ALGEBRA

A DEFINITIONS

Definition Constant

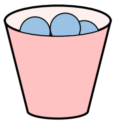
A **constant** is a number.

Ex: $0,\ 3,\ \pi$

Definition Variable

A **variable** is a quantity which we represent by a letter.

Ex:

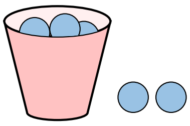


The variable x is the number of marbles inside the cup.

Definition Expression

An **expression** is an algebraic form consisting of constants, variables, and operation signs such as $+, -, \times, \div$ and $\sqrt{\ }$.

Ex:



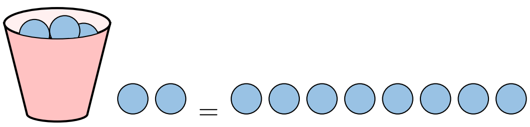
A cup contains x marbles. Next to the cup, there are 2 marbles outside. The expression for the number of marbles is

$$x+2$$

Definition Equation

An **equation** is a mathematical statement consisting of two expressions, the **left-hand side** and the **right-hand side**, separated by an equal sign $=$.

Ex:

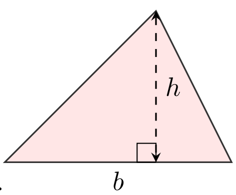


A cup contains x marbles. The equation for the number of marbles is

$$x+2=8$$

Definition Formula

A **formula** is an equation, often related to the real world, to physics or to geometry.

Ex: For a triangle: 

, $A=\dfrac{b\times h}{2}$ is the formula for the area.

B NOTATIONS

Definition Product notation

We can omit the $\times$ sign when it is followed by a variable or a parenthesis.

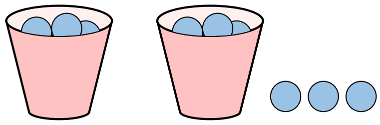
Ex:

- $2 \times x = 2x$
- $2 \times (L + l) = 2(L + l)$

Definition Repeated addition

$$\overbrace{x + x + \ldots + x}^{n \text{ terms}} = n \times x$$

Ex:



Each cup contains x marbles. Simplify the expression for the number of marbles:

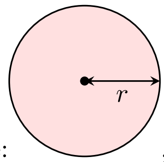
$$x + x + 1 + 1 + 1$$

Answer:

$$x + x + 1 + 1 + 1 = 2x + 3$$

Definition Repeated multiplication

$$\overbrace{x \times x \times \ldots \times x}^{n \text{ factors}} = x^n$$

Ex: For a circle: 

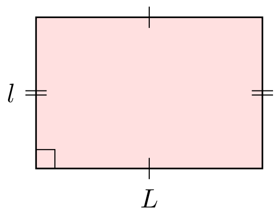
, simplify the formula for the area $A = \pi \times r \times r$.

Answer:

$$\begin{aligned} A &= \pi \times r \times r \\ &= \pi r^2 \end{aligned}$$

C IDENTITY

Discover: Three students were asked to find the formula for the perimeter of the rectangle:



They wrote:

- Su: $P = 2(l + L)$
- Louis: $P = l + L + l + L$
- Hugo: $P = 2l + 2L$

Which students are correct?

Answer: They are all correct. These three expressions $2(l+L)$, $l+L+l+L$ and $2l+2L$ produce the same result for the perimeter of the rectangle for all values of l and L. They are called identities.

Definition Identity

An **identity** is an equality between two expressions such that their evaluations produce the same value for all values of the variables.

Identities are fundamental in algebra: they allow us to transform and simplify expressions and are the foundation for solving equations and manipulating formulas.

Proposition 0 and 1 Identities

$$1 \times x = x \qquad \text{and} \qquad 0 \times x = 0$$

Proposition Commutativity Identities

$$a + b = b + a \qquad \text{and} \qquad a \times b = b \times a$$

Proposition Associativity Identities

$$(a + b) + c = a + (b + c) \qquad \text{and} \qquad (a \times b) \times c = a \times (b \times c)$$

Ex: Show that $l + L + l + L = 2l + 2L$.

Answer:

$$\begin{aligned} l + L + l + L &= l + l + L + L && \text{(collecting terms)} \\ &= 2l + 2L && \text{(repeated addition)} \end{aligned}$$

Method Simplifying by Collecting Like Terms

Simplifying an expression by collecting like terms involves combining terms that have the same variables raised to the same powers.

1. **Identify like terms**: Like terms are terms that have the same variable(s) raised to the same power. For example, $3x$ and $5x$ are like terms, but $3x$ and $3x^2$ are not.
2. **Regroupe like terms**: Add or subtract the coefficients (numerical parts) of the like terms. The variable part remains the same.

Ex: Simplify the expression: $2x + 4 + x - 2$

Answer:

$$\begin{aligned} 2x + 4 + x - 2 &= 2x + 4 + x - 2 && \text{(identifying like terms)} \\ &= (2+1)x + 4 - 2 && \text{(combining like terms)} \\ &= 3x + 2 && \text{(simplifying)} \end{aligned}$$

D SUBSTITUTING

Definition Substituting

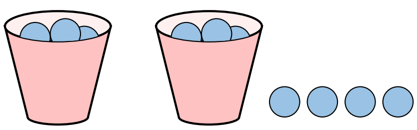
Substituting is replacing a variable in an expression or equation with a specific value.

To avoid confusion with signs, especially when substituting negative values, we usually write substitutions in parentheses.

Method Evaluating

To **evaluate** an expression, substitute a number for each variable and perform the arithmetic operations.

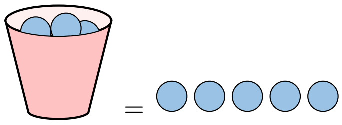
Ex:



Each cup contains x marbles. The expression for the number of marbles is

$$2x+4$$

Evaluate this expression when $x=5$ (meaning there are 5 marbles in each cup):



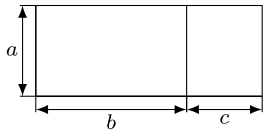
Answer:

$$\begin{aligned} 2x+4 &= 2\times(5)+4 \quad \text{(substituting } x=5\text{)} \\ &= 10+4 \\ &= 14 \end{aligned}$$

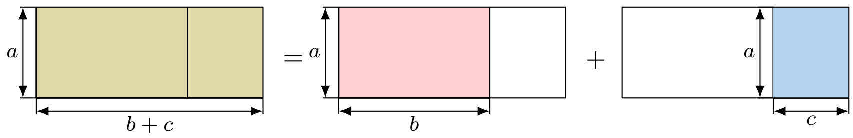
There are 14 marbles.

E DISTRIBUTIVE IDENTITIES

Discover: Find two formulae for the area of the whole rectangle and write the identity.



Answer:



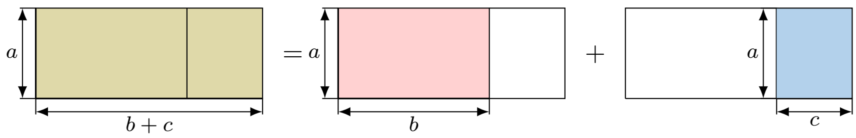
$$a(b+c) \quad = \quad ab \quad + \quad ac$$

Proposition Distributive Identity

Multiplication is distributive over addition and subtraction:

- **Addition:**

$$a\,(\,b+c\,)=ab+ac$$



- **Subtraction:**

$$a\,(\,b-c\,)=ab-ac$$

Ex: Show that $2(l+L)=2l+2L$.

Answer:

$$\begin{aligned} 2\,(\,l+L\,) &= 2\times l + 2\times L \\ &= \;\;2l \;\;+\;\; 2L \end{aligned}$$

Definition Expanding

Expanding is the process of writing an expression as a sum of terms.

Ex: Expand $2(2x+3)$

Answer:

$$\begin{aligned} 2\,(\,2x+3\,) &= 2\times 2x + 2\times 3 \\ &= 4x + 6 \end{aligned}$$