

# ALGEBRA

## A DEFINITIONS

### Definition Constant

A **constant** is a number.

Ex: 0, 3,  $\pi$

### Definition Variable

A **variable** is a quantity which we represent by a letter.

Ex:

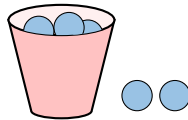


The variable  $x$  is the number of marbles inside the cup.

### Definition Expression

An **expression** is an algebraic form consisting of constants, variables, and operation signs such as  $+$ ,  $-$ ,  $\times$ ,  $\div$  and  $\sqrt{\phantom{x}}$ .

Ex:



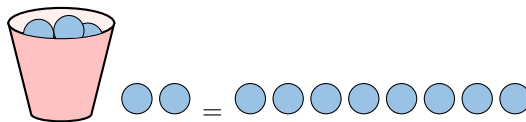
A cup contains  $x$  marbles. Next to the cup, there are 2 marbles outside. The expression for the number of marbles is

$$x + 2$$

### Definition Equation

An **equation** is a mathematical statement consisting of two expressions, the **left-hand side** and the **right-hand side**, separated by an equal sign  $=$ .

Ex:

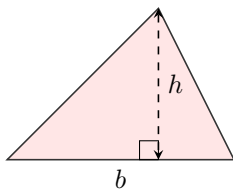


A cup contains  $x$  marbles. The equation for the number of marbles is

$$x + 2 = 8$$

### Definition Formula

A **formula** is an equation, often related to the real world, to physics or to geometry.



Ex: For a triangle:  $A = \frac{b \times h}{2}$  is the formula for the area.

## B NOTATIONS

### Definition Product notation

We can omit the  $\times$  sign when it is followed by a variable or a parenthesis.

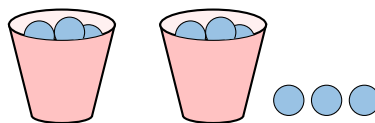
Ex:

- $2 \times x = 2x$
- $2 \times (L + l) = 2(L + l)$

### Definition Repeated addition

$$\overbrace{x + x + \dots + x}^{n \text{ terms}} = n \times x$$

Ex:



Each cup contains  $x$  marbles. Simplify the expression for the number of marbles:

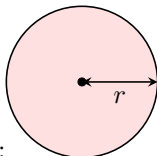
$$x + x + 1 + 1 + 1$$

Answer:

$$x + x + 1 + 1 + 1 = 2x + 3$$

### Definition Repeated multiplication

$$\overbrace{x \times x \times \dots \times x}^{n \text{ factors}} = x^n$$



Ex: For a circle: , simplify the formula for the area  $A = \pi \times r \times r$ .

Answer:

$$\begin{aligned} A &= \pi \times r \times r \\ &= \pi r^2 \end{aligned}$$

## C IDENTITY

### Definition Identity

An **identity** is an equality between two expressions such that their evaluations produce the same value for **all** values of the variables.

Identities are fundamental in algebra: they allow us to transform and simplify expressions and are the foundation for solving equations and manipulating formulas.

### Proposition Properties of Multiplying by 1 and 0

$$1 \times x = x \quad \text{and} \quad 0 \times x = 0$$

### Proposition Commutativity Identities

$$a + b = b + a \quad \text{and} \quad a \times b = b \times a$$

### Proposition Associativity Identities

$$(a + b) + c = a + (b + c) \quad \text{and} \quad (a \times b) \times c = a \times (b \times c)$$

**Ex:** Show that  $l + L + l + L = 2l + 2L$ .

*Answer:*

$$\begin{aligned} l + L + l + L &= l + l + L + L && \text{(collecting terms)} \\ &= 2l + 2L && \text{(repeated addition)} \end{aligned}$$

### Method Simplifying by Collecting Like Terms

**Simplifying an expression by collecting like terms** involves combining terms that have the same variables raised to the same powers.

1. **Identify like terms:** Like terms are terms that have the same variable(s) raised to the same power. For example,  $3x$  and  $5x$  are like terms, but  $3x$  and  $3x^2$  are not.
2. **Combine like terms:** Add or subtract the coefficients (numerical parts) of the like terms. The variable part remains the same.

**Ex:** Simplify the expression:  $2x + 4 + x - 2$

*Answer:*

$$\begin{aligned} 2x + 4 + x - 2 &= 2x + 4 + x - 2 && \text{(identifying like terms)} \\ &= (2 + 1)x + 4 - 2 && \text{(combining like terms)} \\ &= 3x + 2 && \text{(simplifying)} \end{aligned}$$

## D SUBSTITUTING

### Definition Substituting

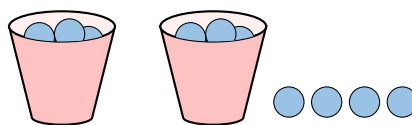
**Substituting** is replacing a variable in an expression or equation with a specific value.

To avoid confusion with signs, especially when substituting negative values, we usually write substitutions in parentheses.

### Method Evaluating

To **evaluate** an expression, substitute a number for each variable and perform the arithmetic operations.

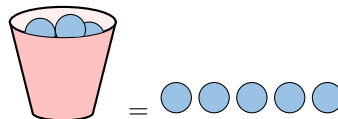
**Ex:**



Each cup contains  $x$  marbles. The expression for the total number of marbles is

$$2x + 4$$

Evaluate this expression when  $x = 5$  (meaning there are 5 marbles in each cup):



*Answer:*

$$\begin{aligned} 2x + 4 &= 2 \times (5) + 4 && \text{(substituting } x \text{ by } 5) \\ &= 10 + 4 \\ &= 14 \end{aligned}$$

There are 14 marbles.

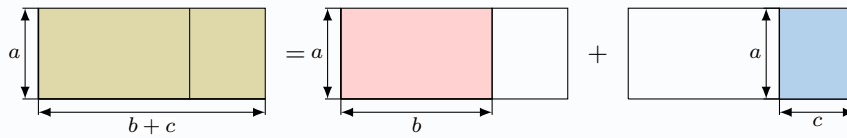
## E DISTRIBUTIVE IDENTITIES

### Proposition Distributive Law

Multiplication is distributive over addition and subtraction:

- **Addition:**

$$a (b + c) = ab + ac$$



- **Subtraction:**

$$a (b - c) = ab - ac$$

**Ex:** Show that  $2(\ell + L) = 2\ell + 2L$ .

*Answer:*

$$2(\ell + L) = 2 \times \ell + 2 \times L = 2\ell + 2L$$

So  $2(\ell + L) = 2\ell + 2L$ .

### Definition Expanding

**Expanding** is the process of using the distributive law to write a product with parentheses as a sum (or difference) of terms.

**Ex:** Expand  $2(2x + 3)$ .

*Answer:*

$$2(2x + 3) = 2 \times 2x + 2 \times 3 = 4x + 6$$

So  $2(2x + 3) = 4x + 6$ .