTION BY A NUMBER

G.1 MULTIPLYING OF ALGEBRAIC FRACTIONS BY NUMBERS

Ex 35: Calculate and simplify:

$$x\times\frac{x}{2} = \boxed{\frac{x^2}{2}}$$

Answer:

$$x \times \frac{x}{2} = \frac{x \times x}{2} \quad \text{(multiply the number by the numerator)}$$
$$= \frac{x^2}{2}$$

Ex 36: Calculate and simplify:

$$3x \times \frac{2x}{9} = \boxed{\frac{2x^2}{3}}$$

Answer:

$$3x \times \frac{2x}{9} = \frac{3x \times 2x}{9} \quad \text{(multiply the number by the numerator)}$$
$$= \frac{6x^2}{9}$$
$$= \frac{\cancel{3} \times 2 \times x^2}{\cancel{3} \times 3} \quad \text{(simplify by canceling common factors)}$$
$$= \frac{2x^2}{3}$$

Ex 37: Calculate and simplify:

$$2x \times \frac{x^3}{6} = \boxed{\frac{x^4}{3}}$$

Answer:

$$2x \times \frac{x^3}{6} = \frac{2x \times x^3}{6} \quad \text{(multiply the number by the numerator)}$$
$$= \frac{2x^4}{6}$$
$$= \frac{\cancel{2} \times x^4}{\cancel{2} \times 3} \quad \text{(simplify by canceling common factors)}$$
$$= \frac{x^4}{3}$$

Ex 38: Calculate and simplify:

$$4x^2 \times \frac{x}{8} = \boxed{\frac{x^3}{2}}$$

Answer:

$$4x^2 \times \frac{x}{8} = \frac{4x^2 \times x}{8} \quad \text{(multiply the number by the numerator)}$$
$$= \frac{4x^3}{8}$$
$$= \frac{\cancel{4} \times x^3}{\cancel{4} \times 2} \quad \text{(simplify by canceling common factors)}$$
$$= \frac{x^3}{2}$$

Ex 39: Calculate and simplify:

$$\frac{5}{x} \times x^2 = \boxed{5x}$$

Answer:

$$\frac{5}{x} \times x^2 = \frac{5 \times x^2}{x} \quad \text{(multiply the numerator and denominator)}$$
$$= \frac{5x^2}{x}$$
$$= 5x \quad \text{(simplify by canceling x in numerator and denominator)}$$

Ex 40: Calculate and simplify:

$$\frac{x^4}{3} \times 6 = \boxed{2x^4}$$

Answer:

$$\frac{x^4}{3} \times 6 = \frac{x^4 \times 6}{3} \quad \text{(multiply the numerator and denominator)}$$
$$= \frac{6x^4}{3}$$
$$= \frac{\cancel{3} \times 2 \times x^4}{\cancel{3}} \quad \text{(simplify by canceling common factors)}$$
$$= 2x^4$$

H MULTIPLICATION OF FRACTIONS

H.1 MULTIPLYING OF ALGEBRAIC FRACTIONS

Ex 41: Calculate and simplify:

$$\frac{2}{3} \times \frac{x}{2} = \boxed{\frac{x}{3}}$$

Answer:

$$\frac{2}{3} \times \frac{x}{2} = \frac{2 \times x}{3 \times 2} \quad \text{(multiply the numerators and denominators)}$$
$$= \frac{2x}{6}$$
$$= \frac{\cancel{2} \times x}{\cancel{2} \times 3} \quad \text{(simplify by canceling common factors)}$$
$$= \frac{x}{3}$$

Ex 42: Calculate and simplify:

$$\frac{5}{2} \times \frac{x}{5} = \boxed{\frac{x}{2}}$$

Answer:

$$\frac{5}{2} \times \frac{x}{5} = \frac{5 \times x}{2 \times 5} \quad \text{(multiply the numerators and denominators)}$$
$$= \frac{5x}{10}$$
$$= \frac{\cancel{5} \times x}{\cancel{5} \times 2} \quad \text{(simplify by canceling common factors)}$$
$$= \frac{x}{2}$$

Ex 43: Calculate and simplify:

$$\frac{x}{3} \times \frac{2}{x} = \boxed{\frac{2}{3}}$$

Answer:

$$\frac{x}{3} \times \frac{2}{x} = \frac{x \times 2}{3 \times x} \quad \text{(multiply the numerators and denominators)}$$
$$= \frac{2x}{3x}$$
$$= \frac{\cancel{x} \times 2}{\cancel{x} \times 3} \quad \text{(simplify by canceling common factors)}$$
$$= \frac{2}{3}$$

Ex 44: Calculate and simplify:

$$\frac{4}{x} \times \frac{1}{2} = \boxed{\frac{2}{x}}$$

Answer:

$$\begin{aligned}
\frac{4}{x} \times \frac{1}{2} &= \frac{4 \times 1}{x \times 2} && \text{(multiply the numerators and denominators)} \\
&= \frac{4}{2x} \\
&= \frac{\cancel{2} \times 2}{\cancel{2} \times x} && \text{(simplify by canceling common factors)} \\
&= \frac{2}{x}
\end{aligned}$$

H.2 MULTIPLYING OF ALGEBRAIC FRACTIONS

Ex 45: Calculate and simplify:

$$\frac{x}{2} \times \frac{2x}{3} = \boxed{\frac{x^2}{3}}$$

Answer:

$$\begin{aligned}
\frac{x}{2} \times \frac{2x}{3} &= \frac{x \times 2x}{2 \times 3} && \text{(multiply the numerators and denominators)} \\
&= \frac{2x^2}{6} \\
&= \frac{\cancel{2} \times x^2}{\cancel{2} \times 3} && \text{(simplify by canceling common factors)} \\
&= \frac{x^2}{3}
\end{aligned}$$

Ex 46: Calculate and simplify:

$$\frac{x}{2} \times \frac{x^2}{3} = \boxed{\frac{x^3}{6}}$$

Answer:

$$\begin{aligned}
\frac{x}{2} \times \frac{x^2}{3} &= \frac{x \times x^2}{2 \times 3} && \text{(multiply the numerators and denominators)} \\
&= \frac{x^3}{6}
\end{aligned}$$

Ex 47: Calculate and simplify:

$$\frac{2}{x} \times \frac{x^2}{3} = \boxed{\frac{2x}{3}}$$

Answer:

$$\begin{aligned}
\frac{2}{x} \times \frac{x^2}{3} &= \frac{2 \times x^2}{x \times 3} && \text{(multiply the numerators and denominators)} \\
&= \frac{2x^2}{3x} \\
&= \frac{\cancel{x} \times 2 \times \cancel{x}}{\cancel{x} \times 3} && \text{(simplify by canceling common factors)} \\
&= \frac{2x}{3}
\end{aligned}$$

Ex 48: Calculate and simplify:

$$\left(\frac{x}{2}\right)^2 = \boxed{\frac{x^2}{4}}$$

Answer:

$$\begin{aligned}
\left(\frac{x}{2}\right)^2 &= \frac{x \times x}{2 \times 2} && \text{(square the numerator and the denominator)} \\
&= \frac{x^2}{4}
\end{aligned}$$

I DIVISION OF FRACTIONS

I.1 DIVIDING ALGEBRAIC FRACTIONS

Ex 49: Calculate and simplify:

$$\frac{2}{3} \div \frac{2}{x} = \boxed{\frac{x}{3}}$$

Answer:

$$\begin{aligned}
\frac{2}{3} \div \frac{2}{x} &= \frac{2}{3} \times \frac{x}{2} && \text{(dividing by a fraction is multiplying by its reciprocal)} \\
&= \frac{2 \times x}{3 \times 2} && \text{(multiply the numerators and denominators)} \\
&= \frac{2x}{6} \\
&= \frac{\cancel{2} \times x}{\cancel{2} \times 3} && \text{(simplify by canceling common factors)} \\
&= \frac{x}{3}
\end{aligned}$$

Ex 50: Calculate and simplify:

$$\frac{2x}{3} \div 2 = \boxed{\frac{x}{3}}$$

Answer:

$$\begin{aligned}
\frac{2x}{3} \div 2 &= \frac{2x}{3} \div \frac{2}{1} && \text{(dividing by a number is multiplying by its reciprocal)} \\
&= \frac{2x}{3} \times \frac{1}{2} && \text{(multiply the numerators and denominators)} \\
&= \frac{2x \times 1}{3 \times 2} \\
&= \frac{2x}{6} \\
&= \frac{\cancel{2} \times x}{\cancel{2} \times 3} && \text{(simplify by canceling common factors)} \\
&= \frac{x}{3}
\end{aligned}$$

Ex 51: Calculate and simplify:

$$\frac{3}{x} \div \frac{6}{x} = \boxed{\frac{1}{2}}$$

Answer:

$$\begin{aligned}
\frac{3}{x} \div \frac{6}{x} &= \frac{3}{x} \times \frac{x}{6} && \text{(dividing by a fraction is multiplying by its reciprocal)} \\
&= \frac{3 \times x}{x \times 6} && \text{(multiply the numerators and denominators)} \\
&= \frac{3x}{6x} \\
&= \frac{\cancel{x} \times 3}{\cancel{x} \times 6} && \text{(simplify by canceling common factors)} \\
&= \frac{3}{6} \\
&= \frac{\cancel{3}}{\cancel{3} \times 2} && \text{(simplify by canceling common factors)} \\
&= \frac{1}{2}
\end{aligned}$$

Ex 52: Calculate and simplify:

$$\frac{4x}{5} \div x = \boxed{\frac{4}{5}}$$

Answer:

$$\begin{aligned}
\frac{4x}{5} \div x &= \frac{4x}{5} \div \frac{x}{1} && \text{(dividing by a number is multiplying by its reciprocal)} \\
&= \frac{4x}{5} \times \frac{1}{x} && \text{(multiply the numerators and denominators)} \\
&= \frac{4x \times 1}{5 \times x} \\
&= \frac{4x}{5x} \\
&= \frac{\cancel{x} \times 4}{\cancel{x} \times 5} && \text{(simplify by canceling common factors)} \\
&= \frac{4}{5}
\end{aligned}$$

Ex 53: Calculate and simplify:

$$\frac{x^2}{2} \div \frac{x}{4} = \boxed{2x}$$

Answer:

$$\begin{aligned}
\frac{x^2}{2} \div \frac{x}{4} &= \frac{x^2}{2} \times \frac{4}{x} && \text{(dividing by a fraction is multiplying by its reciprocal)} \\
&= \frac{x^2 \times 4}{2 \times x} && \text{(multiply the numerators and denominators)} \\
&= \frac{4x^2}{2x} \\
&= \frac{\cancel{2} \times 2 \times x \times \cancel{x}}{\cancel{2} \times x} && \text{(simplify by canceling common factors)} \\
&= \frac{2x}{x} \\
&= 2
\end{aligned}$$

I.2 DIVIDING ALGEBRAIC FRACTIONS

Ex 54: Calculate and simplify:

$$\frac{\frac{2}{3}}{\frac{2}{x}} = \boxed{\frac{x}{3}}$$

Answer:

$$\begin{aligned}
\frac{\frac{2}{3}}{\frac{2}{x}} &= \frac{2}{3} \times \frac{x}{2} && \text{(dividing by a fraction is multiplying by its reciprocal)} \\
&= \frac{2 \times x}{3 \times 2} && \text{(multiply the numerators and denominators)} \\
&= \frac{2x}{6} \\
&= \frac{\cancel{2} \times x}{\cancel{2} \times 3} && \text{(simplify by canceling common factors)} \\
&= \frac{x}{3}
\end{aligned}$$

Ex 55: Calculate and simplify:

$$\frac{\frac{2x}{3}}{2} = \boxed{\frac{x}{3}}$$

Answer:

$$\begin{aligned}
\frac{\frac{2x}{3}}{2} &= \frac{2x}{3} \div \frac{2}{1} && \text{(dividing by a number is multiplying by its reciprocal)} \\
&= \frac{2x}{3} \times \frac{1}{2} && \text{(multiply the numerators and denominators)} \\
&= \frac{2x \times 1}{3 \times 2} \\
&= \frac{2x}{6} \\
&= \frac{\cancel{2} \times x}{\cancel{2} \times 3} && \text{(simplify by canceling common factors)} \\
&= \frac{x}{3}
\end{aligned}$$

Ex 56: Calculate and simplify:

$$\frac{\frac{3}{x}}{\frac{6}{x}} = \boxed{\frac{1}{2}}$$

Answer:

$$\begin{aligned}
\frac{\frac{3}{x}}{\frac{6}{x}} &= \frac{3}{x} \times \frac{x}{6} && \text{(dividing by a fraction is multiplying by its reciprocal)} \\
&= \frac{3 \times x}{x \times 6} && \text{(multiply the numerators and denominators)} \\
&= \frac{3x}{6x} \\
&= \frac{\cancel{x} \times 3}{\cancel{x} \times 6} && \text{(simplify by canceling common factors)} \\
&= \frac{3}{6} \\
&= \frac{\cancel{3}}{\cancel{3} \times 2} && \text{(simplify by canceling common factors)} \\
&= \frac{1}{2}
\end{aligned}$$

Ex 57: Calculate and simplify:

$$\frac{\frac{4x^2}{5}}{2} = \boxed{\frac{2x^2}{5}}$$

Answer:

$$\begin{aligned}
\frac{\frac{4x^2}{5}}{2} &= \frac{4x^2}{5} \div \frac{2}{1} && \text{(dividing by a number is multiplying by its reciprocal)} \\
&= \frac{4x^2}{5} \times \frac{1}{2} && \text{(multiply the numerators and denominators)} \\
&= \frac{4x^2 \times 1}{5 \times 2} \\
&= \frac{4x^2}{10} \\
&= \frac{\cancel{2} \times 2 \times x^2}{\cancel{2} \times 5} && \text{(simplify by canceling common factors)} \\
&= \frac{2x^2}{5}
\end{aligned}$$

Ex 58: Calculate and simplify:

$$\frac{\frac{4x}{5}}{x} = \boxed{\frac{4}{5}}$$

Answer:

$$\begin{aligned}\frac{\frac{4x}{5}}{x} &= \frac{4x}{5} \div \frac{x}{1} && \text{(dividing by a number is multiplying by its reciprocal)}\\ &= \frac{4x}{5} \times \frac{1}{x} && \text{(multiply the numerators and denominators)}\\ &= \frac{4x \times 1}{5 \times x}\\ &= \frac{4x}{5x}\\ &= \frac{\cancel{x} \times 4}{\cancel{x} \times 5} && \text{(simplify by canceling common factors)}\\ &= \frac{4}{5}\end{aligned}$$

Ex 59: Calculate and simplify:

$$\frac{\frac{x^2}{2}}{\frac{x}{4}} = \boxed{2x}$$

Answer:

$$\begin{aligned}\frac{\frac{x^2}{2}}{\frac{x}{4}} &= \frac{x^2}{2} \times \frac{4}{x} && \text{(dividing by a fraction is multiplying by its reciprocal)}\\ &= \frac{x^2 \times 4}{2 \times x} && \text{(multiply the numerators and denominators)}\\ &= \frac{4x^2}{2x}\\ &= \frac{\cancel{2} \times 2 \times x \times \cancel{x}}{\cancel{2} \times x} && \text{(simplify by canceling common factors)}\\ &= \frac{2x}{x}\\ &= 2\end{aligned}$$

J SIGN RULES

J.1 SIMPLIFYING ALGEBRAIC FRACTIONS WITH RELATIVE NUMBERS

Ex 60: Simplify:

$$\frac{-15x}{-30} = \boxed{\frac{x}{2}}$$

Answer:

$$\begin{aligned}\frac{-15x}{-30} &= \frac{\cancel{-15} \times x}{\cancel{-15} \times 2} && \text{(simplify by canceling common factors)}\\ &= \frac{x}{2}\end{aligned}$$

Ex 61: Simplify:

$$\frac{-6x}{12} = \boxed{-\frac{x}{2}}$$

Answer:

$$\begin{aligned}\frac{-6x}{12} &= \frac{-\cancel{6} \times x}{\cancel{6} \times 2} && \text{(simplify by canceling common factors)}\\ &= \frac{-x}{2}\\ &= -\frac{x}{2}\end{aligned}$$

Ex 62: Simplify:

$$\frac{-12x^4}{-2x^2} = \boxed{6x^2}$$

Answer:

$$\begin{aligned}\frac{-12x^4}{-2x^2} &= \frac{\cancel{-2} \times 6 \times x^2 \times \cancel{x^2}}{\cancel{-2} \times 1 \times \cancel{x^2}} && \text{(simplify by canceling common factors)}\\ &= \frac{6x^2}{1}\\ &= 6x^2\end{aligned}$$

Ex 63: Simplify:

$$\frac{3x^3}{-9x} = \boxed{-\frac{x^2}{3}}$$

Answer:

$$\begin{aligned}\frac{3x^3}{-9x} &= \frac{\cancel{3} \times x^2 \times \cancel{x}}{\cancel{3} \times -3 \times \cancel{x}} && \text{(simplify by canceling common factors)}\\ &= \frac{x^2}{-3}\\ &= -\frac{x^2}{3}\end{aligned}$$

Ex 64: Simplify:

$$\frac{-21x^3}{-7x} = \boxed{3x^2}$$

Answer:

$$\begin{aligned}\frac{-21x^3}{-7x} &= \frac{\cancel{-7} \times 3 \times x^2 \times \cancel{x}}{\cancel{-7} \times 1 \times \cancel{x}} && \text{(simplify by canceling common factors)}\\ &= \frac{3x^2}{1}\\ &= 3x^2\end{aligned}$$

Ex 65: Simplify:

$$\frac{-4x^4}{8x^2} = \boxed{-\frac{x^2}{2}}$$

Answer:

$$\begin{aligned}\frac{-4x^4}{8x^2} &= \frac{\cancel{-4} \times x^2 \times \cancel{x^2}}{\cancel{4} \times 2 \times \cancel{x^2}} && \text{(simplify by canceling common factors)}\\ &= \frac{x^2}{2}\\ &= -\frac{x^2}{2}\end{aligned}$$

K ORDER OF OPERATIONS

K.1 CALCULATING ALGEBRAIC EXPRESSIONS

Ex 66: Calculate and simplify:

$$\frac{x + 7x}{3 \times 4} = \boxed{\frac{2x}{3}}$$

Answer:

$$\begin{aligned}\frac{x+7x}{3\times 4} &= \frac{8x}{12} && \text{(simplify numerator and denominator)}\\ &= \frac{\cancel{4}\times 2\times x}{\cancel{4}\times 3} && \text{(simplify by canceling common factors)}\\ &= \frac{2x}{3}\end{aligned}$$

Ex 67: Calculate and simplify:

$$x\times\frac{3x+x}{4+2}=\boxed{\frac{2x^2}{3}}$$

Answer:

$$\begin{aligned}x\times\frac{3x+x}{4+2} &= x\times\frac{4x}{6} && \text{(simplify the numerator and the denominator)}\\ &= \frac{x\times 4x}{6} && \text{(multiply the numerators and denominators)}\\ &= \frac{4x^2}{6}\\ &= \frac{\cancel{2}\times 2\times x^2}{\cancel{2}\times 3} && \text{(simplify by canceling common factors)}\\ &= \frac{2x^2}{3}\end{aligned}$$

Ex 68: Calculate and simplify:

$$\frac{2x^3}{2x-x}=\boxed{2x^2}$$

Answer:

$$\begin{aligned}\frac{2x^3}{2x-x} &= \frac{2x^3}{x} && \text{(combine like terms in the denominator)}\\ &= \frac{2\times x^2\times \cancel{x}}{\cancel{x}} && \text{(simplify by canceling common factors)}\\ &= 2x^2\end{aligned}$$

Ex 69: Calculate and simplify:

$$4x\times\frac{6x-2x}{2\times 8}=\boxed{x^2}$$

Answer:

$$\begin{aligned}4x\times\frac{6x-2x}{2\times 8} &= 4x\times\frac{4x}{16} && \text{(simplify the numerator and the denominator)}\\ &= \frac{4x\times 4x}{16} && \text{(multiply the numerators and denominators)}\\ &= \frac{16x^2}{16}\\ &= \frac{\cancel{16}\times x^2}{\cancel{16}\times 1} && \text{(simplify by canceling common factors)}\\ &= x^2\end{aligned}$$

K.2 CALCULATING ALGEBRAIC EXPRESSIONS

Ex 70: Write as a single fraction:

$$2-\frac{x+1}{3}=\boxed{\frac{5-x}{3}}$$

Answer:

$$\begin{aligned}2-\frac{x+1}{3} &= \frac{2\times 3}{3}-\frac{x+1}{3} && \text{(common denominator=3)}\\ &= \frac{6}{3}-\frac{x+1}{3}\\ &= \frac{6-(x+1)}{3} && \text{(combine fractions with common denominator)}\\ &= \frac{6-x-1}{3} && \text{(distribute the negative sign)}\\ &= \frac{5-x}{3}\end{aligned}$$

Ex 71: Write as a single fraction:

$$3x-\frac{2-x}{4}=\boxed{\frac{13x-2}{4}}$$

Answer:

$$\begin{aligned}3x-\frac{2-x}{4} &= \frac{3x\times 4}{4}-\frac{2-x}{4} && \text{(common denominator =4)}\\ &= \frac{12x}{4}-\frac{2-x}{4}\\ &= \frac{12x-(2-x)}{4} && \text{(combine fractions with common denominator)}\\ &= \frac{12x-2+x}{4} && \text{(distribute the negative sign)}\\ &= \frac{13x-2}{4}\end{aligned}$$

Ex 72: Write as a single fraction:

$$\frac{x}{2}-\frac{x+1}{3}=\boxed{\frac{x-2}{6}}$$

Answer:

$$\begin{aligned}\frac{x}{2}-\frac{x+1}{3} &= \frac{x\times 3}{2\times 3}-\frac{(x+1)\times 2}{3\times 2} && \text{(common denominator=6)}\\ &= \frac{3x}{6}-\frac{2(x+1)}{6}\\ &= \frac{3x-2(x+1)}{6} && \text{(combine fractions with common denominator}\\ &= \frac{3x-2x-2}{6} && \text{(distribute the negative sign)}\\ &= \frac{x-2}{6} && \text{(combine like terms)}\end{aligned}$$

Ex 73: Write as a single fraction:

$$\frac{x+1}{3}-\frac{x+4}{2}=\boxed{\frac{-x-10}{6}}$$

Answer:

$$\begin{aligned}\frac{x+1}{3}-\frac{x+4}{2} &= \frac{(x+1)\times 2}{3\times 2}-\frac{(x+4)\times 3}{2\times 3} && \text{(common denominator=6)}\\ &= \frac{2(x+1)}{6}-\frac{3(x+4)}{6}\\ &= \frac{2(x+1)-3(x+4)}{6}\\ &= \frac{2x+2-3x-12}{6} && \text{(distribute the negative sign)}\\ &= \frac{-x-10}{6} && \text{(combine like terms)}\end{aligned}$$